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THE
MATHEMATICAL
REPOSITORY.

CONTAINING
Analytical Solutions
OF
Five Hundred Questions,
MOSTLY SELECTED FROM
SCARCE and VALUABLE AUTHORS.

DESIGNED 12D
To conduct BEGINNERS to the more difficult
Properties of Numbers.

By JAMES DODSON, *K*
Accomptant, and Teacher of the MATHEMATICS.

L O N D O N :

Printed for J. Nourse, at the *Lamb*, opposite
Katherine-Street, in the *Strand*.

MDCCXLVIII.

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Lincoln's Inn, in the Strand.

MDCCLXXIII.

TO
Mr. Abraham De Moivre,

FELLOW of the ROYAL SOCIETIES of

LONDON and BERLIN.

SIR,



It is not a new custom for authors to dedicate their mathematical works, to gentlemen who are the most illustrious ornaments of mathematical sciences; and as the learned world have long since thought it justice to rank You among that number, it will, I hope, sufficiently apologize for this address, which I flatter myself you will favourably receive, on account of the likelihood there is, that these sheets will prove beneficial to students.

BESIDES the help that your celebrated works in general have afforded me, the appendix to your *Miscellanea Analytica* has furnished me with the method of approximating the sums of such series as are the reciprocals of the powers of arithmetical progressions.

IV DEDICATION.

M. *James Bernoulli* could not sum the series of the reciprocals of square numbers *; this was discovered by M. *John Bernoulli*, who, on that occasion, expresses a singular satisfaction, but confesses his inability to sum the series of the reciprocals of cube numbers †: The sum of both these series, and many others, naturally flow from an approximation you have given to sum the series $\frac{1}{1^m} + \frac{1}{2^m} + \frac{1}{3^m}$, &c. *ad infinitum*; and although you well knew that the *desiderata* of Messrs *Bernoulli* were included therein, yet you waved, not only the application, but the assumption of the honour thereof.

THIS, Sir, as it sufficiently shews the modest opinion you have of your own excellent performances, so it gives me this opportunity of declaring to the public, that I think myself singularly happy in being permitted to subscribe myself,

SIR,

Your most obliged,

And most humble Servant,

James Dodson.

* See his WORKS, Vol. I. Page 398.

† See his WORKS, Vol. IV. Page 20, &c.

THE PREFACE.



THE following sheets contain a large collection of questions in common Algebra; which (some few excepted) are disposed in the following order.

- 1. Such questions, in the solutions of which, only Addition and Subtraction of quantities are used.*
- 2. Questions, which beside the former operations, require the use of Multiplication and Division.*
- 3. Questions, wherein the doctrine of Proportion is requisite.*
- 4. Such questions, producing simple Equations, wherein a more complex process is necessary: Where any of these seemed likely to occur in practice frequently, a general Solution is given.*

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5. *Questions, producing Equations of simple Powers.*

6 *Questions, that produce adfected quadratic Equations, solved by compleating the Square : With a few Examples of Dr. Halley's Method of finding their Roots by a logarithmical Process.*

7. *Questions, that produce adfected Equations of higher Powers.*

THE method of finding divisors, delivered in Sir Isaac Newton's UNIVERSAL ARITHMETIC is here principally used ; because it seems best adapted to find the roots of such equations, when those roots are whole numbers, or rational fractions : There are, however, some examples of finding the roots of such cubic equations as want the second term by a method similar to *Cardan's*, or by Dr. *Halley's* logarithmical process depending thereon ; but where neither of these methods appeared practicable, without a previous reduction, one of Dr. *Halley's* approximations to the root (called his *rational* and *irrational* theorems) is used, according as *this*, or *that*, seemed most applicable to the given numbers.

8. In-

8. Indetermined questions : as well those that are capable of innumerable answers, as those where the number of answers in integers is limited.

THE solutions of these questions are attempted, for the most part, in a manner different from what has been commonly used; and some solutions are given at large, which other writers have thought too oporose to be inserted : A few of the questions usually called *Diaphantine* are introduced toward the end of these, and are solved by the same principles.

9. Questions, relating to Arithmetical Progressions, and other series derived from them : such as, their squares, cubes, &c. the different Series of Figurate numbers ; and of those Numbers whose second, third, fourth, &c. Differences are equal ; the Combinations, Elections, Permutations of Quantities, &c.

10. Questions, relating to Simple Interest, Discompt, &c.

11. Questions, relating to Geometrical Progressions.

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12. *Questions, relating to Compound Interest; and to the values of Annuities for Time certain; both in Possession and Reversion.*

13. *Questions, relating to Geometrical Progressions infinitely decreasing; and to series of Fractions, the Numerators of which are Numbers, whose 1st, 2d, 3d, &c. Differences are equal, and their Denominators a Geometrical Progression.*

14. *The Summation of the several series of the Reciprocals of Figurate Numbers; and of other Series which can be obtained by a similar Process.*

15. *The several Series that are commonly used for making Logarithms, investigated by common Algebra.*

THESE Series were first exhibited by *Mercator*, from the Quadrature of the *Hyperbola*; afterward by *Dr. Halley*, from the Doctrine of *Ratiunculae*, and by extracting the Root of an *Infinite Power*; by others, from the Doctrine of *Fluxions*, &c. Now, as Beginners in Mathematical Learning cannot soon be acquainted with any of the above Principles; and since a Table of *Logarithms* is useful,
even

The P R E F A C E. ix

even at the first Entrance into these Studies, this Solution obtained by the Assumption of a Series, altho' it may not give sufficient Satisfaction as to the Invention, will prove the Truth of those Methods which are used in constructing such Tables; and by Consequence enable the Operator to correct them if erroneous.

16. Approximations to the Sums of the several Series of the Reciprocals of the 1st, 2d, 3d, &c. Powers of an arithmetical Progression.

This manner of ranging the questions has been sometimes dispensed with, when there was an Opportunity thereby of bringing more Matter into less Space.

Variety of Authors have been carefully examined to collect Materials for this work; viz. Oughtred, Leybourn, Moore, Kersey, Wallis, Harriott, Parsons, Newton, Halley, Jones, De Moivre, Ward, Ronayne, Simpson, Farrer, &c. of our own Nation; and Descartes, Alexander, Ozanam, John and James Bernoulli, Wolfius, Euler, &c. of other Nations.

From

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From these Authors, and many others, has been selected much the greater Part of this Work. 'Tis true, neither the Words made use of by an Author in expressing a Question, nor his Manner of Solution, have been strictly retained; it having been endeavoured, in the Diction, to avoid both Ambiguity, and Superfluity; and in the Operation, to use that Kind of Process, which appeared to be most conducive to render the Solution intelligible, elegant, and concise; but how well this has been performed, must be left to superior Judgment.

In the Division and Arrangement of the several Solutions, the Author has endeavoured to imitate the Manner he has observed in the Papers of the most excellent Mathematician William Jones, Esq.

Thus are Students furnished with a great Number of Examples, applicable to any Book of Theory, so disposed, that they may be gradually applied, and by that Means the Learner may be relieved, at proper Intervals, from the Fatigue of going thro' a whole System of Operations, before he can have so much as a Glimpse of their Application or Use; a Disadvantage very common to Books of Theory.

After

The P R E F A C E. xi

Those Persons, also, whose rapid Genius carries them, almost as soon as they can solve an Equation, to the sublimer Speculations of Fluxions, Quadratures, &c. will possibly find here, something, even in Common Algebra worthy Notice; which they may have hitherto either neglected, or overlooked.

And if what is here done, should meet with a favourable Reception, from those, who (having duly examined the several Parts of Mathematical Learning) are the proper Judges of Performances of this Kind, all the Purposes of this Publication will be answered; and Encouragement thereby given, for the farther Prosecution of a Scheme which the Author, and some of his Mathematical Friends have long since projected; and which, when accomplished, will be truly (what this is but in Part) The MATHEMATICAL REPOSITORY.

Bell-Dock, WAPPING,
January 14, 1747.

THE

After the 56th Question of the Second Part of the following Work, should have been inserted the underwritten

Practical Rule

To find, how many terms of a series ($a, b, c, d, \&c.$ whose n^{th} differences are equal, have been added together to compose a number s .

1. Let $p =$ the continual product of m terms of the series 2, 3, 4, 5, &c.

2. And let $pa = A; A - pa = B; B - pb = C; C - pc = D, \&c.$

3. Find the divisors of $A, B, C, \&c.$ ranging them against their respective numbers.

4. Among the divisors of A find n ; so that $n-1$ may be a divisor of B ; $n-2$, of C ; $n-3$, of $D, \&c.$

5. Then will n , be the number of terms required.

The reason of this will appear, if the equation expressing the value of s in quest. 56, be multiplied by $(2 \cdot 3 \cdot 4 \cdot \&c. =) p$. For then n (being a factor in every term on the right side of the equation) must be a divisor of $(p =) A$; and when the number of terms is lessened by one, then $n-1$ must be a divisor of $B, \&c.$

MATHEMATICAL REPOSITORY.

QUESTION I.

TWO travellers set out at the same time from London and York, whose distance is 150 miles; one of them goes 8, the other 7 miles a day: In what time will they meet?

Let - - - x = the time of their meeting;
Then - - - $8x$ = the miles which the first travels;
And - - - $7x$ = the miles which the second travels;
But $\left. \begin{array}{l} \text{But} \\ \text{(their sum)} \end{array} \right\} 15x = 150$ (the distance of the two cities) by question:
Therefore $x = 10$.

QUESTION II.

One bought 12 yards of cloth at a certain price; and at another time 18 yards thereof at the same price; at which last time he paid 48 shillings more than before: What did he pay a yard?

Let - - - x = the price of 1 yd of cloth;
Then - - - $12x$ = the sum paid for 12 yds;
And - - - $18x$ = the sum paid for 18 yds;
But $\left. \begin{array}{l} \text{But (the difference} \\ \text{of those sums)} \end{array} \right\} 6x = 48$ by question:
Therefore $x = 8$.

MATHEMATICAL

QUESTION III.

Four days after a courier, who travels 30 miles a day, had been dispatched, a second was sent with orders to overtake him; in order to which, the latter goes 42 miles a day: In what time will he overtake the former?

Let x = the time of overtaking;

Then $42x$ = miles the second travelled,

And $30x$ = miles travelled by the first in the same time;

But $(30 \times 4 =) 120$ = miles the first had gone before the second set out.

Th. $30x + 120$ = miles the first travelled in all;

But $30x + 120 = 42x$ by question,

Or $120 = 12x$ by subtraction;

Th. $10 = x$.

QUESTION IV.

Four men, A , B , C , and D , built a ship, which cost 2607 £ . whereof B paid twice as much as A ; C paid as much as A and B ; and D paid as much as C and B . What did each pay?

Suppose A paid x , pounds;

Then B paid $2x$,

C , $3x$,

And D , $5x$;

Whence (the whole sum paid) $11x = 2607 \text{ £}$ by quest.

Therefore $x = 237$.

Q U E S.

QUESTION V

A charitable lady, relieving four poor persons, gave among them 6s. 8d. to the second she gave, twice; to the third, thrice; and to the fourth, four times as much as to the first: What did she give to each?

Suppose she gave x pence to the first;
Then she gave $2x$ pence to the second;
And $3x$ pence to the third;
And $4x$ pence to the fourth:

And (she gave in all) $10x (= 6s. 8d.) = 80$ by question;
Therefore $x = 8$ pence.

QUESTION VI.

A cask which held 126 gallons, was filled with a mixture of brandy, wine, and cyder so that there were 13 gallons of wine, more than there were of brandy; and as much cyder as of both wine and brandy: What quantity was there of each?

Suppose there were x gallons of brandy;
Then there were $x + 13$ of wine;
And $2x + 13$ of cyder;

Whence (their sum) $4x + 26 = 126$ by question;
Therefore (by subtraction) $4x = 100$,
And $x = 25$.

QUESTION VII.

In a lump of mixed metal, weighing 29 lb. there were 2 lb. of silver more than of gold; 4 lb. of copper more than of silver; and 3 lb. of brass more than of copper: How many pounds were there of each?

Suppose there were x lb. of gold;
 Then there were $x + 2$ of silver;
 And $x + 6$ of copper;
 Also $x + 9$ of brass:

But (their sum) $4x + 17 = 29$ by question;
 Therefore (by subtraction) $4x = 12$,
 And $x = 3$.

QUESTION VIII.

A detachment of four regiments consisted of 5219 men; colonel A's regiment exceeded, colonel B's by 22 men; colonel C's by 73 men; and colonel D's by 130 men: How many men were in each regiment?

Let x the men in col. A's regiment;
 Then $x - 22 =$ those in col. B's;
 And $x - 73 =$ those in col. C's;
 Also $x - 130 =$ those in col. D's;
 But (their sum) $4x - 225 = 5219$ by question;
 Th. (by addition) $4x = 5444$;
 And $x = 1361$.

ARITHMETIC

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QUESTION IX.

Six men were employed at the same kind of work ; of whom, the second, earned 13 pence ; the third, 14 pence ; the fourth, 17 pence ; the fifth, 23 pence ; and the sixth 29 pence ; respectively, less than the first ; also the five last earned, in all, three times as much as the first : What did each earn ?

Suppose the first earned x pence ;

Then the second earned $x - 13$ pence,

the third $x - 14$,

the fourth $x - 17$,

the fifth $x - 23$,

the sixth $x - 29$;

But (the sum of the five last) $5x - 96 = 3x$ by qu.

Therefore (by transposition) $2x = 96$,

And $x = 48$.

QUESTION X.

Being to buy a suit of cloaths for each of my six children, I propose to lay out four times as much on the eldest as I do on the youngest ; and to bestow 12 shillings a suit less, on each, than on the next elder : What will each suit cost ?

Suppose the youngest's suit cost x shillings,

Then the second's will cost $x + 12$,

the third's $x + 24$,

the fourth's $x + 36$,

the fifth's $x + 48$,

And the eldest's $x + 60$;

But $x + 60 = 4x$ by qu.

Therefore (by subtraction) $60 = 3x$,

And $20 = x$.

QUESTION XI.

A and *B* began trade with equal stocks; *A* in the first year tripled his stock, all but 30/. *B* doubled his stock, and had 50/. to spare; now the amount of both their gains was 4 times the stock of each: That stock is required?

Let $x =$ required stock.

Now $3x - 30 =$ *A*'s stock,

And $2x - 30 =$ *A*'s gain,

Also $2x + 50 =$ *B*'s stock, } at the year's end.

And $x + 50 =$ *B*'s gain,

Then $3x + 20 = 4x$ (the sum of the gains) by quest.

Therefore $20 = x$.

QUESTION XII.

At a certain election 375 persons voted, and the candidate chosen had a majority of 91: How many voted for each?

Suppose the person chosen had x votes,

And the other y votes;

Then (all the persons who voted) $x + y = 375$.

And (the majority) $x - y = 91$.

Now (the sum of these 2 equations is) $2x = 466$.

Or (the difference of them) $2y = 284$.

Therefore $x = 233$.

And $y = 142$.

Q. E. D.

QUESTION XIII.

Two men, who between them had 35 guineas, play'd together, till one of them had won 4 guineas of the other; and then the winner had twice as many guineas as the loser had at first: How many had each?

$$\begin{array}{ll}
 \text{Let} & x = \text{the guineas which the winner had;} \\
 \text{And} & y = \text{those which the loser had;} \\
 \text{Then} & x + y = 35 \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{by question.} \\
 \text{And} & x + 4 = 2y \\
 \text{Th.} & x = 35 - y \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{by transposition.} \\
 \text{And} & x = 2y - 4 \\
 \text{Th.} & 2y - 4 = 35 - y. \\
 \text{Th.} & 3y = 39. \\
 \text{Th.} & y = 13. \\
 \text{Wh.} & x = 22.
 \end{array}$$

QUESTION XIV.

A gentleman being asked the age of his two sons, replied, that, if to the sum of their ages 25 be added, the number arising will be double the age of the eldest; but if 8 be taken from the difference of their ages, the remainder will be the youngest's age: How old was each?

$$\begin{array}{ll}
 \text{Let} & x = \text{eldest's, and } y = \text{youngest's age;} \\
 \text{Then} & x + y + 25 = 2x \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{by question.} \\
 \text{And} & x - y - 8 = y \\
 \text{Th.} & y + 25 = x, \quad \text{in 1st.} \\
 \text{And} & x = 2y + 8, \quad \text{in 2d.} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{step.} \\
 \text{Hence} & 2y + 8 = y + 25. \\
 \text{Th.} & y = 17. \\
 \text{And} & x = 42.
 \end{array}$$

MATHEMATICAL

QUESTION XV.

A merchant received a bill of exchange in pistoles (at 16s. 6d.), guineas (at 21s.), and moidores (at 27s.); the sum of the pistoles and guineas was 40; the sum of the pistoles and moidores was 36; and the sum of the guineas and moidores was 30: What was the value of the bill?

Let the pistoles received be $= x$,

the guineas $= y$,

the moidores $= z$,

Then first, $x + y = 40$;
secondly, $x + z = 36$;
thirdly, $y + z = 30$; } by question:

Whence (by taking the second from the first) $y - z = 4$;

And (by adding the two last) $2y = 34$;

Therefore $y = 17$;

Now by first $x = (40 - 17 =) 23$;

And by second $z = (36 - 23 =) 13$;

Now 23 pistoles $= L. 18\ 19\ 6$,

17 guineas $= 17\ 17\ 0$,

13 moidores $= 17\ 11\ 0$;

Therefore the bill was for $L. 54\ 07\ 6$.

QUEST.

QUESTION XVI.

The stock of three traders amounted to 780/. the shares of the first and second exceeded the third's by 220/. and the shares of the second and third was 380/. more than the first's : Each share is required ?

Let x , y , and z , represent the required shares ;

Then first, $x + y + z = 780$,
secondly, $x + y = z + 220$,
thirdly, $y + z = x + 380$,

Whence (by taking the difference of the first and second equations - - -) $z = 560 - x$,

Or $2x = 560$; therefore $x = 280$;

And (by taking the difference of the first and third equation - - -) $x = 400 - z$,

Or $2z = 400$; whence $z = 200$;

And $y = (780 - 280 - 200 =) 300$;

QUESTION XVII.

Two persons began play with equal sums of money ; the first won 11 shillings, the other lost 7 shillings ; and then the first had twice as many shillings as the second : what sum had each at first ?

Suppose they had x shillings each ;

Then $x + 11$, } were the sums they had after play -
And $x - 7$, } ing ;

Therefore $x + 11 = (x - 7 \times 2 =) 2x - 14$ by qu.

Whence $25 = x$ (by transposition.

TO MATHEMATICAL

QUESTION XVIII.

A mercer having cut x yards off from each of 3 equal pieces of silk, found that the remnants taken together were 126 yards: What was the length of each piece?

Let x = length of each piece;
 Then $x - 12$ = length of each remnant;
 Now $3 \times x - 12 = 126$ by quest.
 Or $3x - 36 = 126$.
 Th. $3x = 162$;
 And $x = 54$.

QUESTION XIX.

After A had won a shilling of B , he had as many shillings as B had left; but had B won a shilling of A , then he would have had twice as many as A would have had left: How many had each?

Suppose A , had x , and B , y shillings;
 Then $x + 1 = y - 1$
 And $y + 1 = (x - 1) \times 2 = 2x - 2$ } by quest.
 Th. $y = x + 2$, in 1st. }
 And $y = 2x - 3$, in 2d. } by transp.
 Th. $2x - 3 = x + 2$;
 Th. $x = 5$;
 And $y = 7$.

Q U E S

QUESTION XX.

The paving of a square at 2 s. a yard, cost as much as the inclosing it at 5 s. a yard : The side of that square is required ?

Let x = side of the square ;

Then $4x$ = yards of inclosure ;

And xx = yards of pavement ;

Whence $20x = (4x \times 5 =)$ price of inclosing ;

And $2xx = (xx \times 2 =)$ price of paving ;

But $2xx = 20x$ by quest.

Th. $xx = 10x$ } by division.

And $x = 10$

QUESTION XXI.

A person, whose ability was known to be greater than his Industry, was hired to work for a year at 7 s. a day ; but with this condition, that for every day he played, he should forfeit 3 s. now at the year's end he had neither money to receive nor to pay : How many days did he work ?

If he worked x days, he played $365 - x$;

Then $7x$ = sum earned ;

And $1095 - 3x = (365 - x \times 3 =)$ sum forfeited ;

But $7x = 1095 - 3x$, by quest.

Th. $10x = 1095$,

And $x = 109\frac{1}{2}$ days.

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QUESTION XXII.

A general, disposing his army into a square battle, finds he has 284 men more than a perfect square; but increasing the side by 1 man, he will want 25 men: How many had he?

Let x = side of first square;

Then $xx + 284$ = army;

And $x + 1 \times x + 1 - 25$ = army;

Hence $xx + 2x - 24 = xx + 284$;

Then $2x = 308$;

Th. $x = 154$;

And he had $(154 \times 154 + 284)$ 24000 men.

QUESTION XXIII.

'Tis required to divide the number 14 into two such parts, that the difference of the squares of those parts may be 56?

Let x represent the greater, and y the lesser of those parts;

Then $xx - yy = 56$ } by quest.

And $x + y = 14$

Whence $x - y = 4$ (for $\frac{xx - yy}{x + y} = x - y$)

Th. $2x = 18$ by adding 2d. and 3d.

Th. $x = 9$.

QUES-

QUESTION XXIV.

A borrowed of *B* as much money as *A* had, and spent 6*d.* to treat him; after which meeting with *C*, *A* borrowed of him twice as much money as he had left, and treated him with 12*d.* lastly, *A* borrowed of *D* three times as much money as he had left, and spent on him 18*d.* after which he had 30*d.* left: What had he at first?

Suppose he had x pence at first;
 Then he borrowed x pence of *B*,
 And (spending 6*d.*) had $2x - 6$ left;
 Then he borrowed $4x - 12$ of *C*,
 And (spending 12*d.*) had $6x - 30$ left;
 Then he borrowed $18x - 90$ of *D*,
 And (spending 18*d.*) had $24x - 138$ left;
 But $24x - 138 = 30$ by quest. J
 Theref. $24x = 168$,
 And $x = 7$.

QUESTION XXV.

Upon measuring the corn produced by a field (being 8 quarters), it appeared that it had yielded but $\frac{3}{4}$ part more than was sown: How much was that?

If x represent the quantity of corn sown;
 Then by question $x + \frac{x}{4} = 8$;
 But (by multiplication) $4x + x = (8 \times 4) = 32$,
 That is $5x = 32$;
 Th. $x = 6\frac{2}{5}$.

QUESTION XXVI.

A gentleman left 210 £. between his two children ; to his daughter he left half as much as he left to his son : What did he leave to each ?

Let the son's legacy $= x$,

Then the daughter's $= \frac{x}{2}$;

Now $210 = x + \frac{x}{2}$ by quest.

And (by multip.) $420 = (2x + x =) 3x$;

Th. $140 = x$.

QUESTION XXVII.

Being sent to market, to buy a certain quantity of meat, I found that if I bought beef (which was then 4d. a lb.) I should lay out all the money I was entrusted with ; but if I bought mutton (then $3\frac{1}{2}$ a lb.) I should have 2 shillings left : How much meat was sent for ?

If x be the lbs. of meat requir'd ;

Then x lb. at 4d. will cost $4x$ pence,

And x lb. at $3\frac{1}{2}$ d. $= \frac{7x}{2}$ pence,

Now $4x = \frac{7x}{2} + 24$ by quest.

Whence $8x = 7x + 48$ (by multiplication)

And $x = 48$ (by subtraction).

QUESTION XXVIII.

A fish was caught whose tail weighed 9 lb. his head weighed as much as his tail and $\frac{1}{2}$ his body; and his body weighed as much as his head and tail: What did the fish weigh?

Suppose the body weighed x lb.

Then $9 + \frac{x}{2} = \text{weight of the head}$

And (head + tail) $9 + 9 + \frac{x}{2} = x$ by quest.

That is $18 + \frac{x}{2} = x$;

But $36 + x = 2x$ by multip.

Th. $36 = x$ by subtraction.

Hence the fish weighed 72 lb.

QUESTION XXIX.

One being asked how old he was, answered; that the product of $\frac{5}{16}$ of the years he had lived, being multiplied by $\frac{1}{4}$ of the same, would be his age: What was it?

Suppose his age was x years:

Then $\frac{x}{20} \times \frac{5x}{8} = x$ by quest.

That is $\left(\frac{5xx}{20 \times 8}\right) \frac{xx}{4 \times 8} = x$;

But (by multiplication) $xx = 32x$,

Th. (by division) $x = 32$.

QUEST

QUESTION XXX

Some persons agreed to give six-pence each to a waterman for carrying them from *London* to *Graysend*; but with this condition, that for every other person taken in by the way, three-pence should be abated in their joint fare; now the waterman took in 3 more than a fourth part of the number of the first passengers, in consideration of which he took of them but five-pence each: How many persons were there at first?

Suppose there were x passengers at first;

Then $\frac{x}{4} + 3$ were taken in afterwards;

And $\frac{x}{4} + 3$ persons at 3d. each comes to $\frac{3x}{4} + 9$;

But $6x - \frac{3x}{4} - 9 = 5x$ by question;

And $24x - 3x - 36 = 20x$ by multiplication,

That is, $24x - 3x - 20x = 36$ by transposition,

Th. $4x = 36$

QUESTION XXXI

From each of 16 pieces of gold, an artist filed the worth of half a crown, and then offered them in payment for their original value; but being detected, and the pieces weighed, they were found to be worth no more than 8 guineas: Their original worth is required?

Suppose they were each worth x shillings,

Then the value of $\left\{ \begin{array}{l} x - 2\frac{1}{2} = x - \frac{5}{2} \end{array} \right\} = \frac{2x - 5}{2}$ each after filing;

But $(8 \text{ guin.} =) 168 = \frac{2x - 5}{2} \times 16 = 2x - 5 \times 8,$

That is $168 = 16x - 40,$

Or (by addit.) $208 = 16x;$

Th. $13 = x.$

QUES-

QUESTION XXXII

What sum of money is that, from which 5% being subtracted, two thirds of the remainder will be 40/3?

Suppose x = the sum required ;

Then $x - 5$ = remainder when 5 is subtracted ;

And $\frac{x - 5 \times 2}{3}$ = two thirds of that remainder ;

But $\frac{x - 5 \times 2}{3} = 40$ by question,

Th. $x - 5 \times 2 = (40 \times 3) = 120$;

And $x - 5 = \frac{120}{2} = 60$,

Th. $x = (60 + 5) = 65$.

QUESTION XXXIII.

One being asked the hour of the day, replied, that the time then passed from noon, was equal to $\frac{29}{43}$ of the time remaining until midnight : What time was that ?

Suppose x = the required time ;

Then $12 - x$ = the remaining time to mid.

And $x = \frac{12 - x \times 29}{43}$ by quest.

Or $43x = (12 - x \times 29) = 348 - 29x$,

Or $(43x + 29x) = 348$;

h. min.

Th. $x = \frac{348}{72} = \frac{29}{6} = 4:50$.

QUES.

QUESTION XXXIV.

'Tis required to divide 55, into two such parts, that the greater of them, divided by their difference, may quote 6?

If x be the greater, and y the lesser part,

Then $x + y = 55$

And $\frac{x}{x - y} = 6$ } by quest.

But (by 2d.) $x = (x - y \times 6) = 6x - 6y$

Or $6y = (6x - x) 5x$

Th. $y = \frac{5x}{6}$

But (by 1st.) $x + \frac{5x}{6} = 55$

Or $(6x + 5x) 11x = 55 \times 6$ by multiplication,

Th. $x = (5 \times 6) 30$

QUESTION XXXV.

A having about him 240, and B having 96 l. were met by some thieves; who took from A twice as much as from B , and left A three times as much as they left B : What sum was each robbed of?

Suppose A was robbed of x , and B , of y , pounds:

Then (by question) $x = 2y$

And $240 - x = 3 \times 96 - y$

And (by writing $2y$ for x in the second equation) $240 - 2y = 288 - y$

Th. (by transp. $3y - 2y = 288 - 240 = 48$

QUESTION XXXVI.

A son asking his father how old he was, the father replied, "My age 7 years ago, was just four times as great as your age, at that time: but 7 years hence, if you and I live, my age will be only double to yours." The age of each person is required?

Suppose the father was x , and the son y years old:

Then first $x - 7 = (y - 7) \times 4 = 4y - 28$ } by qu.

And secondly $x + 7 = (y + 7) \times 2 = 2y + 14$ }

Th. $x = 4y - 21$ } by transposition;

And $x = 2y + 7$ }

But $4y - 21 = 2y + 7$,

And $2y = 28$ by transposition,

Th. $y = 14$.

QUESTION XXXVII.

A company of 18 persons, men and women, clubbing for a reckoning of 97. 18 s. paid each as many shillings as there were men in company: How many were there?

Let the number of men $= x$,

And the number of women $= y$.

Then the sum paid by the men $= xx$ shillings,

And xy women.

Now first $x + y = 18$

And secondly $xx + xy = 198 (= 97. 18s.)$

Whence (by dividing the 2d. equation by the 1st.) $\frac{xx + xy}{x + y} = \frac{198}{18}$

That is $x + y = 11$

And $x = 7$

QUES.

QUESTION XXXVHI.

At an election the number of voters was three times the majority by which the choice was carried; and the product of the numbers which voted for each, was 122 times the said majority! How many votes had each?

Let x be the greater, and y the lesser of the numbers required.

Then the whole number of voters $= x + y$,

And the majority $= x - y$,

Now $(x - y) \times 3 = 3x - 3y = x + y$, } by qu.

And $(x - y) \times 122 = 122x - 122y = xy$, }

But (by transposing the first) $2x = 4y$,

Th. $x = 2y$,

And (by writing $2y$ for x) $244y - 122y = 2y$ by second;

Or (divid. by y) $244 - 122 = 122 = 2y$,

Th. $61 = y$.

$x = 122$

QUESTION XXXV.

A countryman bought 8 yards of cloth for 62 shillings; for part of it, I gave 9s. a yard; and for the rest 7s. How much was bought of each?

Suppose x yards at 9s. and y yards at 7s. were bought;

Then x yards cost $9x$,

And y yards cost $7y$;

But by question $x + y = 8$,

And the first multiplied by 7 produces $7x + 7y = 56$,

Th. (by taking the difference of the two last) $2x = 6$;

Whence $x = 3$,

And $y = 5$.

QUEST.

QUESTION XL.

A person paid a bill of 50 l. with half guineas and crowns, using 101 pieces in all: How many of each sort did he pay?

Let x represent the number of $\frac{1}{2}$ guineas,

And y - - - the number of crowns;

Then $x + y = 101$,

And because

50 l. is 2000 } $21x + 10y = 2000$;

six-pences

The first e-

quation mul-

tiplied by 10

produces

Whence (by subtraction) $11x = (2000 - 1010) = 990$;

Th. $x = 90$,

And $y = 11$.

QUESTION XLI.

Two remnants of cloth, which together measure 40 yards, were of equal value; and the one sold at 3 s. the other at 7 s. a yard: How many yards were there of each?

Suppose there were x yards at 3 s. a yard,

And y yards at 7 s.

Then the value of the first was $3x$,

And $7y$ second

Whence 1. $x + y = 40$ } by question:

And 2. $3x = 7y$

Th. (from first) $x = 40 - y$,

But $3(40 - y) = 7y$ by 2d.

And (by transposition) $120 = 10y$,

Th. $12 = y$.

QUESTION XLII.

Having laid out 37 shillings in brandy at 2 s. and rum at 3 s. a quart; I find that I could have bought as many quarts of rum as I now have of brandy, and as many of brandy as now of rum, for 4 shillings less: How much was bought?

If x , quarts of brandy; and y , of rum were bought,
Then $2x + 3y = 37$,

And $2y + 3x = (37 - 4) = 33$ } by qu.

But (by subtraction) $y - x = 4$,

Th. $y = x + 4$

And (by multiplication) $3y = 3x + 12$

But $2x + 3x + 12 = 37$ by first,

And (by transposition) $5x = 25$,

Th. $x = 5$

And $y = 9$.

QUESTION XLIII.

A person exchanges 6 French crowns and 2 dollars for 45 shillings; and at another time 9 French crowns, 5 dollars, for 76 shillings: What were the values of the crown and dollar?

Suppose a French crown is worth x , } pence;
And a dollar y ,

Then (by question) $\begin{cases} 6x + 2y = 45 \times 12, \\ 9x + 5y = 76 \times 12, \end{cases}$

The difference of } which equation is $3x + 3y = 31 \times 12$,

And (by division) $x + y = (31 \times 4) = 124$,

Th. $x = (124 - y)$

But (by the 1st.) $124 - y \times 6 + 2y = 45 \times 12 = 540$;

That is $(744 - 6y + 2y) = 540$,

And (by transf. $744 - 540 = 204 = 4y$,

Th. $51 = y$;

And $(124 - 51) = 73 = x$.

QUESTION XLIV.

A gentleman gave to 3 persons 56 l. the second received $\frac{2}{3}$ of the sum given to the first, and the third $\frac{1}{3}$ of what the second had: How much had each?

Suppose the first received x l.

Then the second received $\frac{2x}{3}$

And the third $\frac{x}{3}$

Th. $(x + \frac{2x}{3} + \frac{x}{3}) = 56$ by quest.

Whence $9x + 5x = 14x = 56 \times 9$

Therefore (divided by 14) $x = 36$

QUESTION XLV.

Out of a cask of liquor, 63 gallons were sold to two persons (between them); to the first $\frac{2}{3}$, and to the second $\frac{1}{3}$, of what the cask contained: What did the cask hold?

If the cask hold x gallons;

Then the first bought $\frac{2x}{3}$ gallons,

And the second $\frac{x}{3}$ ditto:

Therefore $\frac{2x}{3} + \frac{x}{3} = 63$ by quest.

Which multiplied $\frac{6x}{3} + \frac{6x}{3} = (63 \times 6) = 378$,
by 6 produces

That is $2x + x = 3x = 378$;

Th. $x = 126$.

QUES.

QUESTION XLVI.

Out of a cask of wine, which had leaked away $\frac{1}{3}$, 21 gallons were drawn; and then being gauged, it appeared to be $\frac{2}{3}$ full: How much did it hold?

Suppose it held x gallons,

Then it had leaked $\frac{x}{3}$ ditto;

And when gauged: $21 + \frac{x}{3}$ had been taken out,

But $21 + \frac{x}{3} = \frac{x}{2}$ by question;

Which multiplied by $(3 \times 2 =) 6$ produces $126 + \frac{6x}{3} = \frac{6x}{2}$,

That is $126 + 2x = 3x$;

Th. (by subtraction $126 = (3x - 2x =) x$).

QUESTION XLVII.

After paying away $\frac{1}{4}$ and $\frac{1}{5}$ of my money, I found 66 guineas left in my bag: What was in it at first?

Suppose x guineas;

Then (by question) $x - \frac{x}{4} - \frac{x}{5} = 66$;

Which multiplied by $(4 \times 5 =) 20$ produces $20x - \frac{20x}{4} - \frac{20x}{5} = 66 \times 20 = 1320$;

That is $(20x - 5x - 4x =) 11x = 1320$;

Th. $x = 120$.

QUES;

QUESTION XLVIII.

A gentleman gave in charity 46 pounds; a part thereof, in equal portions, to 5 poor men; and the rest, in equal portions, to 7 poor women; now a man and a woman had between them 8 pounds: What was given to the men, and what to the women?

Suppose the men had - - - x l.

Then the women had $46 - x$;

Whence one man had - - $\frac{x}{5}$;

And one woman - $\frac{46 - x}{7}$;

Now by question - - - $\frac{x}{5} + \frac{46 - x}{7} = 8$;

Which multiplied by $\left. \begin{array}{l} 7x + 46 - x \times 5 = (8 \times 35 =) 280, \\ (5 \times 7 =) 35 \text{ produces} \end{array} \right\}$

That is $(7x + 230 - 5x =) 2x + 230 = 280$,

Or (by transposition) - - - - - $2x = 50$;

Th. - - - - - $x = 25$.

QUESTION XLIX.

What fraction is that, to the numerator of which if 1 be added, the value will be $\frac{1}{3}$; but if 1 be added to the denominator, its value will be $\frac{1}{4}$?

Let $\frac{x}{y}$ represent the fraction required;

Then $\frac{x + 1}{y} = \frac{1}{3}$; Or $3x + 3 = y$,

And $\frac{x}{y + 1} = \frac{1}{4}$; Or $4x = y + 1$,

But by writing $3x + 3$ for y in the second $\left. \begin{array}{l} 4x = \{ 3x + 3 + 1 = \\ 3x + 4; \end{array} \right\}$ by question:

Th. (by subtraction) - - - - - $x = 4$,

And - - - - - $y = (3 \times 4 + 3 =) 15$.

QUESTION LI.

Two remnants of cloth were bought, which measured one 7, the other 5 yards; the first and 1 yard of the second cost 3*l*. 8*s*. also the second and 1 yard of the first cost the same sum: What were each valued at a yard?

Suppose the first remnant cost *y**s*. a yard,

And the second *z**s*. a yard;

Then the value of the first remnant = 7*y*,

And ditto second = 5*z*;

But by question $\begin{cases} 7y + z = 68, \\ 5z + y = 68; \end{cases}$

Th. (by transposing the first) $z = 68 - 7y$,

And $(68 - 7y \times 5 =) 340 - 35y = 5z$;

Th. (by the second $5z + y = 340 - 35y + y = 68$,

That is $340 - 34y = 68$,

Or (by transposition $340 - 68 = 272 = 34y$;

Th. $(\frac{272}{34} =) 8 = y$.

QUESTION LI.

A man dying left his wife big with child, ordering by will, that if the child proved a daughter, then his wife should have $\frac{2}{3}$ and the child $\frac{1}{3}$ of his estate; but if it was a son, then he should have $\frac{2}{3}$ and the mother $\frac{1}{3}$ thereof; now it happened that the mother was delivered of a son and a daughter: How must the estate (which was 6300*l*.) be divided between them?

Suppose the daughter's share was *x**l*.

Then the mother's would be 2*x*,

And the son's = 4*x*;

For then $\begin{cases} \text{the son's share is to the mother's} \\ \text{the mother's to the daughter's} \end{cases}$ as $\frac{2}{3}$ to $\frac{1}{3}$:

But (the whole estate) $6300 = 7x$, by question;

Th. $(\frac{6300}{7} =) 900 = x$.

QUES.

QUESTION LII.

There is a cask containing 120 gallons, filled with brandy, white wine, cyder, and water; the brandy and wine (taken together) make $\frac{1}{2}$ the content of the cask; the brandy and cyder make $\frac{2}{3}$ of the content; and the brandy and water $\frac{3}{4}$ thereof: The quantities of each are required?

Let x , y , u , and z , be the quantities of brandy, wine, cyder, and water:

$$\text{Then } x + y + u + z = 120,$$

$$x + y = \left(\frac{120}{2}\right) = 60,$$

$$x + u = \left(\frac{120 \times 2}{3}\right) = 80,$$

$$x + z = \left(\frac{120 \times 3}{4}\right) = 90,$$

} by question:

$$\text{(By Addition) } 4x + 2y + 2u + 2z = 350;$$

$$\text{(By first) } 2x + 2y + 2u + 2z = 240;$$

$$\text{Th. } 2x = 110 \text{ (by subtraction)}$$

$$\text{And } x = 55.$$

$$y = (60 - 55) = 5,$$

$$u = (80 - 55) = 25,$$

$$z = (90 - 55) = 35.$$

QUESTION LIII.

The expence of building a ship (which cost 3900*l*.) was defrayed by 2 merchants, whose shares in her were as 4 to 9: What did each pay?

Suppose one of them paid x .

$$\text{Then } (4:9::x:) \frac{9x}{4} = \text{the other's payment:}$$

$$\text{And } x + \frac{9x}{4} = 3900 \text{ by question;}$$

$$\text{Or } (4x + 9x) = 13x = 3900 \times 4;$$

$$\text{Th. } x = (3900 \times 4) = 1200.$$

QUESTION LIV.

A draper sold $\frac{3}{4}$ of a piece of cloth at 5s. $\frac{1}{2}$ of it at 4s. and $\frac{1}{4}$ of it at 4s. 6d. a yard; by doing which he took 8 guineas: How many yards did the piece contain?

If it contained x yards;

$$\text{Then } 5 \times \frac{x}{4} + 4 \times \frac{x}{5} + \frac{9}{2} \times \frac{x}{6} = (8 \times 21 =) 168 \text{ by q.}$$

$$\text{Or } \frac{5x}{4} + \frac{4x}{5} + \frac{3x}{4} = 168;$$

$$\text{And (multi-plied by 20)} \left\{ \frac{100x}{4} + \frac{80x}{5} + \frac{60x}{4} = 168 \times 20, \right.$$

$$\text{Or } 25x + 16x + 15x = 3360,$$

$$\text{Or } 56x = 3360;$$

$$\text{Th. } x = \left(\frac{3360}{56} = \right) 60.$$

QUESTION LV.

The governors of *Christ's-Hospital* in *London*, bestowed $\frac{3}{4}$ of a legacy committed to their trust, among 3 of their boys who were sent to the university; $\frac{1}{4}$ of it, among 7 boys sent to sea; and $\frac{1}{8}$ of it, among 11 boys bound to trades; now 1 scholar, 1 sea-boy, and 1 apprentice, received among them 10l. 1s. What was the legacy?

If the legacy was x shillings;

$$\text{Then } \frac{3}{4} \text{ of } \frac{x}{2} + \frac{1}{4} \text{ of } \frac{x}{4} + \frac{1}{8} \text{ of } \frac{x}{6} = (10 : 1 =) 201$$

$$\text{That is } \frac{x}{6} + \frac{x}{28} + \frac{x}{66} = 201 \text{ by quest.}$$

Which being multiplied by $2 \times 14 \times 33$ produces

$$\frac{2 \times 14 \times 33x}{6} + \frac{2 \times 14 \times 33x}{28} + \frac{2 \times 14 \times 33x}{66} = 201 \times 2 \times 14 \times 33,$$

$$\text{Or } 14 \times 11x + 33x + 14x = 201 \times 2 \times 14 \times 33,$$

$$\text{Or } (154x + 33x + 14x =) 201x = 201 \times 2 \times 14 \times 33;$$

$$\text{Th. } x = (2 \times 14 \times 33 = 924 =) 46l. 4s.$$

QUESTION LVI.

Some boys *A, B, C, D, E, F, G, H, and I*, robbed an orchard; *A* had for his share $\frac{1}{5}$, *B* $\frac{1}{12}$, *C* $\frac{1}{8}$, *D* $\frac{1}{20}$, *E* $\frac{1}{7}$, and *F* $\frac{1}{4}$ part of the whole; *G* had 310, *H* 425, and *I* 140 apples: How many apples had they in all?

Suppose the whole number of apples was x ,

$$\text{Then } \frac{x}{5} + \frac{x}{12} + \frac{x}{8} + \frac{x}{20} + \frac{x}{7} + \frac{x}{4} + 310 + 425 + 140 = x;$$

$$\text{Or } \frac{x}{5} + \frac{x}{12} + \frac{x}{8} + \frac{x}{20} + \frac{x}{7} + \frac{x}{4} + 875 = x;$$

Which multiplied by $(5 \times 3 \times 8 \times 7 =) 840$,
 produces, $168x + 70x + 105x + 42x + 120x + 210x + 875 \times 840 = 840x$;

That is $715x + 735000 = 840x$;

And (by transposit.) $735000 = (840x - 715x =) 125x$;

Th. $\left(\frac{735000}{125} = \frac{147000}{25} = \frac{29400}{5} \right) 5880 = x$,

QUESTION LVII.

A person at play won twice as much as he began with, and then lost 16 shillings; after which he lost $\frac{1}{5}$ of what remained; lastly, he won as much he began with; then counting his money found he had 80 shillings: I demand what sum he began with?

Suppose he began with x shillings;

Then he had $3x$, after he had won $2x$;

And $3x - 16$, when 16s. was lost;

Also $\left(3x - 16 - \frac{3x - 16 \times 4}{5} = \right) \frac{3x - 16}{5}$ remained when $\frac{1}{5}$ of the former was lost:

Lastly by question $\frac{3x - 16}{5} + x = 80$,

And $(3x - 16 + 5x =) 8x - 16 = 80 \times 5$;

Now (dividing by 8) $x - 2 = (10 \times 5 =) 50$,

Th. (by addition) $x = 52$.

QUESTION LVIII

A man and his wife did usually drink out a vessel of beer in 12 days; but when the man was out, the vessel lasted the woman 30 days: In how many days would the man alone be drinking it out?

Suppose the man could drink it out in x days;

D. V. D.
Then $(x : 1 :: 12 :) \frac{12}{x} = \left\{ \begin{array}{l} \text{the part drank by the} \\ \text{man in 12 days;} \end{array} \right.$

And $(30 : 1 :: 12 :) \frac{12}{30} = \text{ditto by the woman;}$

But $\frac{12}{x} + \frac{12}{30} = 1$ by question;

Or $360 + 12x = 30x;$

Or $360 = (30x - 12x) = 18x;$

Th. $20 = x.$

QUESTION LIX.

A cistern into which water may be let by two cocks A and B , will be filled by them both running together in 12 hours; and by the cock A alone in 20 hours: In what time will it be filled by the cock B alone?

Suppose in x time;

H. Cist. H.
Then $(x : 1 :: 12 :) \frac{12}{x} = \left\{ \begin{array}{l} \text{the quantity of wa-} \\ \text{ter supplied in 12} \\ \text{hours by} \end{array} \right. \left\{ \begin{array}{l} B, \\ A: \end{array} \right.$

But $\frac{12}{20} + \frac{12}{x} = 1$ (cistern full) by question;

That is $12x + 240 = 20x;$

Or $240 = (20x - 12x) = 8x;$

Th. $30 = x.$

Q U E S.

QUESTION LX.

A sum of money is to be shared between two persons, A and B ; so that as often as A takes 9 ℓ . B is to take 4 ℓ . now it happened, that A received 15 ℓ . more than B : Their respective shares are required?

Suppose A received $x\ell$.

Then B received $x - 15$;

Thence $x : x - 15 :: 9 : 4$ by question;

But the product of the extremes of 4 proportionals, is equal to the product of the means;

Th. $4x = (x - 15 \times 9) = 9x - 135$;

And (by transposition) $135 = (9x - 4x) = 5x$;

Th. $27 = x$.

QUESTION LXI.

A footman, who contracted for 8 ℓ . a year, and a livery suit, was turned away at the end of 7 months, and received only 2 ℓ . 13s. 4d. and his livery; What was its value?

Suppose it was worth x pounds;

Then (because 2 ℓ . 13s. 4d. = $\frac{13}{6}\ell$.)

Mo. ℓ . Mo. ℓ .

$12 : 8 + x :: 7 : \frac{13}{6} + x$ by question;

That is $(12 \times \frac{13}{6} + x) = 26 + 12x = (8 + x \times 7) = 56 + 7x$;

Or $(12x - 7x) = 5x = (56 - 26) = 30$;

Th. $x = (\frac{30}{5}) = 6$.

QUESTION LXII.

A market-woman bought a certain number of eggs, at 2 a penny, and as many at 3 a penny, and sold them all out again at the rate of 5 for two-pence; after which she found that (instead of making just her money again, as she expected) she had lost 4 pence by them: What number of eggs had she?

Suppose she bought x eggs of each sort;

Then x eggs at 2 a penny cost $\frac{x}{2}$ pence;

And x ditto at 3 a penny cost $\frac{x}{3}$ pence;

Also $2x$ eggs at 5 for two-pence sold for $\frac{4x}{5}$ pence;

(For $5 : 2 :: 2x : \frac{4x}{5}$);

Now $\frac{x}{2} + \frac{x}{3} - \frac{4x}{5} = 4$ by question;

That is $15x + 10x - 24x = (4 \times 30 =) 120$;

Th. $x = 120$.

QUESTION LXIII.

What number is that, to which if 3, 5, and 8, be severally added, the first sum shall be to the second, as the second to the third?

If x = the number sought?

Then $x+3 : x+5 :: x+5 : x+8$ by question;

Th. $x+3 \times x+8 = x+5 \times x+5$.

Or $xx + 11x + 24 = xx + 10x + 25$,

Or $11x + 24 = 10x + 25$;

Th. $(11x - 10x =) x = (25 - 24 =) 1$.

QUESTION LXIV.

A hare 50 of her leaps before a greyhound, takes 4 leaps to the greyhound's 3; but 2 of the greyhound's leaps are as much as 3 of the hare's: How many leaps must the greyhound take to catch the hare?

Suppose the greyhound must take x leaps,
and in that time the hare takes y leaps;

Then $3 : 4 :: x : y$ by question;

Th. $3y = 4x$, And $y = \frac{4x}{3}$;

Now the hare takes $50 + \frac{4x}{3}$ leaps in the whole;

Also $2 : 3 :: x : 50 + \frac{4x}{3}$ by question;

That is $(50 + \frac{4x}{3} \times 2 =) 100 + \frac{8x}{3} = 3x$;

Or $300 + 8x = 9x$;

Th. (by subtraction) $300 = x$.

QUESTION LXV.

The joint stock of two partners, (whose particular shares differed by 40 l.) was to the share of the lesser; as 14 to 5: Their particular shares are desired?

Suppose $x =$ the lesser share;

Then $x + 40 =$ the greater;

And $2x + 40 =$ their joint stock;

Whence $2x + 40 : x :: 14 : 5$ by question;

Th. $(2x + 40 \times 5 =) 10x + 200 = 14x$;

And $200 = (14x - 10x =) 4x$;

Th. $(\frac{200}{4} =) 50 = x$.

QUESTION LXVI.

A bankrupt owed to two creditors 140/ the difference of which debts, was to the greater of them, as 4 to 9: What were those debts?

Suppose x = the greater, and y the lesser debt;

Then $x - y : x :: 4 : 9$ by question;

That is $(x - y) \times 9 = 4x - 9y = 4x$;

Or (by transposition $9x - 4x = 9y$);

Th. $x = \frac{9y}{5}$;

But $(x + y = 140$; that is) $\frac{9y}{5} + y = 140$ by quest.

And (by multip.) $(9y + 5y =) 14y = 140 \times 5$;

Th. $y = (10 \times 5 =) 50$.

QUESTION LXVII.

~~A, B, and C, make a joint stock; A put in 17l.~~
~~less than B, and 34l. less than C; and the sum of the~~
~~shares of A and B, is to the sum of the shares of B~~
~~and C, as 6 to 7: What did each put in?~~

Suppose ~~A put in~~ x l.

Then ~~B put in~~ $x + 17$;

And ~~C~~ $x + 34$;

Whence the sum of the shares $\begin{cases} A \text{ and } B = 2x + 17, \\ C \text{ and } B = 2x + 51: \end{cases}$

But $2x + 17 : 2x + 51 :: 6 : 7$ by question;

That is $2x + 17 \times 7 = 2x + 51 \times 6$,

Or $14x + 119 = 12x + 306$,

Or $(14x - 12x =) 2x = (306 - 119 =) 187$;

Th. $x = \left(\frac{187}{2} =\right) 93\text{l. } 10\text{s.}$

QUESTION LXVIII.

'Tis required to find two numbers, the greater where-
of shall be to the lesser, as their sum is to 54; and, as
their difference is to 9?

Let x , represent the greater; and y the lesser number;

Then $x : y :: x + y : 45$ } by question;

And $x : y :: x - y : 9$

Th. $45x = (x + y \times y) = xy + y^2$

And $9x = (x - y \times y) = xy - y^2$

The sum of which $54x = 2xy$,

Whence $27x = xy$,

And $27 = y$

Now $9x = 27x - 27 \times 27$,

And $x = 3x - 3 \times 27$;

Th. $81 = 2x$,

And $40\frac{1}{2} = x$.

QUESTION LXIX.

'Tis required to divide 60*l.* so, between two men A
and B , that the difference between A 's share and 31, shall
be to the difference between 31 and B 's share, as 6 to 7?

Suppose A 's share to be x *l.* and B 's y *l.*

Then $x - 31 : 31 - y :: 6 : 7$ by question;

That is $(x - 31 \times 7) = 7x - 217 = (31 - y \times 6) = 186 - 6y$,

Or (by transposition) $7x = 403 - 6y$;

Th. $x = \frac{403 - 6y}{7}$.

But $(x + y = 60$; that is) $\frac{403 - 6y}{7} + y = 60$ by question;

And (by multip.) $403 - 6y + 7y = (60 \times 7) = 420$;

Th. (by transposition) $y = (420 - 403) = 17$;

And $x = \left(\frac{403 - 6 \times 17}{7} = \frac{301}{7} \right) = 43$.

QUESTION LXX

Sold a quantity of tobacco for 19 shillings, part at 1s. a lb. and the rest at 15d. now the first part was to the latter, as $\frac{3}{4}$ to $\frac{2}{3}$: How much was sold of each?

Let x = the quantity sold at 1s. a lb.

Then $(\frac{3}{4} : \frac{2}{3} :: x :) \frac{2 \times 4x}{3 \times 3}$ = the other ::

Also $12x$, is the value of x lb. at 12d.

And $(\frac{2 \times 4 \times 15x}{3 \times 3} =) \frac{2 \times 4 \times 5x}{3}$, the val. of $\frac{2 \times 4x}{3 \times 3}$ lb. at 15d.

Now $12x + \frac{2 \times 4 \times 5x}{3} = (19 \times 12 =) 228$ by question;

Or $(36x + 40x =) 76x = 228 \times 3$;

Th. $x = (3 \times 3 =) 9$.

QUESTION LXXI.

What two numbers are as 3 to 1; and the sum of their squares is to their sum as 15 to 1?

Let x = the greater number;

Then $(3 : 1 :: x :) \frac{x}{3}$ = the lesser number;

Whence $(x + \frac{x}{3} =) \frac{4x}{3}$ = their sum;

And $(xx + \frac{xx}{9} =) \frac{10xx}{9}$ = sum of their squares;

Now $\frac{10xx}{9} : \frac{4x}{3} :: 15 : 1$ by question;

That is $\frac{10xx}{9} = \frac{4 \times 15x}{3}$;

Or $10xx = 3 \times 4 \times 15x$;

Or $10x = 3 \times 4 \times 15$;

Th. $x = (3 \times 2 \times 3 =) 18$.

QUESTION LXXII.

What 2 numbers are as 3 to 2; whose sum is equal to the square of their difference?

Let x represent one of those numbers;

Then $(3 : 2 :: x :) \frac{2x}{3} = \text{the other};$

Now $(x + \frac{2x}{3} = \frac{3x + 2x}{3} = \frac{5x}{3} = \text{their sum};$

And $(x - \frac{2x}{3} = \frac{3x - 2x}{3} = \frac{x}{3} = \text{their difference};$

But (by question) $\frac{5x}{3} = \frac{x}{3} \times 3;$

And (by division) $5 = \frac{x}{3};$

Th. (by multiplication) $15 = x.$

QUESTION LXXIII.

What 2 numbers are as 2 to 3, to each of which if 4 be added, the sums will be as 5 to 7?

Let x equal one of the required number;

Then $(2 : 3 :: x :) \frac{3x}{2} = \text{the other number};$

Also $x + 4 : \frac{3x}{2} + 4 :: 5 : 7$ by question;

Th. $(x + 4 \times 7 =) 7x + 28 = \frac{3x}{2} + 4 \times 5,$

That is $7x + 28 = \frac{15x}{2} + 20,$

Or $7x + 8 = \frac{15x}{2};$

But $14x + 16 = 15x;$

Th. $16 = x.$

QUEST

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QUESTION LXXIV.

There are three numbers whose differences are equal (that is, the second exceeds the first, as much as the third exceeds the second); and the first is to the third as 5 to 7; also the sum of the three numbers is 324: What are those numbers?

Let the second number be represented by y ,

And the difference of those numbers by x ;

Then $y - x =$ the first number,

$y =$ the second,

And $y + x =$ the third;

But (their sum) $3y = 324$ by question;

Therefore $y = 108$ (by division);

But $y - x : y + x :: 5 : 7$ by question;

That is $108 - x : 108 + x :: 5 : 7$;

Th. $108 - x \times 7 = 108 + x \times 5$,

Or $756 - 7x = 540 + 5x$,

Or $(756 - 540) 216 = (7x + 5x) 12x$;

Th. $\left(\frac{216}{12} =\right) 18 = x$;

And the numbers required are 90, 108, and 126.

QUESTION LXXV.

What two numbers are those, whose difference sum and product are to each other, as the numbers 2, 3, and 5, respectively?

If $x =$ the greater, and $y =$ the lesser of the numbers required,

Then $x - y : x + y :: 2 : 3$ } by question:

And $x + y : xy :: 3 : 5$

That is $3x - 3y = 2x + 2y$; Or $x = 5y$;

And $5x + 5y = 3xy$; Or $5 \times 5y + 5y = 3y \times 5y$,

That is $(25y + 5y) 30y = 15y$;

And (by division) $30 = 15y$.

Th. $\left(\frac{30}{15} =\right) 2 = y$;

And $x = (5 \times 2) 10$.

Q U E S.

QUESTION LXXVI.

A butcher being asked, what number of calves and sheep he had bought? replied, if I had bought 4 more of each, I should have had 4 sheep for every 3 calves; and if I had bought 4 less of each, I should have had 3 sheep for every 2 calves: How many of each did he buy?

If he bought x sheep, and y calves;

Then $x + 4 : y + 4 :: 4 : 3$ } by question;
And $x - 4 : y - 4 :: 3 : 2$ }

That is $\begin{cases} (x + 4 \times 3 =) 3x + 12 = (y + 4 \times 4 =) 4y + 16, \\ (x - 4 \times 2 =) 2x - 8 = (y - 4 \times 3 =) 3y - 12; \end{cases}$

Whence $\begin{cases} x = \frac{4y + 4}{3}, \\ x = \frac{3y - 4}{2}; \end{cases}$

But $\frac{3y - 4}{2} = \frac{4y + 4}{3},$

Or $9y - 12 = 8y + 8;$

Th. $(9y - 8y =) y = (8 + 12) 20:$

$x = \left(\frac{3 \times 20 - 4}{2} = \frac{56}{2} = \right) 28.$

QUESTION LXXVII.

One has 2 sorts of wine: A and B , the wine A is worth $6d$ a quart, and B , $10d$. a quart; he would mix 100 quarts of those, so that each quart of the mixture may be afforded for $7d$. How many quarts of each may be taken?

Suppose x quarts of A , and y quarts of B ;

Also put $6 = a$, $10 = b$, $100 = t$, and $7 = m$:

Then $x + y = t$ } by question;

And $ax + by = mt$ }

But $ax + ay = at$ by multiplication;

And $by - ay = mt - at$ by subtraction;

Th. $y = \left(\frac{mt - at}{b - a} = \right) \frac{m - a \times t}{b - a}.$

In this examp. $y = \left(\frac{7 - 6}{10 - 6} \times 100 = \frac{1}{4} \times 100 = \right) 25.$

Q U E S.

QUESTION LXXVIII.

Four persons, *A*, *B*, *C*, *D*, spent 20 shillings in company together; whereof *A* proposed to pay $\frac{1}{4}$, *B* $\frac{1}{5}$, *C* $\frac{1}{6}$, and *D* $\frac{1}{8}$ part; but, when the Money came to be collected, they found it was not sufficient to answer the intended purpose: How much must each person contribute to make up the whole reckoning, supposing their several shares to be, still, to each other, in the proportion above specified?

Suppose *A* was to pay x shillings;

Then $(\frac{1}{4} : \frac{1}{5} :: x :) \frac{3x}{4} = \left\{ \begin{array}{l} \text{the sum } B \\ \text{must pay;} \end{array} \right.$

And $(\frac{1}{4} : \frac{1}{6} :: x :) \frac{3x}{5} = \left\{ \begin{array}{l} \text{ditto } C \text{ must} \\ \text{pay;} \end{array} \right.$

And $(\frac{1}{4} : \frac{1}{8} :: x :) \frac{x}{2} = \left\{ \begin{array}{l} \text{ditto } D \text{ must} \\ \text{pay;} \end{array} \right.$

But $x + \frac{3x}{4} + \frac{3x}{5} + \frac{x}{2} = 20$ by question

Or $20x + 15x + 12x + 10x = (20 \times 20 =) 400,$

That is $57x = 400;$

Th $x = (\frac{400}{57} =) 7s. \frac{1}{37};$

B must pay $(\frac{400}{57} \times \frac{3}{4} = \frac{300}{57} =) 5 \frac{11}{37};$

C ditto $(\frac{400}{57} \times \frac{3}{5} = \frac{80 \times 3}{57} = \frac{240}{57} =) 4 \frac{12}{37};$

D ditto $(\frac{400}{57} \times \frac{1}{2} = \frac{200}{57} =) 3 \frac{29}{37}.$

Q U E S.

QUESTION LXXIX.

What two numbers are as 7 to 5, whose product is to their sum as 35 to 3?

If x be the greater required number;

Then $(7 : 5 :: x : \frac{5x}{7})$ the lesser;

Now $(\frac{5x}{7} \times x = \frac{5x^2}{7})$ their product;

And $(\frac{5x}{7} + x = \frac{5x + 7x}{7} = \frac{12x}{7})$ their sum;

But $\frac{5x^2}{7} : \frac{12x}{7} :: 35 : 3$ by question;

That is $(\frac{5x^2}{7} \times 3 = \frac{15x^2}{7} = \frac{12x}{7} \times 35)$

Or (by division) $\frac{15x^2}{7} = \frac{12x}{7} \times 35$

Th. $x = (4 \times 7 =) 28.$

QUESTION LXXX.

A and B severally cut the cards, so as to cut off less than they left; now what A left, added to what B cut off, make 50; also the cards left by both exceed those cut off by 64: How many did each cut off?

Suppose A cut off x , and B y cards;

Then A left $52 - x$, and B $52 - y$;

And the cards left by both $(= 52 - x + 52 - y = 104 - x - y)$;

From which if the cards cut off by both be taken; $104 - 2x - 2y$ remains:

But $52 - x + y = 50$ by question;

And $104 - 2x - 2y = 64$

But (from first) $104 - 2x + 2y = 100$, by multiplication,

And (by subtraction) $4y = (100 - 64 =) 36$

Th. $y = 9$;

And $(52 - 50 + 9 =) x = 11.$

QUESTION LXXXI.

Two pieces of cloth, of equal goodness, but of different lengths, were bought, the one for 5*l.* and the other for 6*l.* 10*s.* now if the lengths of both pieces were increased by 10, the numbers resulting will be in proportion as 5 to 6: How long was each piece, and how much did they cost a yard?

Suppose the price of 1 yard of each was x shillings;

Then $\frac{100}{x}$ will be the length of the least piece,

And $\frac{130}{x}$ ditto - - - - - greatest;

Now $\frac{100}{x} + 10 : \frac{130}{x} + 10 :: 5 : 6$ by question;

$$\text{Th. } \frac{600}{x} + 60 = \frac{650}{x} + 50;$$

$$\text{Or } 600 + 60x = 650 + 50x;$$

$$\text{Or } (60x - 50x) = (650 - 600) = 50;$$

$$\text{Th. } - - - - x = \left(\frac{50}{10}\right) = 5;$$

And $\left\{ \begin{array}{l} \frac{100}{5} = 20 \\ \frac{130}{5} = 26 \end{array} \right\}$ were the lengths of the pieces.

QUESTION LXXXII.

A vintner having 75 quarts of wine worth 6*d.* a quart, would mix as much of another sort of wine (which is worth 10*d.* a quart) with it, as is requisite to make the mixture worth 7*d.* a quart: How much must he take?

Suppose he must use y quarts;

Also let $6 = a$; $10 = b$; $75 = p$; and $7 = m$:

$$\text{Then } ap + by = (p + y \times m) = mp + my,$$

$$\text{Or } by - my = mp - ap;$$

$$\text{Th. } - - - - y = \left(\frac{mp - ap}{b - m}\right) = \frac{m - a}{b - m} \times p.$$

$$\text{In this example } y = \left(\frac{7 - 6}{10 - 7}\right) \times 75 = \frac{1}{3} \times 75 = 25.$$

QUES-

QUESTION LXXXIII.

What 3 numbers are those whose sum is 78; and of the first is to $\frac{1}{3}$ of the second as 1 to 2; also $\frac{1}{4}$ of the second is to $\frac{1}{5}$ of the third as 2 to 3?

If x , y , and z be the numbers required,

Then $\frac{x}{3} : \frac{y}{4} :: 1 : 2$; Th. $\frac{2x}{3} = \frac{y}{4}$;

And $\frac{y}{4} : \frac{z}{5} :: 2 : 3$; Th. $\frac{3y}{4} = \frac{2z}{5}$;

Whence $\frac{8x}{3} = y$; And $y = \frac{8z}{15}$;

But $\frac{8x}{3} = \frac{8z}{15}$; Th. $5x = z$;

But $x + y + z = 78$ by question;

That is $x + \frac{8x}{3} + 5x = 78$,

Or $3x + 8x + 15x = (78 \times 3 =) 234$;

Or $26x = 234$;

Th. $x = \left(\frac{234}{26} =\right) 9$.

QUESTION LXXXIV.

'Tis required to find the value of y in the following equations, viz.

$\left. \begin{array}{l} ax + by = m \\ cx + dy = n \end{array} \right\} \left\{ \begin{array}{l} \text{where } a, b, c, d, m, \text{ and } n, \text{ are} \\ \text{known numbers?} \end{array} \right.$

$\left. \begin{array}{l} \text{AND } \left\{ \begin{array}{l} cax + cby = cm \\ acx + ady = an \end{array} \right\} \text{ by multiplication;}$

And $cby - ady = cm - an$ by subtraction;

Th. $y = \frac{cm - an}{cb - ad}$.

QUES-

QUESTION LXXXV.

There are four numbers, the first of which is to the second, as the third to the fourth; the sum of the first and second is 20, and of the third and fourth 12; but the difference of the second and third is 4: What are those numbers?

If $x, y, u,$ and z be the numbers required;

Then $x : y :: u : z$; Or $xz = yu$;

$x + y = 20$; Or $y = 20 - x$;

$u + z = 12$; Or $z = 12 - u$;

$y - u = 4$; Or $y = u + 4$;

Now $u + 4 = 20 - x$;

Th. $x = 16 - u$;

But ($yu = xz$; That is) $u + 4 \times u = 16 - u \times 12 - u$,

Or $uu + 4u = 192 - 28u + uu$,

Or $4u = 192 - 28u$,

Or $(28u + 4u =) 32u = 192$;

Th. $u = 6$.

QUESTION LXXXVI.

Suppose that for every 10 sheep a farmer kept, he should plow an acre of land; and be allowed one acre of pasture for every 4 sheep: How many sheep may that person keep, who farms 700 acres of land?

If x be the number of sheep required,

Then $(10 : 1 :: x :) \frac{x}{10} = \text{acres plowed};$

And $(4 : 1 :: x :) \frac{x}{4} = \text{acres of pasture};$

Now $\frac{x}{10} + \frac{x}{4} = 700$ by question;

Or $(4x + 10x =) 14x = 700 \times 40$;

Th. $x = (50 \times 40 =) 2000$.

QUES-

QUESTION LXXXVII.

Fifteen guineas are to be divided among 4 persons, *A*, *B*, *C*, and *D*, in such a manner, that *A*'s share may be to *B*'s, as 2 to 3, *B*'s share to *C*'s as 4 to 5, and *C*'s share to *D*'s as 6 to 7: What had each?

If $x = A$'s share;

Then $(2 : 3 :: x : \frac{3x}{2}) = B$'s share;

And $(4 : 5 :: \frac{3x}{2} : \frac{3 \times 5x}{2 \times 4}) = C$'s share,

Also $(6 : 7 :: \frac{3 \times 5x}{2 \times 4} : \frac{3 \times 5 \times 7x}{2 \times 4 \times 6}) = \frac{5 \times 7x}{4 \times 4} = D$'s;

Now $x + \frac{3x}{2} + \frac{15x}{8} + \frac{35x}{16} = (15 \times 21 =) 315$;

And $(16x + 24x + 30x + 35x =) 105x = 315 \times 16$;

Therefore $x = (3 \times 16 =) 48$.

QUESTION LXXXVIII.

The value of y in the following equations is required, viz.

$$\left. \begin{aligned} \frac{x}{a} + \frac{y}{b} &= m \\ \frac{x}{c} + \frac{y}{d} &= n \end{aligned} \right\} ?$$

Now $\left\{ \begin{aligned} x + \frac{ay}{b} &= am \\ x + \frac{cy}{d} &= cn \end{aligned} \right\}$ by multiplication;

And $\frac{ay}{b} - \frac{cy}{d} = am - cn$ by subtraction;

Or $day - bcy = \overline{am - cn} \times bd$;

Th. $y = \frac{am - cn}{da - bc} \times bd$.

QUES.

QUESTION LXXXIX.

'Tis required to divide the number 128 into 4 such parts, that the first added to 7, the second less 7, the third multiplied by 7, and the fourth divided by 7; shall be equal among themselves?

Let $x, y, z,$ be the parts required;

Then $u + 7 = x - 7 = 7y = \frac{z}{7}$ by question;

Th. $u + 14 = x; \frac{u + 7}{7} = y; \text{ and } u + 7 \times 7 = z;$

But $u + x + y + z = 128$ by question;

That is $u + u + 14 + \frac{u + 7}{7} + 7u + 49 = 128;$

Or $u + u + \frac{u + 7}{7} + 7u = (128 - 14 - 49 =) 65;$

Or $9u + \frac{u + 7}{7} = 65,$

Or $63u + u + 7 = (65 \times 7 =) 455,$

Or $64u = (455 - 7 =) 448;$

Th. $u = \left(\frac{448}{64}\right) = 7;$

Whence $x = 7 + 14 = 21;$

And $y = \left(\frac{7 + 7}{7}\right) = 2;$

Also $z = (7 + 7 \times 7) = 98.$

QUES.

QUESTION XC.

A gentleman had three horses, *A*, *B*, and *C*, and a saddle with its furniture worth 55*l*. now if the saddle be put on *A*'s back, he will be worth as much as *B* and *C*; if the saddle be put on *B*'s back, he will be worth twice as much as *A* and *C*; and if the saddle be put on *C*'s back, he will be worth thrice as much as *A* and *B*. What is each horse worth?

Suppose *x*, *y*, and *z*, to be the value of the three horses;

Then $x + 55 = y + z$,
 $y + 55 = (x + z) \times 2 = 2x + 2z$,
 $z + 55 = (x + y) \times 3 = 3x + 3y$ } by quest.

And $x = y + z - 55$

By equation, { first $x = y + z - 55$
 second $x = \frac{y + 55 - 2z}{2}$

third $x = \frac{z + 55 - 3y}{3}$

Th. $y + z - 55 = \frac{y + 55 - 2z}{2}$

Or $2y + 2z - 110 = y + 55 - 2z$

Whence $y = 165 - 4z$

Also $y + z - 55 = \frac{z + 55 - 3y}{3}$

Or $3y + 3z - 165 = z + 55 - 3y$

Whence $y = \frac{220 - 2z}{6}$

Now $165 - 4z = \frac{220 - 2z}{6}$

Or $990 - 24z = 220 - 2z$

Or $770 = 22z$

Th. $35 = z$

And $y = (165 - 140) = 25$
 $x = (25 + 35 - 55) = 5$

Q U E S -

QUESTION XC.

QUESTION XCI.

A and *B*, being at play, severally cut the cards, so as to take off more than they left; now it happened, that *A* cut off twice as many as *B* left; and *B* cut off 7 times as many as *A* left: How were the cards cut by each?

Suppose *A* cut off x cards, and *B* y cards;

Then *A* left $52 - x$;

And *B* $52 - y$.

But by question $\begin{cases} x = (52 - y) \times 2 = 104 - 2y, \\ y = (52 - x) \times 7 = 364 - 7x; \end{cases}$

Now from the second $7x = 364 - y$;

Th. $x = \frac{364 - y}{7}$;

But $\frac{364 - y}{7} = 104 - 2y$;

And $364 - y = (104 - 2y) \times 7 = 728 - 14y$;

Or $(14y - y) = 13y = (728 - 364) = 364$;

Th. $y = \left(\frac{364}{13}\right) = 28$;

And $x = \left(\frac{364 - 28}{7}\right) = \frac{336}{7} = 48$.

QUES.

QUESTION XCII.

Having first lost $\frac{1}{3}$ of my money at play, I won 3 times as much as I had left, $\frac{1}{2}$ as much money as I began with, and 50/. and then found, that I had as much above 100/. as the sum I began with was below 100/. What sum did I begin with?

Suppose x the sum began with;

Then $(x - \frac{x}{3}) = \frac{2x}{3}$ was left after $\frac{1}{3}x$ was lost;

And $(\frac{2x}{3} \times 3 + \frac{x}{2} + 50 =) \frac{5x}{2} + 50$ was afterwards won;

Th. $(\frac{2x}{3} + \frac{5x}{2} + 50 =) \frac{19x}{6} = 50$ } was the sum in hand after winning;

But $\frac{19x}{6} + 50 - 100 = 100 - x$ by question;

Or $-\frac{19x}{6} - 50 = 100 - x$;

Or $-\frac{19x}{6} = 150 - x$;

And (by multip.) $19x = (150 - x \times 6 =) 900 - 6x$;

Or $(19x + 6x =) 25x = 900$;

Th. $x = 36$.

QUESTION XCIII.

What 3 numbers are those, the difference of whose differences is 2, the sum of the two first numbers is equal to the third, and the sum of the first and third is to the second as 17 to 8?

If x , y , and z , be the numbers required;

Then $x + z : y :: 17 : 8$ by question;

That is $8x + 8z = 17y$;

But $x + y = z$ by question;

And $8x + 8y = 8z$;

Th. $8x + 8x + 8y = 17y$;

Or $16x = 9y$;

Th. $x = \frac{9y}{16}$;

And $\left(\frac{9y}{16} + y\right) = \frac{25y}{16} = z$;

Now $z - y = \left(\frac{25y}{16} - y\right) = \frac{9y}{16}$;

And $y - x = \left(y - \frac{9y}{16}\right) = \frac{7y}{16}$;

But (their diff.) $\frac{2y}{16} = 2$ by question;

Th. $y = 16$;

$$x = \frac{9 \times 16}{16} = 9;$$

And $z = \left(\frac{25 \times 16}{16}\right) = 25.$

QUESTION XCIV.

A lady being asked, how many yards of silk she had in her gown, replied, that if she had bought 2 yards more, at a price greater by three shillings a yard, the gown would have cost 64 shillings more than it did; and if she had bought 3 yards more at an advance of 2 shillings a yard, it would have cost 68 shillings more than it did: How many yards were in it?

If x be the yards in the gown, and y shillings the price a yard;

Then the gown cost xy shillings;

But $(x+2) \times (y+3) = xy + 3x + 2y + 6 = xy + 64$;

And $(x+3) \times (y+2) = xy + 2x + 3y + 6 = xy + 68$;

That is $3x + 2y = (64 - 6) = 58$,

And $2x + 3y = (68 - 6) = 62$;

Wh. (by sub.) $y - x = (62 - 58) = 4$;

Th. $y = x + 4$;

But $3x + x + 4 \times 2 = 58$;

That is $5x + 8 = 58$;

Or $5x = 50$;

Th. $x = 10$.

QUESTION XCV.

Three men, *A*, *B*, and *C*, enter partnership; *A* paid in as much as *B* and $\frac{1}{3}$ of *C*; *B* paid in as much as *C* and $\frac{1}{3}$ of *A*; and *C* paid in 10% and $\frac{1}{3}$ of *A*: What did each man contribute to the stock?

Suppose *A*, paid x ; *B*, y ; and *C*, z pounds;

$$\text{Then (by question) } \begin{cases} x = (y + \frac{z}{3}) = \frac{3y + z}{3}; \\ y = (z + \frac{x}{3}); \text{ Or } 3y - 3z = x; \\ z = (10 + \frac{x}{3}); \text{ Or } 3z - 30 = x; \end{cases}$$

$$\text{Whence } \frac{3y + z}{3} = 3y - 3z,$$

$$\text{Or } - 3y + z = 3y - 3z \times 3 = 9y - 9z;$$

$$\text{Th. } - z = \frac{6y}{10};$$

$$\text{Also } - \frac{3y + z}{3} = 3z - 30,$$

$$\text{Or } - 3y + z = 9z - 90,$$

$$\text{Th. } \frac{3y + 90}{8} = z;$$

$$\text{But } - \frac{3y + 90}{8} = \frac{6y}{10},$$

$$\text{Or } 30y + 900 = 48y;$$

$$\text{Th. } (\frac{900}{18} = 50) = y;$$

$$z = (\frac{6 \times 50}{10} =) 30;$$

$$\text{And } x = (3 \times 30 - 30 =) 60.$$

QUESTION XCVI.

Three persons, *A*, *B*, and *C*, being at play, *A* won $\frac{1}{2}$ the money that *B* and *C* had, and carried off 153*l*. now if *B* had won $\frac{1}{3}$ of what *A* and *C* had, or if *C* had won $\frac{1}{4}$ of what *A* and *B* had, they would have carried off the same sum? How much had each?

If *x* be *A*'s money; *y*, *B*'s; and *z*, *C*'s:

$$\text{Then (by question) } \left\{ \begin{array}{l} x + \frac{y+z}{2} = 153; \text{ Or } 2x + y + z = 306; \\ y + \frac{x+z}{3} = 153; \text{ Or } 3y + x + z = 459; \\ z + \frac{x+y}{4} = 153; \text{ Or } 4z + x + y = 612; \end{array} \right.$$

$$\text{Th. } \frac{306-y-z}{2} = (x=) 459-3y-z,$$

$$\text{Or } 306-y-z = 918-6y-2z,$$

$$\text{Th. } - - y = \frac{612-z}{5};$$

$$\text{Also } \frac{306-y-z}{2} = (x=) 612-4z-y,$$

$$\text{Or } 306-y-z = 1224-8z-2y,$$

$$\text{Th. } - - y = 918-7z;$$

$$\text{But } \frac{612-z}{5} = 918-7z,$$

$$\text{Or } 612-z = 4590-35z,$$

$$\text{Th. } - - z = \left(\frac{3978}{34} = \frac{1989}{17} = \right) 117;$$

$$y = (918-819 =) 99;$$

$$x = 459-297-117 = 45.$$

QUESTION XC VII.

To find three numbers, so that $\frac{1}{2}$ the first, $\frac{1}{3}$ of the second, and $\frac{1}{4}$ of the third, shall be equal to 62; $\frac{1}{3}$ of the first, $\frac{1}{4}$ of the second, and $\frac{1}{5}$ of the third, equal to 47; and $\frac{1}{4}$ of the first, $\frac{1}{5}$ of the second, and $\frac{1}{6}$ of the third, equal to 38?

If x , y , and z , represent the numbers required;

$$\text{Th. } \frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 62; \text{ or } 6x + 4y + 3z = (62 \times 12 =) 744;$$

$$\frac{x}{3} + \frac{y}{4} + \frac{z}{5} = 47; \text{ or } 20x + 15y + 12z = (47 \times 60 =) 2820;$$

$$\frac{x}{4} + \frac{y}{5} + \frac{z}{6} = 38; \text{ or } 15x + 12y + 10z = (38 \times 60 =) 2280;$$

$$\text{But } 120x + 80y + 60z = (744 \times 20 =) 14880$$

$$100x + 75y + 60z = (2820 \times 5 =) 14100$$

$$\text{And } 90x + 72y + 60z = (2280 \times 6 =) 13680$$

$$\text{Th. } 20x + 5y = (14880 - 14100 =) 780 \quad \left. \begin{array}{l} \text{by sub.} \\ \text{by sub.} \end{array} \right\}$$

$$\text{And } 10x + 3y = (14100 - 13680 =) 420$$

$$\text{Now } - - 20x + 6y = (420 \times 2 =) 840 \text{ by mul.}$$

$$\text{Th. } - - - y = (840 - 780 =) 60 \text{ by sub.}$$

$$\text{But } - - 10x + (3 \times 60 =) 180 = 420,$$

$$\text{Or } - - - 10x = (420 - 180 =) 240;$$

$$\text{Th. } - - - x = \left(\frac{240}{10} = \right) 24;$$

$$\text{Also } - - 6 \times 24 + 4 \times 60 + 3z = 744,$$

$$\text{Or } - - - 144 + 240 + 3z = 744,$$

$$\text{Or } - 3z = (744 - 144 - 240 =) 360;$$

$$\text{Th. } - - - z = \left(\frac{360}{3} = \right) 120.$$

QUES-

QUESTION XCVIII.

A gentleman left some money to be divided among four servants, so that the share of the first should be equal to $\frac{1}{2}$ the sum of the shares of the other three; that the share of the second should be $\frac{1}{3}$ of the sum of the shares of the other three; and that the share of the third should be $\frac{1}{4}$ of the sum of the shares of the other three; now, upon dividing the money in this manner, it was found, that the share of the first exceeded that of the last by 14 pounds: What was the share of each person?

If x , y , u , and z , represent the shares required?

Then $x = \frac{y+u+z}{2}$; or $2x = y+u+z$,
 $y = \frac{x+u+z}{3}$; or $3y = x+u+z$,
 $u = \frac{x+y+z}{4}$; or $4u = x+y+z$,
 $x - z = 14$; or $x = 14 + z$,
 by question.

Now by adding $\left\{ \begin{matrix} x \\ y \\ u \end{matrix} \right\}$ the $\left\{ \begin{matrix} \text{first} \\ \text{second} \\ \text{third} \end{matrix} \right\}$ equat. $\left\{ \begin{matrix} 3x = x+y+u+z \\ 4y = x+y+u+z \\ 5u = x+y+u+z \end{matrix} \right.$

Th. $3x = 47$, And $3x = 5u$;

Whence $\frac{3x}{4} = y$, And $\frac{3x}{5} = u$;

But (by the fourth) $x - 14 = z$;

And (by the first) $2x = \frac{3x}{4} + \frac{3x}{5} + x - 14$,

Which (multiplied by 20) $40x = 15x + 12x + 20x$
 $[-280$;

Or (by transposition) $280 = (47x - 40x) 7x$;

Th. $40 = x$.

QUESTION XCIX.

A shepherd, in time of war, was plundered by a party of soldiers, who took $\frac{1}{4}$ of his flock, and $\frac{1}{4}$ of a sheep; another party took from him $\frac{1}{3}$ of what he had left, and $\frac{1}{3}$ of a sheep; also a third party took $\frac{1}{2}$ of what were then left, and $\frac{1}{2}$ a sheep; which being done, he had but 25 sheep left: How many had he at first?

Suppose he had x sheep at first;

Then the 1st party took $\left(\frac{x}{4} + \frac{1}{4}\right) = \frac{x+1}{4}$;

And there remained $\left(x - \frac{x+1}{4} = \frac{3x-1}{4}\right)$;

The 2d party took $\left(\frac{3x-1}{3 \times 4} + \frac{1}{3} = \frac{3x+3}{3 \times 4}\right) = \frac{x+1}{4}$;

And then remained $\left(\frac{3x-1}{4} - \frac{x+1}{4} = \frac{2x-2}{4}\right) = \frac{x-1}{2}$;

The 3d party took $\left(\frac{x-1}{2 \times 2} + \frac{1}{2}\right) = \frac{x+1}{4}$;

And then remained $\left(\frac{x-1}{2} - \frac{x+1}{4}\right) = \frac{x-3}{4}$;

But (by question) $\frac{x-3}{4} = 25$,

Or $x-3 = (25 \times 4) = 100$;

Th. $x = 103$.

Q U E S.

QUESTION C.

A trader maintained himself for 3 years, at the expence of 50*l*. a year; and in each of those years augmented that part of his stock, which was not so expended, by $\frac{1}{3}$ thereof; at the end of the third year his original stock was doubled: What had he at first?

Suppose his original stock was x *l*.

$$\text{Then } x - 50 = \left. \begin{array}{l} \text{the sum} \\ \text{traded with} \\ \text{the 1st year,} \end{array} \right\}$$

$$\text{And } \frac{x - 50}{3} = \left. \begin{array}{l} \text{the sum} \\ \text{gained} \\ \text{therein;} \end{array} \right\}$$

$$\text{Th. } (x - 50 + \frac{x - 50}{3}) = \frac{4x - 200}{3} = \left. \begin{array}{l} \text{sum pos-} \\ \text{sessed at the} \\ \text{end of it;} \end{array} \right\}$$

$$\text{Also } (\frac{4x - 200}{3} - 50) = \frac{4x - 350}{3} = \left. \begin{array}{l} \text{sum traded} \\ \text{with the} \\ \text{2d year,} \end{array} \right\}$$

$$\text{And } \frac{4x - 350}{9} = \left. \begin{array}{l} \text{sum gained} \\ \text{therein;} \end{array} \right\}$$

$$\text{Th. } (\frac{4x - 350}{3} + \frac{4x - 350}{9}) = \frac{16x - 1400}{9} = \left. \begin{array}{l} \text{sum pos-} \\ \text{sessed at the} \\ \text{end of it;} \end{array} \right\}$$

$$\text{Again } (\frac{16x - 1400}{9} - 50) = \frac{16x - 1850}{9} = \left. \begin{array}{l} \text{sum traded} \\ \text{with the} \\ \text{3d year,} \end{array} \right\}$$

$$\text{And } \frac{16x - 1850}{27} = \left. \begin{array}{l} \text{sum gained} \\ \text{therein;} \end{array} \right\}$$

$$\text{Th. } \frac{16x - 1850}{9} + \frac{16x - 1850}{27} = \left. \begin{array}{l} \text{sum pos-} \\ \text{sessed at the} \\ \text{end of it;} \end{array} \right\}$$

$$\text{That is } \frac{64x - 7400}{27} = 2x \text{ by quest.}$$

$$\text{Or } 64x - 7400 = 54x,$$

$$\text{Or } 10x = 7400;$$

$$\text{Th. } x = 740.$$

QUESTION XII.

A, *B*, and *C*, playing together, the first game *A* lost to *B*, as much as *B* began with, and to *C*, as much as *C* began with; the second game *B* lost to *C*, as much as *C* then had, and to *A*, as much as *A* had left; the third game, *C* lost to *A*, as much as *A* then had, and to *B*, as much as *B* had left; and then each had 8 shillings: What had each at first?

Suppose *A* had x ; *B*, y ; and *C*, z shillings;

Then
after the 1st game
 $\left\{ \begin{array}{l} A \text{ had } x-y-z, \\ B \text{ had } 2y, \\ C \text{ had } 2z; \end{array} \right.$

after the 2d game
 $\left\{ \begin{array}{l} A \text{ had } 2x-2y-2z, \\ B \text{ had } (2y-2z-x+y+z) = 3y-z-x, \\ C \text{ had } 4z; \end{array} \right.$

after the 3d game
 $\left\{ \begin{array}{l} A \text{ had } 4x-4y-4z, \\ B \text{ had } 6y-2z-2x, \\ C \text{ had } (4z-2x+2y+2z-3y+z+x) = 7z-x-y; \end{array} \right.$

but by quest.
 $\left\{ \begin{array}{l} 4x-4y-4z=8; \text{ Or } x=2+y+z; \\ 6y-2z-2x=8; \text{ Or } x=3y-z-4; \\ 7z-x-y=8; \text{ Or } x=7z-y-8; \end{array} \right.$

Whence $2+y+z=3y-z-4$.

Th. $z=y-3$;

Also $2+y+z=7z-y-8$,

Or $10+2y=6z$,

Th. $\frac{5+y}{3}=z$;

But $\frac{5+y}{3}=y-3$,

Or $5+y=3y-9$;

Th. $7=y$;

And $z=(7-3)=4$;
 $x=(2+7+4)=13$.

QUESTION CHL.

A , B , and C , (who had among them 480 shillings) went to play; the first game A lost $\frac{1}{2}$ his money equally to B and C ; the second game B lost $\frac{1}{3}$ of the money he then had, equally to A and C ; the third game C lost 40 shillings to A , and 40 shillings to B ; now upon counting their money, it appeared, that they had each an equal sum: What had each at first?

Suppose A had x ; B , y ; and C , z shillings;

Then after the 1st game

$$\left\{ \begin{array}{l} A \text{ had } \frac{x}{2}, \\ B \text{ had } y + \frac{x}{4}, \\ C \text{ had } z + \frac{x}{4}; \end{array} \right.$$

After the 2d game

$$\left\{ \begin{array}{l} A \text{ had } \left(\frac{x}{2} + \frac{y}{6} + \frac{x}{24} \right) = \frac{13x}{24} + \frac{y}{6}, \\ B \text{ had } \left(y + \frac{x}{4} - \frac{y}{3} - \frac{x}{12} \right) = \frac{2y}{3} + \frac{x}{6}, \\ C \text{ had } \left(z + \frac{x}{4} + \frac{y}{6} + \frac{x}{24} \right) = z + \frac{7x}{24} + \frac{y}{6}; \end{array} \right.$$

After the 3d game

$$\left\{ \begin{array}{l} A \text{ had } \left(\frac{13x}{24} + \frac{y}{6} + 40 \right) = \frac{13x + 4y + 960}{24}, \\ B \text{ had } \left(\frac{2y}{3} + \frac{x}{6} + 40 \right) = \frac{16y + 4x + 960}{24}, \\ C \text{ had } \left(z + \frac{7x}{24} + \frac{y}{6} - 80 \right) = \frac{24z + 7x + 4y - 1920}{24}; \end{array} \right.$$

But by quest.

$$\left\{ \begin{array}{l} \frac{13x + 4y + 960}{24} = \frac{16y + 4x + 960}{24}, \\ \frac{13x + 4y + 960}{24} = \frac{24z + 7x + 4y - 1920}{24}; \end{array} \right.$$

Or - - - $\left\{ \begin{array}{l} 13x + 4y = 16y + 4x, \\ 13x + 4y = 24z + 7x + 4y - 2880; \end{array} \right.$

That

QUESTION CIII. [Continued.]

That is $\begin{cases} 9x = 12y; & \text{Or } \frac{3x}{4} = y; \\ 6x = 24z - 2880; & \text{Or } \frac{480+x}{4} = z; \end{cases}$

But by quest. $480 = x + y + z;$

That is $480 = x + \frac{3x}{4} + \frac{480+x}{4},$

Or $1920 = (4x + 3x + 480 + x) 8x + 480,$

Or $1440 = 8x;$

Th. $180 = x,$

$y = \left(\frac{3 \times 180}{4} \right) = 135;$

And $z = \left(\frac{480 + 180}{4} \right) = 165.$

QUESTION CIV.

There are two numbers, in proportion as 3 to 5; the sum of whose squares is 306: What are those numbers?

Suppose x the lesser of those numbers;

Then $(3 : 5 :: x :) \frac{5x}{3} = \text{the greater};$

But $x^2 + \frac{5 \times 5 \times x^2}{3 \times 3} = 306 \text{ by question};$

That is $\left(\frac{9x^2 + 25x^2}{9} \right) \frac{34x^2}{9} = 306,$

Or $\frac{xx}{9} = \left(\frac{306}{34} \right) = 9;$

Th. $x^2 = (9 \times 9) = 9^2,$

And (the root thereof) $x = 9.$

QUES.

QUESTION CV.

If A and B together, can perform a piece of work in 8 days; A and C together in 9 days; and B and C in 10 days: How many days will it take each person alone to perform the same work?

Suppose A could perform it in x days, B in y , and C in z days; and because the work to be performed by all is the same, let it be represented by unity:

$$\begin{array}{lcl} \text{Then } (x : 1 :: 8 :) \frac{8}{x} = & \left. \begin{array}{l} \frac{8}{y} = \\ \frac{9}{x} = \\ \frac{9}{z} = \end{array} \right\} \text{the work performed by} & \begin{array}{l} A \text{ in 8 days;} \\ B \text{ in 8 days;} \\ A \text{ in 9 days;} \end{array} \\ \text{And } (y : 1 :: 8 :) \frac{8}{y} = & & B \text{ in 8 days;} \\ \text{Also } (x : 1 :: 9 :) \frac{9}{x} = & & A \text{ in 9 days;} \\ \text{And } (z : 1 :: 9 :) \frac{9}{z} = & & C \text{ in 9 days;} \\ \text{Again } (y : 1 :: 10 :) \frac{10}{y} = & & B \text{ in 10 days;} \\ \text{And } (z : 1 :: 10 :) \frac{10}{z} = & & C \text{ in 10 days;} \end{array}$$

$$\begin{array}{lcl} \text{But } \frac{8}{x} + \frac{8}{y} = 1; & \text{Or } \frac{1}{x} + \frac{1}{y} = \frac{1}{8} & \\ \text{And } \frac{9}{x} + \frac{9}{z} = 1; & \text{Or } \frac{1}{x} + \frac{1}{z} = \frac{1}{9} & \\ \text{Also } \frac{10}{y} + \frac{10}{z} = 1; & \text{Or } \frac{1}{y} + \frac{1}{z} = \frac{1}{10} & \end{array} \left. \vphantom{\begin{array}{l} \frac{8}{x} + \frac{8}{y} = 1 \\ \frac{9}{x} + \frac{9}{z} = 1 \\ \frac{10}{y} + \frac{10}{z} = 1 \end{array}} \right\} \text{by question:}$$

$$\text{Th. (their sum)} \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \times 2 = \frac{1}{8} + \frac{1}{9} + \frac{1}{10};$$

$$\text{Or (by division)} \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{16} + \frac{1}{18} + \frac{1}{20};$$

But

QUESTION CVI. [Continued.]

But because $\begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{1}{8} \\ \frac{1}{x} + \frac{1}{z} = \frac{1}{9} \\ \frac{1}{y} + \frac{1}{z} = \frac{1}{10} \end{cases}$ Therefore $\begin{cases} \frac{1}{x} = \frac{1}{16} + \frac{1}{18} + \frac{1}{20} - \frac{1}{9} \\ \frac{1}{y} = \frac{1}{16} + \frac{1}{18} + \frac{1}{20} - \frac{1}{9} \\ \frac{1}{z} = \frac{1}{16} + \frac{1}{18} + \frac{1}{20} - \frac{1}{9} \end{cases}$

Or - $\begin{cases} \frac{1}{x} = \left(\frac{1}{18} + \frac{1}{20} - \frac{1}{16} \right) = \frac{10 \times 8 + 9 \times 8 - 9 \times 10}{9 \times 10 \times 8 \times 2} \\ \frac{1}{y} = \left(\frac{1}{16} + \frac{1}{20} - \frac{1}{18} \right) = \frac{10 \times 9 + 8 \times 9 - 8 \times 10}{9 \times 10 \times 8 \times 2} \\ \frac{1}{z} = \left(\frac{1}{16} + \frac{1}{18} - \frac{1}{20} \right) = \frac{9 \times 10 + 8 \times 10 - 8 \times 9}{9 \times 10 \times 8 \times 2} \end{cases}$

Th. $\begin{cases} z = \left(\frac{9 \times 10 \times 8 \times 2}{10 \times 8 + 9 \times 8 - 9 \times 10} \right) = 23 \frac{7}{11} \\ y = \left(\frac{9 \times 10 \times 8 \times 2}{10 \times 9 + 8 \times 9 - 8 \times 10} \right) = 17 \frac{2}{11} \\ x = \left(\frac{9 \times 10 \times 8 \times 2}{9 \times 10 + 8 \times 10 - 8 \times 9} \right) = 14 \frac{4}{11} \end{cases}$

QUESTION CVI.

How many yards were there in a piece of cloth, that cost 15 guineas, the price of a yard being to the number of yards as 5 to 7?

Suppose there was x yards;

Then - $(7:5::x:)\frac{5x}{7}$ = price of a yard;

And - $(x \times \frac{5x}{7}) = \frac{5xx}{7} = 15 \times 21$ by question;

Th. (by division) - $\frac{xx}{7} = (3 \times 21 =) 3^2 \times 7$.

And (by multiplication) $xx = 3^2 \times 7^2$;

Th. (the root thereof) $x = (3 \times 7 =) 21$.

Q U E S.

[QUESTION CVII.]

If a vintner mixes sherry and sack in quantities proportional as 3 to 1, the mixture will be worth but 56 pence a gallon; but if the proportion were as 5 to 3, it would be worth 60 pence a gallon: Required the price of each wine?

Suppose there were x gallons of sherry,
And that the sherry was y , and the sack z pence a gallon;

Then $(3 : 1 :: x :) \frac{x}{3} = \left. \begin{array}{l} \text{gallons of sack in the first} \\ \text{mixture,} \end{array} \right\}$

Th. $(x + \frac{x}{3}) = \frac{4x}{3} = \text{gallons of both in ditto;}$

Also $(5 : 3 :: x :) \frac{3x}{5} = \left. \begin{array}{l} \text{gallons of sack in the second} \\ \text{mixture,} \end{array} \right\}$

Th. $(x + \frac{3x}{5}) = \frac{8x}{5} = \text{gallons of both in ditto:}$

Whence $- xy + \frac{xz}{3} = (\frac{4x}{3} \times 56 =) \frac{224x}{3},$

And $- xy + \frac{3xz}{5} = (\frac{8x}{5} \times 60 = \frac{480x}{5} =) 96x;$

That is $- y + \frac{z}{3} = \frac{224}{3} \left. \begin{array}{l} \\ \end{array} \right\} \text{by division;}$

And $- y + \frac{3z}{5} = 96 \left. \begin{array}{l} \\ \end{array} \right\}$

Also $- \frac{3z}{5} - \frac{z}{3} = 96 - \frac{224}{3} \text{ by subtraction;}$

Or $- \frac{4z}{15} = \frac{64}{3};$

Th. $- z = \frac{64 \times 15}{4 \times 3} = 16 \times 5 = 80;$

But $- y + \frac{3 \times 80}{5} = 96; \text{ Or } y + 3 \times 16 = 96;$

Th. $- y = (96 - 48 =) 48.$

Q U E S.

QUESTION CVIII.

Three workmen can severally do a piece of work in the following times, *viz.* *A* can perform it once in 3 weeks, *B* thrice in 8 weeks, and *C* five times in 12 weeks : In what time will it be done if they all work thereat ?

Or generally,

The forces of several agents being given, to determine (*x*) the time wherein they will jointly produce a given effect *E* ?

If $\left\{ \begin{array}{l} A \\ B \\ C \\ \&c. \end{array} \right\}$ can perform $E \left\{ \begin{array}{l} a \\ b \\ c \end{array} \right\}$ times in the time $\left\{ \begin{array}{l} a. \\ b. \\ c. \end{array} \right\}$

Then $\left\{ \begin{array}{l} a : a :: x : \frac{ax}{a} = \\ b : b :: x : \frac{bx}{b} = \\ c : c :: x : \frac{cx}{c} = \end{array} \right\}$ the effect which will be produced in the time *x* by $\left\{ \begin{array}{l} A. \\ B. \\ C. \end{array} \right\}$

That is $\frac{ax}{a} + \frac{bx}{b} + \frac{cx}{c} = E;$

Th. $x = \frac{E}{\frac{a}{a} + \frac{b}{b} + \frac{c}{c}}.$

In this example $x = \left(\frac{1}{\frac{1}{3} + \frac{1}{8} + \frac{1}{12}} \right) = \frac{24}{8+9+10} = \frac{24}{27} = \frac{8}{9}$

QUES.

QUESTION CIX.

If two post-boys *A* and *B*, being 59 miles asunder, set out to meet each other; *A* going 7 miles in 2 hours, and *B* (who began his journey 1 hour later than *A*) going 8 miles in 3 hours: How many miles will each travel before they meet?

Or generally,

The velocities or celerities of two moveable bodies *A* and *B*, tending, in opposite directions, to the same place; together with the distance or interval of the places; and times, from, and in which, they begin to move, being given: To find the place of their meeting?

Let the distance of *A* from the place of meeting be x ;

————— *A* from *B* ————— d ;

And the interval of time between their setting out be t ;

Now if *A* move thro' the space a , in the time a ;

And *B* ————— b , ————— b :

Then $(a : a :: x : \frac{ax}{a})$ = the time *A* moves,

And $(b : b :: d - x : \frac{d - x \times b}{b})$ = ditto *B*:

But $\frac{ax}{a} - \frac{d - x \times b}{b} = t$ by question;

Or $bax - abd + abx = abt$ by multiplication;

Or (by addition) $bax + abx = abt + abd$;

Th. $x = \frac{bt + bd}{ba + ab} \times a$.

In the above example $x = \left(\frac{8 \times 1 + 3 \times 59}{8 \times 2 + 7 \times 3} \times 7 \right) = 35$.

QUES-

QUESTION CX.

The velocities or celerities of 2 moveable bodies A and B , tending in the same direction, to the same place; together with the distance or interval, of the places, and times, from, and in which, they begin to move, being given: To find the place where they will be together?

Let $\left\{ \begin{array}{l} \text{the distance of } A \text{ (the hindermost body) from} \\ \text{the required place be } x, \\ \text{the distance of } A \text{ from } B \text{ be } d, \\ \text{the interval of time between their setting out be } t; \end{array} \right.$

Now if A moves thro' the space $\left\{ \begin{array}{l} a \\ b \end{array} \right\}$ in the time $\left\{ \begin{array}{l} a \\ b \end{array} \right\}$:
And B moves thro' the space $\left\{ \begin{array}{l} a \\ b \end{array} \right\}$ in the time $\left\{ \begin{array}{l} a \\ b \end{array} \right\}$:

(But $\frac{a}{a} - \frac{b}{b}$ or the question will be impossible.)

Then $(a : a :: x : \frac{ax}{a}) = \text{the time } A \text{ moves,}$

And $(b : b :: x - d : \frac{x-d}{b} \times b) = \text{ditto } B:$

Now CASE I. If A sets out before B ;

Then $\frac{ax}{a} - \frac{x-d}{b} \times b = t;$

Or $bax - abx + abd = abt;$

Or $bax - abx = abt - abd;$

Th $x = \frac{bt - bd}{ba - ab} \times a.$

But CASE II. If B sets out before A ;

Then $\frac{x-d}{b} \times b - \frac{ax}{a} = t,$

Or $abx - bax = abt + abd;$

Th $x = \frac{bt + bd}{ab - ba} \times a.$

QUES.

QUESTION CX. [Continued.]

EXAMPLE to CASE I.

B, travelling 5 miles in 2 hours, sets out 4 hours after *A*, who was 59 miles behind him, and goes 10 miles in 3 hours: How many miles must *A* go to overtake *B*?

$$\text{Here } x = \left(\frac{5 \times 4 - 2 \times 59}{5 \times 3 - 10 \times 2} \right) \times 10 = \frac{-98}{-5} \times 10 = 98 \times 2 = 196.$$

EXAMPLE to CASE II.

B, travelling 5 miles in 2 hours, sets out 4 hours before *A*, who was 59 miles behind him, and goes 10 miles in 3 hours: How many miles must *A* travel to overtake *B*?

$$\text{Here } x = \left(\frac{5 \times 4 + 2 \times 59}{10 \times 2 - 5 \times 3} \right) \times 10 = \frac{138}{5} \times 10 = 138 \times 2 = 276.$$

QUES.

QUESTION CXI.

The weight (w) of any mixture, together with the specific gravity thereof (s), and of each of the two things mixed (a , b), being given: To find the quantity of each of the things mixed?

Let x be the weight of that simple whose specific gravity a is greatest;

Then $w-x$ = the weight of the other simple;

$$\left. \begin{array}{l} \text{But } \frac{x}{a} = \\ \frac{w-x}{b} = \end{array} \right\} \begin{array}{l} \text{the magnitude of the body} \\ \text{whose weight is} \end{array} \left\{ \begin{array}{l} x, \\ w-x, \\ w; \end{array} \right.$$

And $\frac{w}{s} =$

Whence $\frac{x}{a} + \frac{w-x}{b} = \frac{w}{s};$

Or $sbx + saw - sax = abw;$

Or $saw - abw = sax - sbx;$

Th. $\frac{saw - abw}{s - b} = x.$

EXAMPLE.

What quantity of gold is there, in a mixture of gold and silver whose weight is 85 ounces, and specific gravity 15; the specific gravity of gold being 19; and of silver 10 $\frac{1}{2}$?

Here $x = \left(\frac{15 - \frac{1}{2} \times 19 \times 85}{19 - \frac{1}{2} \times 15} = \frac{9 \times 19 \times 85}{17 \times 15} = 19 \times 3 = \right) 57.$

QUES-

QUESTION CXII.

Two partners, *A* and *B*, dividing their gain (60*l*.) *B* took 20; *A*'s money continued in trade 4 months, and if the number 50 be divided by *A*'s money, the quotient will be the time that *B*'s money (*viz.* 100*l*.) continued in trade: What was *A*'s money?

Suppose *A*'s money was *x* pounds;

Then $\frac{50}{x}$ = the time *B*'s money was in trade;

And $4x + \frac{50}{x} \times 100 : 60 :: \frac{50}{x} \times 100 : 20$;

That is $80x + \frac{100000}{x} = \frac{300000}{x}$,

Or - $80xx + 100000 = 300000$,

Or - - - - $80xx = (300000 - 100000) = 200000$,

Or - - - - $xx = \frac{200000}{80} = 2500$;

Th. (the root) - $x = 50$.

QUESTION CXIII.

What 2 numbers are those, whose sum is to the greater as 11 to 7; and the difference of their squares is 132?

If *x* = the greater, and *y* the lesser number;

Then $\frac{x+y}{x} : x :: 11 : 7$

That is $(x+y) \times 7 = 11x$ } by question :

And - - - - $xx - yy = 132$

But (by the first) - - $7y = 11x - 7x = 4x$

Th. - - - - $y = \frac{4x}{7}$;

And - - - - $yy = \frac{16xx}{49}$;

Now (by the second) $xx - \frac{16xx}{49} = 132$,

Or $(49xx - 16xx) = 132 \times 49$,

Or (by division) - - $xx = (4 \times 49) = 2^2 \times 7^2$;

Th. (the root thereof) - $x = (2 \times 7) = 14$.

Q U E S.

QUESTION CXIV.

'Tis required to find two numbers, whose difference may be to the lesser, as 48 is to the greater; and to the greater, as 3 is to the lesser?

Suppose x the greater, and y the lesser;

Then $x - y : y :: 48 : x$ } by question :

And $x - y : x :: 3 : y$ }

That is $x - y \times x = 48y$;

And $x - y \times y = 3x$;

The product of which two equations will be

$$(x - y)^2 \times xy = (48 \times 3)xy;$$

Whence $x - y = (48 \times 3) = 144$;

And $x - y = 12$ by extraction;

Th. $x = 12 + y$;

(From first) $144 + 12y = 48y$;

Whence $144 = 36y$;

And $4 = y$;

Th. $x = (12 + 4) = 16$.

QUESTION CXV.

'Tis required to divide 21 into two such parts, that the quotient of the greater part, divided by the lesser, may be to the quotient of the lesser part, divided by the greater, as 25 to 4?

If x be the greater part,

Then $21 - x$ is the lesser;

And $\frac{x}{21 - x} : \frac{21 - x}{x} :: 25 : 4$ by question;

That is $\frac{4x}{21 - x} = \frac{25 \times 21 - x}{x}$;

Or $4xx = 25 \times 21 - x^2$;

Whence $2x = (5 \times 21 - x) = 105 - 5x$;

And $7x = 105$;

Th. $x = 15$.

QUES-

QUESTION CXVI.

What two numbers are those, whose sum is to the greater as 7 to 5, and their sum multiplied by the lesser produces 126?

If x be the greater, and y the lesser number;

Then $x + y : x :: 7 : 5$

That is $\frac{x}{x+y} = \frac{5}{7}$ } by question :

And $x \times y = 126$

The product of which two equations is $xy = (\frac{5}{7} \times 126 = 5 \times 18 =) 90 :$

But $(y \times x + y =) xy + yy = 126 ;$

Whence $90 + yy = 126,$

Or $yy = (126 - 90 =) 36 ;$

Th. (its root) $y = 6.$

QUESTION CXVII.

What two numbers are those, whose sum, multiplied by the greater, produces 120; and by the lesser, 105?

If x be the greater, and y the lesser number;

Then $(x + y \times x =) xx + xy = 120$ } by quest.

And $(x + y \times y =) xy + yy = 105$

The sum of which equations is $xx + 2xy + yy = 225 ;$

Th. its square root $x + y = 15 ;$

But (by first) $(x + y \times x =) 15x = 120 ;$

Th. $x = 8 ;$

And $y = 7.$

QUES-

QUESTION CXVIII.

What two numbers are those, whose product is 63; and the square of their sum is to the square of their difference, as 64 to 1?

Suppose x = the greater, and y = the lesser of those numbers;

Then $xy = 63$, } by question.

And $x+y : x-y :: 64 : 1$,

Or $x+y : x-y :: \sqrt{64} : \sqrt{1}$;

By last - $x+y = (x-y \times 8) = 8x - 8y$,

Or - $9y = 7x$;

Th. - $y = \frac{7x}{9}$;

But $(xy) = \frac{7xx}{9} = 63$ by first,

Or - $xx = 9 \times 9$,

Th. - $x = 9$.

QUESTION CXIX.

What two numbers are those, whose sum, multiplied by the greater produces 209, and by their difference 57?

If x be the greater, and y the lesser number;

Then - $(x+y \times x) = xx + xy = 209$ } by qu

And - $(x+y \times x-y) = xx - yy = 57$

The diff. of which equations is $xy + yy = 152$;

And the sum of the } $xx + 2xy + yy = 361$;

first and last - } $x + y = 19$;

Th. the root thereof - $x + y = 19$;

But by first - $(x+y \times x) = 19x = 209$;

Th. - $x = \left(\frac{209}{19}\right) = 11$;

And - $y = 8$.

QUESTION CXX.

'Tis required to divide 16 into 2 such parts, that the squares of those parts may be in proportion as 25 to 9?

Let x be the greater part;

Then $16 - x$ will be the lesser;

Whence $xx : 16 - x^2 :: 25 : 9$;

That is $9xx = 25 \times 16 - x^2$;

Th. $- - 3x = (5 \times 16 - x) = 80 - 5x$;

And $(3x + 5x) = 8x = 80$,

Th. $- - x = 10$.

QUESTION CXXI.

What 2 numbers are those, whose sum being multiplied by the greater, and the product divided by the lesser, quotes 24; but if their sum be multiplied by the lesser, and the product divided by the greater, the quotient will be but 6?

Suppose $x =$ the greater, and $y =$ the lesser number;

Then by question $\begin{cases} x+y \times \frac{x}{y} = 24, \\ x+y \times \frac{y}{x} = 6; \end{cases}$

The product of these equations is $\overline{x+y}^2 = (24 \times 6) = 144$;

Th. $- - - - - x+y = 12$;

But $- (by\ first\ \overline{x+y} \times \frac{x}{y} =) \frac{12x}{y} = 24$;

Th. $- - - - - x = 2y$;

But $- - (x+y = 2y+y =) 3y = 12$;

Th. $- - - - - y = 4$.

QUES.

QUESTION CXXII.

'Tis required to find three numbers, x , y , and z , so that, x multiplied by y , may be 12; x , multiplied by z , 18; and y , multiplied by z , 24?

$$\left. \begin{array}{l} \text{That is } xy = 12, \quad \text{Th. } x = \frac{12}{y} \\ \quad \quad \quad xz = 18, \quad \text{Th. } x = \frac{18}{z} \\ \quad \quad \quad yz = 24, \quad \text{Th. } y = \frac{24}{z} \end{array} \right\} \text{by question :}$$

But $\frac{12}{y} = \frac{18}{z}$; Or $\frac{z}{y} = \frac{3}{2}$:

And $2z = 3y$; Or $\frac{2z}{3} = y$:

Also $\frac{24}{z} = \frac{2z}{3}$; Or $\frac{12}{z} = \frac{z}{3}$:

Th. $36 = zz$; And $6 = z$.

QUESTION CXXIII.

The sum of three numbers (whose differences are equal) is 21; and the sum of the squares of those numbers is 155: What are those numbers?

If y equal the second number, and $x =$ the difference of those numbers;

Then $\left\{ \begin{array}{l} y-x \\ y \\ y+x \end{array} \right\}$ will be the numbers required; $\left\{ \begin{array}{l} yy-2xy+xx \\ yy \\ yy+2xy+xx \end{array} \right\}$ their squares:

And $3y =$ their sum; Also $3yy+2xx =$ sum of squares:

But $3y = 21$ by qu. And $3yy+2xx = 155$:

Th. $y = 7$; And $3 \times 49 + 2xx = 155$,

Or $2xx = (155 - 147) = 8$;

Th. $x = 2$.

QUESTION CXXIV.

What three numbers are those, whose differences are equal, whose sum is 15, and the sum of their cubes 495?

Let y represent the second of those numbers;

And x their difference;

Then $\begin{cases} y-x \\ y \\ y+x \end{cases}$ will be the numbers; $\begin{cases} y^3-3y^2x+3yx^2-x^3 \\ y^3 \\ y^3+3y^2x+3yx^2+x^3 \end{cases}$ their cubes.

And $3y = \text{sum}$; Also $3y^3 + 6yx^2 = \text{sum of cubes.}$

But $3y = 15$; And $3y^3 + 6yx^2 = 495$ by quest.

Th. $y = 5$; And $3 \times 125 + 30x^2 = 495$;

Now (by transp.) $30x^2 = (495 - 375) = 120$,

Or (by division) $x^2 = 4$;

Th. $x = 2$.

QUESTION CXXV.

The sum of three numbers, which have equal differences, is 21; and the sum of their squares is equal to their product? What are those numbers?

If $y = \text{the second number, and } x \text{ the difference of them;}$

Then $\begin{cases} y-x \\ y \\ y+x \end{cases}$ will be the numbers $\begin{cases} y^2-2xy+xx \\ y^2 \\ y^2+2xy+xx \end{cases}$ their squares required; and

And $3y = \text{their sum}$; Also $3y^2 + 2xx = \text{sum squares:}$

Again $(y-x \times y \times y+x) = y^2y - yxx = \text{their product:}$

But $3y = 21$ by quest. And $3y^2 + 2xx = y^3 - yx^2$:

Th. $y = 7$; And $3 \times 49 + 2xx = 343 - 7x^2$:

Or (by transposition) $7xx + 2xx = (343 - 147) = 196$;

Th. $3x = 14$;

And $x = \frac{14}{3}$.

QUES:

QUESTION CXXVI.

A, sets out from *London* for *York*; and at the same time, *B*, sets out from *York* for *London*; they meet in the road, and find *A*, had travelled 30 miles more than *B*, had; *A*, expected to reach *York* in 4 days, and *B*, to reach *London* in 9 days, each travelling at the same rate he had hitherto done: Required the distance of *London* and *York*?

Suppose - - - - - $x =$ miles *A* } had travelled;
 Then - - - - - $x - 30 =$ ditto *B* }
 And - - - - - $2x - 30 =$ distance of the two cities:
 Now - - - - - $\left\{ \begin{array}{l} x - 30 = \text{miles } A \\ x = \text{ditto } B \end{array} \right\}$ had yet to travel;

Therefore - - - - - $\left\{ \begin{array}{l} \frac{x - 30}{4} = \text{miles } A \\ \frac{x}{9} = \text{ditto } B \end{array} \right\}$ travels a day;

And $\left\{ \begin{array}{l} \left(\frac{x - 30}{4} \right) \frac{x}{1} (= \frac{4x}{x - 30} = \text{time } A) \\ \left(\frac{x}{9} \right) \frac{x - 30}{1} (= \frac{x - 30 \times 9}{x} = \text{time } B) \end{array} \right\}$ spent before they met;

Th. $\left\{ \begin{array}{l} \text{because those} \\ \text{times were} \\ \text{equal,} \end{array} \right\} \frac{4x}{x - 30} = \frac{x - 30 \times 9}{x}$ by question;

Or - - - - - $4xx = x - 30 \times 9$;

Th. - - - - - $2x = (x - 30 \times 3) = 3x - 90$,

And - - - - - $90 = x$.

QUESTION CXXVII

A and *B*, set out the one from *London* the other from *Lincoln* at the same time; they met in the road, and at that time *A* had travelled 20 miles more than *B*, and had gone in $6\frac{2}{3}$ days as far as *B* had gone in all; lastly, *B* continuing the same pace would get to *London* in 15 days: Required the distance of the two cities?

Let x = the time of their meeting,
And y = the miles travelled by *B* in 1 day;

Then the miles tra-
velled by

$$\left. \begin{array}{l} B = xy, \\ A = 15y \end{array} \right\} \begin{array}{l} \text{for } A \text{ had already} \\ \text{gone those miles} \\ \text{which } B \text{ could go in} \\ 15 \text{ days;} \end{array}$$

Also $\frac{15y}{x}$ = miles *A* went per day;

But $6\frac{2}{3} = \frac{20}{3}$;

Th. $\left(\frac{20}{3} \times \frac{15y}{x}\right) = \frac{100y}{x} = xy$ $\left\{ \begin{array}{l} \text{for } A \text{ went in } 6\frac{2}{3} \text{ days} \\ \text{as far as } B \text{ went in all;} \end{array} \right.$

Or $\frac{100}{x} = x$ by division,

Or $100 = xx$;

Th. $10 = x$;

But $15y = xy + 20$ $\left\{ \begin{array}{l} \text{for } A \text{ travelled 20} \\ \text{miles more than } B; \end{array} \right.$

That is $15y = 10y + 20$,

Or $(15y - 10y) = 5y = 20$;

Th. $y = 4$.

Now $(xy + 15y) = 10 \times 4 + 15 \times 4 = 25 \times 4 =$
100 is the distance required.

QUES.

QUESTION CXXVIII.

A prize being taken at sea, was equally divided among the ship's company; and each man received 1*l*. and the $\frac{1}{100}$ part of the remaining sum; now if the number of men, be added to the pounds that each man received, the square of that sum will be equal to four times the value of the prize: That value is required?

If the number of men $- - - = x,$

And the share of each $- - - = y;$

Then the prize was $- - - = xy:$

But $(x+y)^2 = xx + 2xy + yy = 4xy$ per quest;

And (subtract. $4xy$) $xx - 2xy + yy = 0,$

Th. $- - - - - x - y = 0;$ or $x = y:$

Now the prize was $- - - (xy =) xx;$

And each man's share $- - - = 1 + \frac{xx-1}{100}$ by quest;

That is $- - - (y =) x = 1 + \frac{xx-1}{100},$

Or $- - - - - x - 1 = \frac{xx-1}{100}:$

Now (dividing by $x-1$) $- - - 1 = \frac{x+1}{100},$

Or $- - - - - 100 = x + 1;$

Th. $- - - - - 99 = x.$

$$\begin{aligned} \frac{1}{2} &= \left(\frac{10}{20}\right) \\ \frac{1}{4} &= \left(\frac{10}{40}\right) \\ \frac{1}{8} &= \left(\frac{10}{80}\right) \end{aligned}$$

R 4

QUES.

QUESTION CXXIX.

What three numbers are those whose sum, multiplied by the lesser, produce $\frac{133}{90}$; by the mean $\frac{133}{80}$; and by the greater $\frac{133}{75}$?

If x = the greatest; y = the mean; and z = the least N^o. required.

$$\begin{cases} z \times x + y + z = \frac{133}{90} \\ \text{Then (by question)} \begin{cases} y \times x + y + z = \frac{133}{80} \\ x \times x + y + z = \frac{133}{75} \end{cases} \end{cases}$$

The sum of which 3 equations is $\left\{ \begin{array}{l} x + y + z \times x + y + z = \frac{133}{90} + \frac{133}{80} + \frac{133}{75} \end{array} \right.$

That is $x + y + z = \frac{17689}{3600}$

Th. $x + y + z = \frac{133}{60}$

Now (by Equation $\left\{ \begin{array}{l} \text{first) } z \times \frac{133}{60} = \frac{133}{90} \\ \text{2d) } y \times \frac{133}{60} = \frac{133}{80} \\ \text{3d) } x \times \frac{133}{60} = \frac{133}{75} \end{array} \right.$

Th. $\left\{ \begin{array}{l} z = \left(\frac{60}{90} \right) = \frac{2}{3} \\ y = \left(\frac{60}{80} \right) = \frac{3}{4} \\ x = \left(\frac{60}{75} \right) = \frac{4}{5} \end{array} \right.$

QUES.

QUESTION CXXX.

There are 3 numbers, x, y, z ; the sum of x and y multiplied by z will produce $\frac{57}{40}$; the sum of x and z multiplied by y , $\frac{13}{10}$; and the sum of y and z multiplied by x , $\frac{11}{8}$: What are those numbers?

$$\left. \begin{array}{l} \text{Now } (x+y) \times z = \frac{57}{40} \\ \text{And } (x+z) \times y = \frac{13}{10} \\ \text{Also } (y+z) \times x = \frac{11}{8} \end{array} \right\} \text{by question;}$$

The sum of which 3 equat. is $2xy + 2xz + 2yz = \frac{57}{40} + \frac{13}{10} + \frac{11}{8}$

That is - $2xy + 2xz + 2yz = \frac{164}{40}$

Th. - $xy + xz + yz = \frac{82}{40}$

If the three given equat. be severally taken from the last

$$\left\{ \begin{array}{l} xz = \left(\frac{82}{40} - \frac{57}{40} \right) = \frac{5}{8} \\ xz = \left(\frac{82}{40} - \frac{13}{10} \right) = \frac{3}{4} \\ yz = \left(\frac{82}{40} - \frac{11}{8} \right) = \frac{27}{40} \end{array} \right.$$

And the product of the 3 last equat. is $x^2 y^2 z^2 = \left(\frac{5}{8} \times \frac{3}{4} \times \frac{27}{40} \right) = \frac{27 \times 3}{8 \times 4 \times 8}$

QUESTION CXXX. [Continued.]

$$\text{Th: } \dots \dots \dots xyz = \left(\sqrt{\frac{27 \times 3}{8^2 \times 4}} \right) \frac{9}{16}$$

$$\text{But (because) } \begin{cases} xy = \frac{5}{8}; \text{ Th. }) \frac{5x}{8} = \frac{9}{16}, \\ xz = \frac{3}{4}; \text{ Th. }) \frac{3x}{4} = \frac{9}{16}, \\ yz = \frac{27}{40}; \text{ Th. }) \frac{27x}{40} = \frac{9}{16}; \end{cases}$$

$$\text{Th: } \dots \dots \dots \begin{cases} z = \left(\frac{8 \times 9}{16 \times 5} \right) \frac{9}{10}, \\ y = \left(\frac{4 \times 9}{3 \times 16} \right) \frac{3}{4}, \\ x = \left(\frac{40 \times 9}{27 \times 16} \right) \frac{5}{6}; \end{cases}$$

QUESTION CXXXI.

What two numbers are those, whose difference is to the greater as 5 to 6; and their product multiplied by the lesser produces 384?

Suppose x = the greater and y the lesser.

Then $x - y : x :: 5 : 6$,

That is $(x - y) \times 6 = 5x$, $6x - 6y = 5x$,

And $xy \times y = 384$;

} by question;

But (by first $6x - 5x = 6y$;

And by second $xyy = 384$,

Or (by division) $y^2 = (64) 4^2$;

Th. $\dots \dots \dots = y = 4$.

QUES;

QUESTION CXXXII.

There are three numbers whose continual product being divided by the sum of the two greater will quote 480; if by the sum of the two lesser 672, and if by the greatest and least 560: What are those numbers?

If x represent the greater, y the mean and z the lesser;

Then by question; $\left\{ \begin{array}{l} \frac{xyz}{x+y} = 480; \\ \frac{xyz}{x+z} = 560; \\ \frac{xyz}{y+z} = 672; \end{array} \right\}$ That is $\left\{ \begin{array}{l} xyz = 480x + 480y, \\ xyz = 560x + 560z, \\ xyz = 672y + 672z, \end{array} \right.$

Th. $480x + 480y = 560x + 560z,$

Or $6x + 6y = 7x + 7z,$

Th. $6y - 7z = (7x - 6x) = x;$

Also $560x + 560z = 672y + 672z,$

Or $5x + 5z = 6y + 6z;$

Th. $x = \frac{6y + z}{5};$

But $6y - 7z = \frac{6y + z}{5},$

Or $30y - 35z = 6y + z;$

Th $\frac{2y}{3} = z;$

Again (bec. $6y - 7z = x$) $6y - \frac{14y}{3} = x,$

Or $18y - 14y = 3x;$

Th. $\frac{4y}{3} = x;$

But (by first $\frac{4y}{3} \times y \times \frac{2y}{3} = \frac{8y^3}{9} = \frac{4y}{3} + y \times 480,$

That is (by division) $\frac{2y}{9} = \left(\frac{4}{3} + 1 \right) \times 60 = \frac{7}{3} \times 60,$

Or $yy = 7 \times 3 \times 60 = 3^2 \times 2^2 \times 35;$

Th. $y = (3 \times 2 \sqrt{35}) = 6\sqrt{35},$

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QUESTION CXXXII. [Continued.]

And $x = \left(\frac{4 \times 6 \sqrt{35}}{3} \right) 8 \sqrt{35}$

Also $x = \left(\frac{2 \times 6 \sqrt{35}}{3} \right) 4 \sqrt{35}$

QUESTION CXXXIII.

What two numbers are as 3 to 2; the sum of whose cubes is 280?

If $x =$ the greater of those numbers

Then $(3 : 2 :: x :) \frac{2x}{3} =$ the lesser;

Whence $\left(x^3 + \frac{8x^3}{27} \right) \frac{35x^3}{27} = 280$, by question;

Or (by division) $\frac{x^3}{27} = (8 \div 3) 2^3$,

Or (by multiplicat.) $x^3 = (2^3 \times 27) 2^3 \times 3^3$;

Th. $x = (2 \times 3) 6$.

QUESTION CXXXIV.

What two numbers are as 5 to 4; and their product multiplied by the lesser produces 2160?

If x be the greater number,

Then $(5 : 4 :: x :) \frac{4x}{5} =$ the lesser number,

Also $\left(x \times \frac{4x}{5} \right) \frac{4xx}{5} =$ their product.

But $\left(\frac{4xx}{5} \times \frac{4x}{5} \right) \frac{16x^3}{25} = 2160$ by question,

Or (by division) $\frac{x^3}{25} = (135 \div 25) 27 \times 5$,

Or (by multip.) $x^3 = (27 \times 5 \times 25) 3^3 \times 5^3$;

Th. $x = (3 \times 5) 15$.

QUES:

QUESTION CXXXV.

Some gentlemen travelling; each had twice as many guineas as there were servants in all, and each had as many servants as there were gentlemen, and all their money was 3456 guineas: How many gentlemen and servants were there?

If the number of gentlemen = x ,
 Then the numb. of servants = x^2 ;
 And $(1 : 2xx :: x :)$ $2x^3$ = number of guineas:
 But - - - - - $2x^3$ = 3456 by question,
 Or - - - - - x^3 = $(1728 =)$ 12^3 ;
 Th. - - - - - x = 12 ($= 12^1$)

QUESTION CXXXVI.

What two numbers are those, the product of the greater and square root of the lesser of which is 18; and the product of the lesser and square root of the greater is 12?

If x = the greater, and y the lesser of those numbers;

Then $x\sqrt{y} = 18$; Or $x = \frac{18}{\sqrt{y}}$ } by question.

And $y\sqrt{x} = 12$; Or $\sqrt{x} = \frac{12}{y}$

Now - - $x = \frac{12^2}{y^2}$; Th. $\frac{18}{\sqrt{y}} = \frac{12^2}{y^2}$

Or - - $\frac{1}{\sqrt{y}} = \frac{8}{y^2}$; Th. $y^2 = 8\sqrt{y}$

Or - - $y^4 = 64y$; Th. $y^3 = (64 =)$ 4^3 ;

And - - - - - $y = 4$

$\frac{10801}{101} = \frac{144}{4}$ QUEST

QUESTION CXXXVII.

What two numbers are those, the square of the greater of which multiplied by the lesser, produces 147; and the square of the lesser multiplied by the greater 63?

If x , be the greater; and y , the lesser;

Then by question $\begin{cases} xxy = 147, \\ xyx = 63: \end{cases}$

And the first equat. divided by the 2d; is $\frac{x}{y} = \left(\frac{147}{63} = \frac{49}{21} = \frac{7}{3} \right)$

Th. $x = \frac{7y}{3}$

But (by 2d $xy = 63$) $\frac{7yy}{3} = 63$

That is $y^2 = (63 \times \frac{3}{7} = 9 \times 3) 3^2$

Th. $y = 3$

QUESTION CXXXVIII.

Several merchants making a joint stock, paid each 65 times as many pounds as there were partners; and having traded therewith, gained as many pounds *per Cent.* as there were partners; now if 10 guineas be added to and taken from their gained in pounds, the product of that sum and difference will be $6491\frac{5}{8}$: How many partners were there?

Suppose there were x partners;

Then each paid $65x$ pounds,

And their whole stock was $65xx$ pounds:

But $\left(100 : x :: 65xx : \frac{13x^3}{20} \right)$ = their gain;

And $\frac{13x^3}{20} + \frac{21}{2} \times \frac{13x^3}{20} - \frac{21}{2} = 6491\frac{5}{16}$ by question,

That is $\frac{169x^3}{400} - \frac{441}{4} = \frac{103861}{16}$

Or

QUESTION CXXXVIII. [Continued.]

$$\begin{aligned} \text{Or } - - - - - \frac{169x^6}{400} &= \frac{103861}{16} + \frac{441}{4}, \\ \text{Or } - - - - - \frac{169x^6}{25} &= 103861 + 1764, \\ \text{Or } - - - - - x^6 &= (105625 \times \frac{25}{169}) 5^6; \\ \text{Th. } - - - - - x &= 5. \end{aligned}$$

QUESTION CXXXIX.

A draper fold a piece of cloth for 24^l and gained as much *per Cent.* as the cloth cost him: What was the price of that cloth?

Suppose the cloth cost x pounds

Then $24 - x$ was gained by selling it for 24^l:

But $100 : x :: x : 24 - x$ by question;

$$\text{Th. } (100 \times 24 - x =) 2400 - 100x = xx,$$

$$\text{Th. } - - - - - 2400 = xx + 100x:$$

$$\text{But } - (2400 + 2500 =) 4900 = xx + 100x + 50 \times 50;$$

$$\text{Th. } - - - - - 70 = x + 50,$$

$$\text{And } = = = (70 - 50 =) 20 = x.$$

QUES

QUESTION CXL

There are four numbers (x, y, u, z); the continual product of x, y , and u , is 252; of x, u , and z , is 756; of u, y , and z , 336; and of y, x , and z , 432: What are those numbers?

$$\begin{aligned} \text{Now } xy &= (252 = 4 \times 63 =) 4 \times 9 \times 7, \\ xuz &= (756 = 4 \times 189 =) 4 \times 9 \times 7 \times 3, \\ yuz &= (336 = 4 \times 84 =) 4 \times 4 \times 7 \times 3, \\ yxz &= (432 = 4 \times 108 =) 4 \times 4 \times 9 \times 3, \end{aligned} \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \text{by quest.}$$

The prod. of which 4 } $x^3 y^3 u^3 z^3 = 4^6 \times 9^3 \times 7^3 \times 3^3$,
equat. is

Th. $xyuz = 4^2 \times 9 \times 7 \times 3:$

Whence $\left\{ \begin{aligned} \left(\frac{xyuz}{xy} \right) &= \left(\frac{4^2 \times 9 \times 7 \times 3}{4 \times 9 \times 7} = 4 \times 3 \right) 12, \\ \left(\frac{xyuz}{xuz} \right) &= \left(\frac{4^2 \times 9 \times 7 \times 3}{4 \times 9 \times 7 \times 3} = 4, \right. \\ \left(\frac{xyuz}{yuz} \right) &= \left(\frac{4^2 \times 9 \times 7 \times 3}{4 \times 4 \times 7 \times 3} = 9, \right. \\ \left(\frac{xyuz}{yxz} \right) &= \left(\frac{4^2 \times 9 \times 7 \times 3}{4 \times 4 \times 9 \times 3} = 7. \end{aligned} \right.$

QUES

Now

EnA

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EnA

QUESTION CXLI. [Continued.]

Now $\frac{c-x}{c-t} = r$ by quest.

Or $\frac{c-x}{c-t} = r$

Th. $\frac{c-x}{c-t} = r$

And $\frac{c-x}{c-t} = r$

In this example $81 - \sqrt[4]{16 \times 81} = (81 - 2 \times 3^3 = 27)$

QUESTION CXLII.

'Tis required to find two numbers x and y in proportion as r to s , so that x^m may be equal to y^n ; (where if $r \sqsubset s$, then m must be $\sqsupset n$; but if $r \sqsupset s$, then $m \sqsubset n$)?

Now $r : s :: x : y$; Th. $x = \frac{r}{s} y$

And $\left(\frac{r}{s} y\right)^m = y^n$ by question;

That is $\frac{r^m y^m}{s^m} = y^n$

Or $\frac{r^m}{s^m} = \left(\frac{y^n}{y^m}\right)^{n-m}$

Or $\frac{r^m}{s^m} = y^{n-m}$

Th. $\frac{r^m}{s^m} = y^{n-m}$

If R , S and T represent the logarithms of r , s and y

Then $\frac{m}{n-m} \times R - S = T$.

QUES:

QUESTION CXLIII.

One bought some oxen for 80 l.; now had he bought four more for the same money, he would have paid 1 l. less for each? How many did he buy?

Suppose he bought x oxen.

Then $\frac{80}{x}$ = the price of an ox,

And $\frac{80}{x+4}$ = the price of d^o if $x+4$ cost 80 l.

But $\frac{80}{x} = \frac{80}{x+4} + 1$;

Th: $80x + 320 = 80x + xx + 4x$,

Th. $320 = xx + 4x$;

But $(320+4) 324 = xx + 4x + 2 \times 2$,

Th. $18 = x + 2$;

And $16 = x$.

QUESTION CXLIV.

There are two numbers whose difference is 3; and the difference of their cubes is 117: What are those numbers?

If the lesser required number = x ,

Then the greater = $x+3$,

And $9x^2 + 27x + 27 = x^3 + 3x^2 - x^3$;

But $9x^2 + 27x + 27 = 117$ by question,

Or $9xx + 27x = 90$,

Th. $xx + 3x = 10$;

But $xx + 3x + \frac{9}{4} = \left(10 + \frac{9}{4}\right) \frac{49}{4}$;

Th. $x + \frac{3}{2} = \frac{7}{2}$; and $x = \left(\frac{7-3}{2}\right) = 2$.

QUES.

QUESTION CXLV.

Bought two remnants of cloth, one of which was six yards longer than the other, for 3*l*. 8*s*.; and each of them cost as many shillings a yard as there were yards therein? What was the length of each?

Suppose x = the length of lesser,
 Then $x + 6$ = ditto the greater;
 And xx = value of the lesser,

Also $(x + 6)xx + 12x + 36$ = ditto of the greater;
 Now (their sum) $2xx + 12x + 36 = 68$ by question,

Or $2xx + 12x = 32$,

Th. $xx + 6x = 16$;

But $xx + 6x + 9 = (16 + 9) = 25$;

Th. $x + 3 = 5$; And $x = 2$.

QUESTION CXLVI.

What two numbers are those, whose difference is 2; and the product of their cubes 42875?

If the lesser number be x ;

Then the greater is $x + 2$;

And $x^3 \times (x + 2)^3 = 42875$ by quest.;

Th. $x \times (x + 2) = \sqrt[3]{42875} = 35$;

That is $xx + 2x = 35$;

But $xx + 2x + 1 = (35 + 1) = 36$;

Th. $x + 1 = 6$; And $x = 5$.

QUES.

QUESTION CXLVII.

One having sold a piece of cloth which cost him 30*l*. found that if the price he sold it for, was multiplied by his gain, the product would be equal to the cube of his gain: What was it?

Suppose he gained x *l*;

Then he sold the cloth for $30 + x$:

But $(30 + x \times x =) 30x + xx = x^3$ by quest.

Or $30 + x = xx$;

Th. $30 = xx - x$:

But $(30 + \frac{1}{4} =) \frac{121}{4} = xx - x + \frac{1}{4}$;

Th. $\frac{11}{2} = x - \frac{1}{4}$;

And $(\frac{11 + 1}{2} =) 6 = x$.

QUESTION CXLVIII.

A company at a tavern had 8*l*. 15*s*. to pay, but before the bill was paid, two of them sneaked away, then those who remained had each 10*s*. more to pay, than before: How many were in company?

Suppose there were x persons in company;

Then $\frac{175}{x} =$ each man's share of the bill;

Also $\frac{175}{x-2} =$ the share of each that remained after the two were gone:

Now $\frac{175}{x-2} = \frac{175}{x} + 10$ by question.

Th. $175x = 175x - 350 + 10x^2 - 20x$,

Or $350 = 10xx - 20x$,

Th. $35 = x^2 - 2x$:

But $(35 + 1 =) 36 = x^2 - 2x + 1$;

Th. $6 = x - 1$,

And $x = 7$.

QUES:

QUESTION CXLIX.

A grazier bought as many sheep as cost him 60 £; out of which he reserved 15, and sold the remainder for 54 £, gaining two shillings a head by them: How many sheep did he buy?

Suppose he bought x Sheep at y Shillings each;

Then he sold $x-15$ Sheep at $y+2$ Shillings each:

Now $\left\{ \begin{array}{l} \frac{xy}{x-15} = \frac{60 \times 20}{54 \times 20} = 1200 \end{array} \right\}$ by quest.

Or $xy - 15y + 2x - 30 = 1080,$

Or $1200 - 15y + 2x - 30 = 1080;$

Th. $- 15y + 2x = (1080 + 30 - 1200) = -90:$

But $- - - - - x = \frac{1200}{y};$ Th. $2x = \frac{2400}{y},$

And $- - 15y + \frac{2400}{y} = -90,$

Or $- - 15y + 2400 = 90y,$

Or $- - - 2400 = 15y - 90y,$

Th. $- - - 160 = y - 6y:$

But $(160 + 9) 169 = y - 6y + 9;$

Tn. $- - - 13 = y - 3; \text{ And } 16 = y:$

Also $- - - x = \left(\frac{1200}{16} \right) 75.$

QUES:

QUESTION CL.

'Tis required to divide 37 into two such parts, that the product of the squares of those parts may be 117964 ?

Suppose - - - - - x = one of those parts;

Then - - - - - $37-x$ = the other part,

And - - - - - $(37-x)^2 \times x^2 = 117964$ by question :

Th. - - - - - $37-x \times x = (\sqrt{117964} =) 342$;

That is, - - - - - $37x - xx = 342$;

Th. - - - - - $xx - 37x = -342$;

But $xx - 37x + \frac{37 \times 37}{4} = \left(\frac{1369}{4} - 342 = \right) \frac{1}{4}$;

Th. - - - - - $x - \frac{37}{2} = \frac{1}{2}$.

And - - - - - $x = \left(\frac{37+1}{2} = \right) 19$.

QUESTION CLI.

'Tis required to divide the number 48, into two such parts, that the sum of their alternate quotients may be $5\frac{1}{3}$?

If x be one of those parts,

Then $48-x$ = will be the other;

Now - - - - - $\frac{x}{48-x} + \frac{48-x}{x} = \left(5\frac{1}{3} = \right) \frac{26}{5}$ by question

Or - - - - - $5xx + 5 \times 48 - x^2 = 48-x \times 26x$;

That is, $11520 - 480x + 10xx = 1248x - 26xx$,

Or - - - - - $36xx - 1728x = -11520$;

Th. - - - - - $xx - 48x = -320$;

But - - - - - $xx - 48x + 24 \times 24 = 576 - 320 = 256$;

Th. - - - - - $x - 24 = 16$; Th. $x = (16 + 24 =) 40$.

QUES.

QUESTION CLII.

'Tis required to find two numbers, the first of which may be to the second as the second is to 20; and the sum of the squares of the numbers sought may be 125?

If x and y represent the numbers required.

Then $xx + yy = 125$; Or $yy = 125 - xx$ } by quest.

And $x : y :: y : 20$; Or $yy = 20x$ }

Th. $125 - xx = 20x$;

Th. $xx + 20x = 125$;

But $xx + 20x + 100 = (125 + 100 =) 225$;

Th. $x + 10 = 15$;

And $x = 5$;

But $yy = (20 \times 5 =) 100$;

Th. $y = 10$.

QUESTION CLIII.

There are two numbers whose difference is 15; and $\frac{1}{2}$ their product is the cube of the lesser number: What are those numbers?

If the lesser of those numbers be $= x$;

Then the greater $= x + 15$;

And $(x + 15 \times x =) xx + 15x =$ their product:

But $\frac{xx + 15x}{2} = xxx$ by question,

And (dividing by $\frac{x}{2}$) $x + 15 = 2xx$;

Th. $\frac{15}{2} = xx - \frac{x}{2}$;

But $\left(\frac{15}{2} + \frac{1}{16} = \frac{120+1}{16} =\right) \frac{121}{16} = xx - \frac{x}{2} + \frac{1}{16}$;

Th. $\frac{11}{4} = x - \frac{1}{4}$;

And $\left(\frac{11}{4} + \frac{1}{4} =\right) 3 = x$.

QUES.

QUESTION CLIV.

There are two numbers whose difference is 7; also their sum multiplied by the greater produces 345: What are those numbers?

Let the greater number $= x$,

Then the lesser $= x - 7$;

And their sum $= 2x - 7$;

But $(2x - 7) \times x = 2x^2 - 7x = 345$ by question,

Th. $xx - \frac{7}{2}x = \frac{345}{2}$;

But $xx - \frac{7}{2}x + \frac{7 \times 7}{4 \times 4} = \left(\frac{345}{2} + \frac{49}{16} \right) = \frac{2809}{16}$;

Th. $x - \frac{7}{4} = \frac{53}{4}$; And $x = \frac{53 + 7}{4} = 15$.

QUESTION CLV.

'Tis required to divide the number 24 into two such parts, that their product may be thirty-five times their difference?

If x be the greater, and y the lesser of the parts required;

Then $x + y = 24$ } by question;

And $(x - y) \times 35 = 35x - 35y = xy$

But $35x + 35y = (24 \times 35) = 840$;

Th. $70x = xy + 840$;

But $y = 24 - x$,

Th. $70x = 24x - xx + 840$,

Th. $xx + 46x = 840$;

But $xx + 46x + 529 = 840 + 529 = 1369$;

Th. $x + 23 = 37$; And $x = (37 - 23) = 14$.

QUESTION CLVI

What two numbers are those, whose sum, product, and difference of their squares, are equal among themselves?

Let x represent the greater, and y the lesser of those numbers

Then $x + y = xy = xx - yy$ by question;

Whence $1 = \left(\frac{xx - yy}{x + y} \right) = x - y$;

And $1 + y = x$;

But $(1 + y + y) = 1 + 2y = (1 + y) \times y = y + yy$;

That is $1 = yy - y$;

But $(1 + \frac{1}{2}) = \frac{5}{4} = yy - y + \frac{1}{4}$;

And $\frac{\sqrt{5}}{2} = y - \frac{1}{2}$;

Th. $\frac{\sqrt{5} + 1}{2} = y$;

And $\frac{\sqrt{5} + 3}{2} = x$.

QUES.

QUESTION CLVII.

A and *B* (who were 120 miles distant) set out to meet each other; *A* travelled 5 miles a day; and the number of days at the end of which they met was greater by 3 than the number of miles which *B* went in a day: How many miles did each go?

Suppose A travelled - - - x } miles ;
Then B went - - - $120 - x$ }

And they met at the end of $\frac{x}{5}$ days; $\left\{ \begin{array}{l} \text{for } A \text{ went} \\ 5 \text{ m. a day;} \end{array} \right.$

Th. B. went - - - $\frac{2}{5}$ - 3 miles a day ;

And B went in all $-\quad-\quad-\quad \frac{x}{5} \times \frac{x}{5} = 3$ miles;

Th. $120 - x = \left(\frac{x}{5} \times \frac{x}{5} - 3 \right) \frac{xx}{25} - \frac{3x}{5}$

$$\text{Or } 120 - x \times 25 = 3000 - 25x = x - 15x,$$
$$3000 = xx + 10x:$$

But $-(3000 + 25) = 3025 = xx + 10x + 25;$

$$55 = x + 5,$$

And - - - 50 = *

QUESTION CLVIII.

To find 4 numbers, the first of which may be to the second as the third to the fourth; that the first may be to the fourth as 1 to 5; that the second may be to the third as 5 to 9; and the sum of the second and fourth may be 20?

If x, y, z , and u be the numbers required,

Then $x : y :: z : u$; or $xz = yu$:

$x : z :: 1 : 5$; or $5x = z$:

$y : z :: 5 : 9$; or $9y = 5z$:

And $y + z = 20$; or $z = 20 - y$:

Now $5x = 20 - y$,

Th. $y = 20 - 5x$:

But $(20 - 5x \times 9 =) 180 - 45x = 5u$,

Th. $36 - 9x = u$:

But $(xz = yu \text{ that is}) 5x \times x = 20 - 5x \times 36 - 9x$,

Or $5xx = 720 - 360x + 45x^2$;

Or $40x^2 - 360x = -720$;

Th. $x^2 - 9x = -18$:

But $x^2 - 9x + \frac{81}{4} = \left(\frac{81}{4} - 18 = \frac{81 - 72}{4} = \frac{9}{4}\right)$;

Th. $\frac{9}{2} - x = \frac{3}{2}$;

And $\left(\frac{9 - 3}{2} = \right) 3 = x$.

QUES.

QUESTION CLIX.

It is required to find the values of x , y , u and z , in the following equations;

Viz.
$$\left\{ \begin{array}{l} x + y = 10 \\ u + z = 20 \\ xy + uz = 120 \\ x + y + u \times u + y + u + z \times y = 360 \end{array} \right\}$$

Now
$$\left\{ \begin{array}{l} \text{(by 1st)} \quad x = 10 - y \\ \text{(by 2d)} \quad z = 20 - u \\ \text{(by 3d)} \quad 10 - y \times y + 20 - u \times u = 120 \\ \text{(by 4th)} \quad 10 + u \times u + y + 20 \times y = 360 \end{array} \right.$$

But by adding the two last $30u + 30y = 480$;

Th. $x + y = \left(\frac{480}{30} =\right) 16$; and $y = 16 - u$;

Now $x = (10 - y = 10 - 16 + u =) u - 6$;

But $(xy + uz =) u - 6 \times 16 - u + u \times 20 - u = 120$ (by 3d),

That is $22u - uu - 96 + 20u - uu = 120$,

Or $42u - 2u^2 - 96 = 120$;

Th. $uu - 21u = -108$;

But $uu - 21u + \frac{21 \times 21}{2 \times 2} = \frac{441}{4} - 108$
 $= \frac{9}{4}$;

Th. $(u - \frac{21}{2})$; Or $\frac{21}{2} - u = \frac{3}{2}$;

And $(\frac{21-3}{2} =) 9 = u$.

QUESTION CLX.

What two numbers are those, whose sum is 140; and their product 3136?

If the given sum (140) $= s$; the given product (3136) $= p$; $x =$ the greater, and $y =$ the lesser number required;

$$\begin{array}{l} \text{Then} \quad x + y = s; \text{ Or } x = s - y \\ \text{And} \quad xy = p; \text{ Or } x = \frac{p}{y} \end{array} \quad \left. \vphantom{\begin{array}{l} \text{Then} \\ \text{And} \end{array}} \right\} \text{by qu.}$$

$$\text{Th.} \quad s - y = \frac{p}{y}; \text{ Or } sy - y^2 = p,$$

$$\text{Or} \quad y^2 - sy = -p:$$

$$\text{But} \quad y^2 - sy + \frac{ss}{4} = \left(\frac{ss}{4} - p \right) = \frac{ss - 4p}{4},$$

$$\text{Th.} \quad y - \frac{s}{2} \left. \vphantom{\begin{array}{l} y - \frac{s}{2} \\ \frac{s}{2} - y \end{array}} \right\} = \frac{1}{2} \sqrt{ss - 4p}$$

$$\text{Or} \quad \frac{s}{2} - y \left. \vphantom{\begin{array}{l} y - \frac{s}{2} \\ \frac{s}{2} - y \end{array}} \right\} = \frac{1}{2} \sqrt{ss - 4p}$$

If y be exterminated instead of x , the equation arising will be $sx - xx = p$, which reduced as above will give

$$\left. \begin{array}{l} x - \frac{s}{2} \\ \frac{s}{2} - x \end{array} \right\} = \frac{1}{2} \sqrt{ss - 4p}.$$

But by the assumption (viz. $x \sqsupset y$) $x \sqsupset \frac{s}{2}$,

$$\text{And } y \sqsubset \frac{s}{2};$$

$$\text{Th.} \quad \left\{ \begin{array}{l} x - \frac{s}{2} \\ \frac{s}{2} - y \end{array} \right\} = \frac{1}{2} \sqrt{ss - 4p},$$

And

QUESTION CLX. [Continued.]

And
$$\begin{cases} x = \frac{s + \sqrt{s^2 - 4p}}{2}, \\ y = \frac{s - \sqrt{s^2 - 4p}}{2}, \end{cases}$$

QUESTION CLXI.

One buys cloth for 33*l*. 15*s*. which he sells again at 48 Shillings per Piece, and gained by the bargain as much as 1 piece cost : Required the number of pieces ?

Suppose he bought x pieces ;

Then each piece cost $\frac{675}{x}$ shillings,

And he sold them for 48*x* shillings,

And gained thereby 48*x* - 675 Do.

But $48x - 675 = \frac{675}{x}$ by quest.

Or $48x^2 - 675x = 675$;

Th. $x^2 - \frac{225}{16}x = \frac{225}{16}$;

But $xx - \frac{225}{16}x + \frac{225}{32} = \left(\frac{225}{16} + \frac{50625}{1024} \right) = \frac{65025}{1024}$;

Th. $x - \frac{225}{32} = \frac{225}{32}$; and $x = \left(\frac{255 + 225}{32} \right) = 15$.

QUES.

QUESTION CLXII.

What two numbers are those, whose difference multiplied by the difference of their squares will produce 576; and whose sum multiplied by the sum of their squares is 2336?

If x , the greater, and y , the lesser, of those numbers;

Then $\frac{x-y}{x+y} \times \frac{xx-yy}{xx+yy} = 576;$

And $\frac{x+y}{x-y} \times \frac{xx+yy}{xx-yy} = 2336;$

That is, $\begin{cases} x^3 - x^2y - xy^2 + y^3 = 576, \\ x^3 + x^2y + xy^2 + y^3 = 2336: \end{cases}$

(By subtr.) $2x^2y + 2xy^2 = 1760;$

And $x^3 + 3x^2y + 3xy^2 + y^3 = 4096$ by addition;

Th. $x + y = \sqrt{4096} = 16:$

But $2x^2y + 2xy^2 = 2xy \times x + y;$

Th. $(16 \times 2xy =) 32xy = 1760$ by fifth,

And $xy = 55:$

Now (by qu. 160) $\begin{cases} x = \left(16 + \sqrt{256 - 4 \times 55 \times \frac{1}{2}} \right) 11, \\ y = \left(16 - \sqrt{256 - 4 \times 55 \times \frac{1}{2}} \right) 5. \end{cases}$

QUESTION CLXIII.

What two numbers are those, whose product is 6; and if their sum be added to the sum of their squares, the number resulting will be 18?

Let the required numbers be denoted by x & y ;

Then $\frac{x+y}{x-y} + \frac{xx+yy}{xx-yy} = 18$ quest.

QUES.

QUESTION CLXIII. [Continued.]

And (since $xy = 6$) $2xy = 12$;

Th. - $x+y + \overline{x+y}^2 = 30$ by addition:

But $\frac{1}{4} + \overline{x+y} + \overline{x+y}^2 = (30 + \frac{1}{4}) = \frac{121}{4}$;

Th. - $\frac{1}{2} + \overline{x+y} = \frac{11}{2}$,

And - $x+y = (\frac{11-1}{2}) = 5$;

Now (by quest. 160) $\begin{cases} x = 5 + \sqrt{25 - 4 \times 6} \times \frac{1}{2} = 3, \\ y = 5 - \sqrt{25 - 4 \times 6} \times \frac{1}{2} = 2. \end{cases}$

QUESTION CLXIV.

What two numbers are those, whose sum taken from the sum of their squares leaves 78; and their sum added to their product makes 39?

If x and y represent the numbers required;

Then - $xx + yy - x + y = 78$, } by quest.
And - $xy + x + y = 39$, }

Or (by second) $2xy + 2x + y = 78$;

The sum of the $\begin{cases} \overline{x+y}^2 \\ \text{first and last is } \end{cases} x+y + x+y = 156$;

But - $\overline{x+y}^2 + \overline{x+y} + \frac{1}{4} = (156 + \frac{1}{4}) = \frac{625}{4}$;

Th. $\overline{x+y} + \frac{1}{2} = \frac{25}{2}$; and $x+y = (\frac{25-1}{2}) = 12$;

Now - $xy + 12 = 39$ by second;

Th. - $xy = (39 - 12) = 27$;

And (by quest. 160) $\begin{cases} x = 12 + \sqrt{144 - 4 \times 27} \times \frac{1}{2} = 9, \\ y = 12 - \sqrt{144 - 4 \times 27} \times \frac{1}{2} = 3. \end{cases}$

QUES-

QUESTION CXLV.

'Tis required to divide the number 7 into two such parts, that the square of thrice the lesser may be 17 more, than the square of twice the greater ?

If the greater part $\equiv x$;

Then the lesser $\equiv 7-x$;

The square of $2 \times x = 4xx$,

The square of $3 \times 7-x = 441-126x+9xx$:

Now $441-126x+9xx = 4xx+17$ by quest.

Or $5xx-126x = (17-441) = -424$,

Or $xx - \frac{126}{5}x = -\frac{424}{5}$;

But $xx - \frac{126}{5}x + \left(\frac{63}{5}\right)^2 = \left(\frac{3969}{25} - \frac{424}{5}\right) = \frac{1849}{25}$;

Th. $\left(x - \frac{63}{5} \text{ or } \frac{63}{5} - x = \frac{43}{5}\right)$,

And $\left(\frac{63-43}{5}\right) 4 = x$.

QUESTION CXLVI.

A sets out from *C*, towards *D*, and travels 8 miles a day, after he had gone 27 miles, *B* sets out from *D*, towards *C*, and goes every Day $\frac{1}{20}$ of the whole journey; and after he had travelled as many days as he goes miles in 1 Day, he met *A*. Required the distance of those places?

Suppose *C* was x miles distant from *D*;

Then *B* went $\frac{x}{20}$ miles a day,

And

QUESTION CLXVI. [Continued.]

And B went $\frac{xx}{400}$ miles in all ;

Also A went $27 + 8 \times \frac{x}{20}$ miles in all $= 27 + \frac{2x}{5}$:

Now $-\frac{xx}{400} + 27 + \frac{2x}{5} = x$ by quest.

Or $-xx + 10800 + 160x = 400x$,

Or $-xx - 240x = -10800$:

But $xx - 240x + \frac{10800}{120} = (14400 - 10800) = 3600$;

Th, $x - 120 = 60$; and $x = (120 + 60) = 180$;

Or $120 - x = 60$; and $x = (120 - 60) = 60$.

QUESTION CLXVII.

Two merchants after a successful voyage shared 854 £ . between them ; the first had laid out 420 £ . in Goods, and the second had gained 52 £ . by what he had laid out. How much did the first gain, and the second advance at first ?

Supposed the second laid out x ? ;

Then $(x : 52 : 420) \frac{420 \times 52}{x} = \text{the first's gain}$:

But the sum at first laid out by both, added to the sum of their gains, is to be shared between them ;

That is $420 + x + 52 + \frac{420 \times 52}{x} = 854$,

Or $420x + x^2 + 52x + 420 \times 52 = 854x$,

QUES-

QUESTION CLXVI. [Continued.]

$$\text{Th. } - - - - - xx - 382x = -21840:$$

$$\text{But } - \quad xx - 382x + 191^2 = 36481 - 21840 \\ = 14641;$$

$$\text{Th. } x - 191 = 121; \text{ and } x = (121 + 191 =) 312.$$

QUESTION CLXVIII.

Three merchants, *A*, *B*, *C*, join stocks; *A*'s stock was 13 *l*. less than *B*'s; the sum of *B*'s and *C*'s stock was 175 *l*. their gain exceeded the whole stock by 48 *l*. and *A*'s share of that gain was 78 *l*. What was each man's part of the stock and gain?

$$\begin{array}{ll} \text{Suppose } A's \text{ stock was} & x \text{ l.} \\ \text{Then } B's & \text{was } x + 13, \\ \text{And } C's & \text{was } 162 - x (= 175 - x - 13): \end{array}$$

$$\begin{array}{ll} \text{Then the joint stock was} & 175 + x, \\ \text{And the gain} & 223 + x (= 48 + 175 + x); \end{array}$$

$$\text{Now } 175 + x : 223 + x :: x : 78 \text{ by quest.}$$

$$\text{That is, } - \quad 13650 + 78x = 223x + xx;$$

$$\text{Th. } - \quad xx + 145x = 13650$$

$$\text{But } xx + 145x + \frac{145^2}{2} = \left(13650 + \frac{21025}{4} \right) \frac{75625}{4};$$

$$\text{Th. } x + \frac{145}{2} = \frac{275}{2}; \text{ and } x = \left(\frac{275 - 145}{2} = \right) 65.$$

QUESTION CLXIX.

Toward the expence of building a ship, *A* paid 2000 *l.* more than *B*, and 9000 more than *C*; also the square of what *A* paid is equal to the sum of the squares of *B*'s and *C*'s payments: How much did each pay?

Suppose <i>A</i> paid	x pounds,
Then <i>B</i> paid	$x - 2000$,
And <i>C</i>	$x - 9000$;
Square of <i>B</i> 's payment	$= xx - 4000x + 4000000$,
Do. of <i>C</i> 's	$= xx - 18000x + 81000000$;
Their sum is	$= 2x^2 - 22000x + 85000000$;
Now $2x^2 - 22000x + 85000000 = xx$ by quest.	
Th.	$x^2 - 22000x = -85000000$;
But	$x^2 - 22000x + 11000^2 = (121000000 - 85000000 =) 36000000$;
Th.	$x - 11000 = 6000$;
And	$x = (11000 + 6000 =) 17000$.

QUES.

QUESTION CLXX.

A draper bought some pieces of two sorts of cloth for 47*l.* 7*s.* 11*d.*; there were as many pieces bought of each sort, and as many pence *per* yard paid for them, as a piece of that sort contained yards; now two pieces, one of each sort, measured together, were 35 yards: How many yards were in each?

$$\begin{array}{l}
 \text{If a piece of one of the sorts} \quad = x \text{ yards,} \\
 \text{Then a piece of the other sort} \quad = 35 - x \text{ yards,} \\
 \text{Now} \quad \left\{ \begin{array}{l} x \text{ pieces, each of } x \text{ yards} = xx \text{ yards;} \\ 35 - x \text{ do.} = 35 - x \text{ do.} = 35 - x \text{ yards;} \end{array} \right. \\
 \text{And} \quad \left\{ \begin{array}{l} xx \text{ yards, at } x \text{ pence a yard,} = x^3 \text{ pence,} \\ 35 - x \text{ do. at } 35 - x \text{ do.} = 35 - x \text{ do.} = 35 - x \text{ pence;} \end{array} \right. \\
 \text{Also} \quad 47 \text{ l. } 7 \text{ s. } 11 \text{ d.} = 11375 \text{ pence;} \\
 \text{Now } (35 - x + x^3) = 42875 - 3675x + 105x^3 = 11375 \text{ by quest.} \\
 \text{Or} \quad 105x^3 - 3675x = -31500, \\
 \text{Or} \quad x^3 - 35x = -300; \\
 \text{But} \quad 35x - 35x + \frac{35 \times 35}{2 \times 2} = \left(\frac{1225}{4} - 300 \right) = \frac{25}{4} \\
 \text{Th.} \quad x - \frac{35}{2} = \frac{5}{2}; \text{ and} \quad x = \left(\frac{35 + 5}{2} \right) = 20.
 \end{array}$$

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Then $xx + yy + zz = 3037.$

The { sum } of the { $2y^2 + x + z = 1748$;
 { diff. } 2 last, { $z - x = 6$;

And $x^2 - 2xx + xx = 36 :$

The diff. of
first and last, $\left\{ \begin{array}{l} - \\ + \end{array} \right. \quad x + 2x^2 = 3001;$

The diff. of
last and 4th, $\left\{ \begin{array}{l} 4x^2 - x - z = 4254; \end{array} \right.$

Th. $4x \times x + 6 - x - x - 6 = 4254;$

$$\text{Or } 4x^2 + 22x = 4260;$$

But - $\sqrt{xx + \frac{11}{x} + \frac{11}{x}} = 1065 + \frac{121}{x}$

Th. $x + \frac{11}{4} = \frac{131}{4}$; and $x = \left(\frac{131 - 11}{4} \right)_{30}:$

$$\begin{aligned} \text{Or } & - \\ & - \end{aligned}$$

$y = 29.$

Q U E S.

QUESTION CLXXII.

Two partners *A* and *B* had gained 18 £ . 15 s . by trade ;
A's money was in trade 12 months, and he received for
his principal and gain 26 £ . also *B*'s money (viz 30 £ .)
was in trade 17 months : what money did *A* put into
trade ?

Suppose *A* advanced x pounds ;

The the product of }
A's stock and time } = $12x$,

B's ditto - - - = $(30 \times 17 =) 510$;

And their sum - - - = $12x + 510$:

Now $(12x + 510 : 18 \frac{3}{4} :: 12x :) \frac{225x}{12x + 510}$ *A*'s gain ;

But $x + \frac{225x}{12x + 510} = 26$ by quest.

Or $12xx + 510x + 225x = 312x + 13260$,

Or - $12xx + 423x = 13260$,

Or - $xx + \frac{141}{4}x = \frac{4420}{4}$;

But $xx + \frac{141}{4}x + \frac{141}{8}x^2 = \left(\frac{4420}{4} + \frac{19881}{64} \right) = \frac{90601}{64}$;

Th. $x + \frac{141}{8} = \frac{301}{8}$; And $x = \left(\frac{301 - 141}{8} \right) = 20$.

QUES-

VIXXIO M O I T B E U O

QUESTION CLXXIII.

Two partners, *A* and *B*, had gained 140 *l.* by trade; *A*'s money was three months in trade, and his gain was 60 *l.* less than his stock: Also *B*'s money, which was 50 *l.* more than *A*'s, was in trade 5 months: What was *A*'s stock?

Suppose *A*'s stock = x *l.*

Then *B*'s stock = $x + 50$,

And *A*'s gain = $x - 60$,

Also *B*'s gain = $200 - x (= 140 - x - 60)$:

Now $3x : x - 60 :: x + 50 \times 5 : 200 - x$ by qu.

That is $3x \times 200 - x = x - 60 \times 5x + 250$,

Or $600x - 3x^2 = 5x^2 - 50x - 15000$,

Th. $xx - \frac{650}{8}x = \frac{15000}{8}$:

But $xx - \frac{650}{8}x + \frac{325}{8} = \frac{15000}{8} + \frac{105625}{64}$
 $= \frac{225625}{64}$;

Th. $x - \frac{325}{8} = \frac{475}{8}$; And $x = \left(\frac{475 + 325}{8} \right) = 100$.

Answered

QUES.

QUES.

QUESTION CLXXIV.

The joint flock of 2 partners *A* and *B* was 165 l. *A*'s money was in trade 12, and *B*'s 8 months : When they shared stock and gain *A* received 67, and *B* 126 l. What was each man's stock ?

Suppose *A* advanced x pounds :

Then *B* advanced 165 — x pounds ;

And the product of *A*'s stock and time

Do of *B*'s $= (165 - x) \times 8 = 1320 - 8x$;

And their sum is $= 4x + 1320$;

Also their gain $= (67 + 126 - 165) = 28$;

But $(4x + 1320) : 28 :: 12x : \frac{336x}{1320 + 4x} = A$'s gain,

And (*A*'s stock and gain $=$) $x + \frac{336x}{1320 + 4x} = 67$ by quest.

Or $1320x + 4x^2 + 336x = 88440 + 268x$;

Th. $x^2 + 347x = 22110$;

But $x^2 + 347x + \frac{347^2}{4} = 22110 + \frac{120409}{4}$,

Or $x^2 + 347x + \frac{347^2}{4} = \frac{208849}{4}$;

Th. $x + \frac{347}{2} = \frac{457}{2}$; And $x = (\frac{457 - 347}{2}) = 55$.

Otherwise

QUESTION CLXXIV. [Continued.]

The root of the equation $xx + 347x = 22110$, may be obtained by the tables of the logarithms of numbers and tangents, according to the method given by the excellent mathematician Dr. Edmund Halley, in his geometrical lectures, published as an appendix to the last edition of *Kersey's algebra*; as follows:

The logarithm of	22110 =	4.3445887;
From half thereof	+ 10 =	12.1722943,
Take the logarithm of	$\frac{347}{2} =$	2.2392995;
Remains the log. tang. of	40° : 35' : 52" =	9.9329948;
To the log. tang. of $\frac{1}{2}$ that arc, viz. of 20 : 17 : 56 =		9.5680714,
Add $\frac{1}{2}$ the logarithm of	22110 =	2.1722943;
The sum (-10) is the log. of	x =	1.7403657.
Therefore x =		55.

QUESTION CLXXV.

'Tis required to divide the number 63 into 5 such parts; that the first *more* 2, the second *less* 2, the third multiplied by 2, the fourth divided by 2, and the square root of the fifth, may be equal among themselves?

If x, y, u, z , and e represent the numbers required;

Then $x + 2 = y - 2 = 2u = \frac{z}{2} = \sqrt{e}$ by qu-

$$\text{Therefore } \left\{ \begin{array}{l} x = x, \\ x + 4 = y, \\ \frac{x + 2}{2} = u, \\ (x + 2 \times 2) = 2x + 4 = z, \\ xx + 4x + 4 = e. \end{array} \right.$$

And $xx + 8x + 12 + \frac{x+2}{2} = (x+y+u+z+e) 63,$

Or $2xx + 16x + 24 + x + 2 = (63 \times 2) 126,$

That is $2x^2 + 17x + 26 = 126;$

Th. $xx + \frac{17}{2}x = 50:$

But $xx + \frac{17}{2}x + \frac{17}{4} = \left(50 + \frac{289}{16}\right) \frac{1089}{16};$

Th. $x + \frac{17}{4} = \frac{33}{4};$ and $x = \left(\frac{33-17}{4}\right) 4.$

QUES-

QUESTION CLXXVI.

What number is that, which divided by the product of its two digits quotes 3; and if 18 be added to it, the digits will be inverted?

If the digits be represented by x and y ;

Then the number will be $10x + y$,

And (by quest.) $\begin{cases} 3 = \frac{10x + y}{xy}, \\ 10y + x = 10x + y + 18; \end{cases}$

But by the last $9y = 9x + 18$;

And $y = x + 2$;

Now (writing $x + 2$ for y in the 1st) $3 = \frac{10x + x + 2}{xx + 2}$,

Or $(3x \times x + 2 =) 3xx + 6x = 11x + 2$;

Th. $xx - \frac{5}{3}x = \frac{2}{3}$;

But $xx - \frac{5}{3}x + \frac{5}{6} = \left(\frac{2}{3} + \frac{25}{36} = \frac{49}{36}\right)$;

Th. $x - \frac{5}{6} = \frac{7}{6}$; and $x = \left(\frac{7+5}{6} = \right) 2$.

QUEST

QUESTION CLXXVII.

A grocer sold 80 lb. of mace, and 100 lb. of cloves for 65 l. but he sold 60 lb. more of cloves for 20 l. than he did of mace for 10 l. What prices did he sell each at?

Suppose he sold the mace at x , } shill. a lb.
And the cloves at y , }

Then $(x : 1 \text{ lb.} :: (10 \text{ l.} =) 200 \text{ s.} ::) \frac{200}{x} = \text{lbs. of m. for } 10 \text{ l.}$

And $(y : 1 \text{ lb.} :: (20 \text{ l.} =) 400 \text{ s.} ::) \frac{400}{y} = \text{lbs. of c. for } 20 \text{ l.}$

Now $(65 \text{ l.} =) 1300 = 80x + 100y$ } by q.
And $\frac{200}{x} + 60 = \frac{400}{y}$

By first $\frac{130 - 8x}{10} = y$,

By second $y = \frac{20x}{10 + 3x}$;

Th. $\frac{130 - 8x}{10} = \frac{20x}{10 + 3x}$

Or $1300 - 80x + 390x - 24x^2 = 200x$,

That is $24x^2 - 110x = 1300$;

Th. $x^2 - \frac{110}{24}x = \frac{1300}{24}$;

But $x^2 - \frac{110}{24}x + \frac{55}{24} = \frac{1300}{24} + \frac{3025}{576}$
 $= \frac{34225}{576}$;

Th. $x - \frac{55}{24} = \frac{185}{24}$; and $x = \left(\frac{185 + 55}{24} = \right) 10$.

QUES-

QUESTION CLXXVII. [Continued.]

Or the root of $xx - \frac{110}{24}x = \frac{1300}{24}$ may be found as follows, by Dr. Halley's method before quoted:

The Logarithm of $1300 = 3,1139434,$

And of Do. $24 = 1,3802112;$

Therefore the Logarithm of $\frac{1300}{24} = 1,7337322;$

Also Log. of (half $110 =$) $55 = 1,7403627,$

And Log. of $24 = 1,3802112,$

Th. Log. of $\frac{55}{24} = 0,3601515;$

From half the Log. of $\frac{1300}{24}, + 10 = 10,8668661,$

Take the Log. of $\frac{55}{24} = 0,3601515;$

Remains Log. Tang. of $72 : 42 : 16 = 10,5067146:$

From half the Log. of $\frac{1300}{24}, + 10 = 10,8668661,$

Take the Log. Tang. of half the Arc. above found $\left. \begin{array}{l} 36 : 21 : 8 = 9,8668617; \end{array} \right\}$

Remains the Log. of $x = 10,0000044.$

Th. $x = 10.$

QUEST

QUESTION CLXXVIII.

There are three numbers, the difference of whose differences is 4; their sum 40; and product 1764: what are those numbers?

If the second number $- - - = z$,

And diff. of the second and least $= x$;

Then the numbers $- - - = \begin{cases} z - x, \\ z, \\ z + x + 4; \end{cases}$

And (their sum) $- - - 40 = 3z + 4$ by quest.

Th. $- - - 36 = 3z$; And $12 = z$:

But $z - x \times z \times z + x + 4 = z^3 + 4z^2 - zx^2 - 4zx$,

That is, $1728 + 4 \times 144 - 12x^2 - 48x = 1764$ by quest.

Or $- - - 540 = 12x^2 - 48x$;

Th. $- - - 45 = x^2 + 4x$;

But $- - - (45 + 4) = 49 = x^2 + 4x + 4$;

Th. $- - - 7 = x + 2$,

And $- - - 5 = x$.

Q U E S.

QUESTION CLXXIX.

There are three numbers the difference of whose differences is 6; whose sum is 42; and the sum of their squares 794: What are those numbers?

Let the numbers required = $\left\{ \begin{array}{l} x-x, \\ x, \\ x+x+6; \end{array} \right\}$ as in the last qu.

Then $- - - - - 42 = 3x + 6;$

Th. $- 36 = 3x;$ and $12 = x;$

Now their squares $\left\{ \begin{array}{l} 12-x^2 = 144 - 24x + xx, \\ 12 \times 12 = 144, \\ 18+x^2 = 324 + 36x + xx; \end{array} \right.$

Th. (their sum) $- 794 = 612 + 12x + 2xx,$

Or $- - - - - 182 = 2xx + 12x;$

Th. $- - - - - 91 = xx + 6x;$

But $- (91 + 9 =) 100 = xx + 6x + 9;$

Th. $- - - - - 10 = x + 3,$

And $- - - - - 7 = x.$

QUESTION CLXXX.

There are three numbers whose differences are equal, the sum of their squares is 93; and if the first be multiplied by 3, the second by 4, and the third by 5, the sum of those products will be 66: What are those numbers?

If x = the second number, and x = their difference;

$$\text{Then the numbs. are } \begin{cases} x-x \\ x \\ x+x \end{cases} \text{ and their squares } = \begin{cases} xx-2xx+xx, \\ xx, \\ xx+2xx+xx; \end{cases}$$

Therefore (by question) $93 = 3x^2 + 2xx:$

But $\begin{cases} x-x \times 3 = 3x-3x, \\ x \times 4 = 4x, \\ x+x \times 5 = 5x+5x; \end{cases}$

Therefore (by question) $66 = 12x + 2x,$

And $33 - 6x = x,$

Also $1089 - 396x + 36x^2 = xx:$

Now $93 = 3x^2 + 2178 - 792x + 72xx,$

Or, $75x^2 - 792x = -2085;$

Th. $x^2 - \frac{264}{25}x = -\frac{695}{25}:$

But $x^2 - \frac{264}{25}x + \frac{132}{25} = \frac{17424}{625} - \frac{695}{25} = \frac{49}{625};$

Th. $\left(x - \frac{132}{25} = \frac{7}{25} \text{ Or, } \right) \frac{132}{25} - x = \frac{7}{25},$

And $x = \left(\frac{132-7}{27} = \right) 5 = x.$

Otherwise, by Dr *Halley's* method :

$$\text{If } x = \frac{264}{25} = \frac{695}{25}$$

$$\text{Then from Log. of } \frac{695}{25} = 2,8419848,$$

$$\text{Take Log. of } \frac{25}{25} = 1,3979400 ;$$

$$\text{Remains Log. of } \frac{695}{25} = 1,4440448,$$

$$\text{And from Log. of } \left(\frac{264}{2} \right)^{132} = 2,1205739,$$

$$\text{Take Log. of } \frac{25}{25} = 1,3979400,$$

$$\text{Remains Log. of } \frac{132}{25} = 0,7226339 ;$$

$$\text{Now from } \frac{1}{2} \text{ the Log. of } \frac{695}{25} + 10 = 10,7220224,$$

$$\text{Take Log. of } \frac{132}{25} = 0,7226339 ;$$

$$\text{Remains Log. Sine of } 86 : 57 : 45 = 9,9993885 ;$$

$$\text{To and from half the Log. of } \frac{695}{25} = 0,7220224,$$

$$\text{Add or subtract the } \left\{ \begin{array}{l} \text{Log. Tang. of } \frac{1}{2} \text{ the} \\ \text{above found Arc,} \end{array} \right. \left\{ \begin{array}{l} 0 : 1 : 11 \\ 43 : 28 : 52 \end{array} \right. = 9,9769646 ;$$

$$\text{The Log. of } x = \left\{ \begin{array}{l} \text{Sum} = -10 \\ \text{Differ.} = +10 \end{array} \right. \begin{array}{l} = 0,6989870, \\ = 0,7450578 : \end{array}$$

$$\text{Th. } x = 5.$$

QUESTION CLXXXI.

There is a number consisting of three digits, the first of which is to the second as the second to the third; the number itself is to the sum of its digits as 124 to 7; and if 594 be added to it, the digits will be inverted: What is that number?

Suppose x , y , and z , were the digits in their order,

Then $100x + 10y + z$ will be the number;

Now 1st $100x + 10y + z : x + y + z :: 124 : 7$ by q.

That is $700x + 70y + 7z = 124x + 124y + 124z$,

Or $576x = 54y + 117z$;

Th. $x = \left(\frac{54y + 117z}{576} \right) \frac{6y + 13z}{64}$.

And 2d $100z + 10y + x = 100x + 10y + z + 594$,

That is $99z - 594 = 99x$,

Th. $z - 6 = x$;

Th. $z - 6 = \frac{6y + 13z}{64}$,

Or $64z - 384 = 6y + 13z$,

Or $51z - 384 = 6y$;

Th. $\frac{17z - 128}{2} = y$;

But

QUESTION CLXXXI. [Continued.]

But $x:y::y:x$; Th. $x=y$,

That is $x-6 \times x = \frac{17x-128}{2} \times \frac{17x-128}{2}$;

Or $4xx-24x=289x^2-4352x+16384$,

Or $285xx-4328x=-16384$;

Th. $xx-\frac{4328}{285}x=-\frac{16384}{285}$;

But $xx-\frac{4328}{285}x+\frac{2164}{285}=\left(\frac{4682896}{285 \times 285}-\frac{16314}{285}\right)=\frac{13456}{285 \times 285}$.

Th. $x-\frac{2164}{285}=\frac{116}{285}$; and $x=\left(\frac{2164+116}{285}=\frac{2280}{285}\right) 8$.

QUES.

QUESTION CLXXXII.

There are three numbers whose squares have equal differences; their sum is 13; and the sum of their squares 75: What are those numbers?

Let x , y , and z , be the numbers required,
And d the difference of their squares;

$$\text{Then } \left\{ \begin{array}{l} yy - d = xx, \\ yy = yy, \\ yy + d = zz; \end{array} \right.$$

And (their sum) $3y = 75$ by quest.

Th. $yy = 25$; and $y = 5$:

Now $(x + y + z =) x + 5 + z = 13$,

And $x + z = 8$,

Th. $xx + 2xz + zz = 64$;

Also $xx + zz = (y^2 - d + y^2 + d =) 2yy$,

That is $xx + zz = (25 \times 2 =) 50$,

And (by subtr.) $2xz = 64 - 50 = 14$;

Th. $xz = 7$:

$$\text{By (quest. 160)} \left\{ \begin{array}{l} z = (8 + \sqrt{64 - 4 \times 7 \times \frac{1}{2}} =) 7, \\ x = (8 - \sqrt{64 - 4 \times 7 \times \frac{1}{2}} =) 1. \end{array} \right.$$

QUES.

QUESTION CLXXXIII.

What two numbers are those, whose sum multiplied by the greater produces 77; and whose difference multiplied by the lesser gives 12?

Let x = greater and y = lesser number;

$$\begin{array}{l} \text{Then } (x+y \times x =) xx + xy = 77 \\ \text{And } (x-y \times y =) xy - yy = 12 \end{array} \left. \vphantom{\begin{array}{l} \text{Then } (x+y \times x =) xx + xy = 77 \\ \text{And } (x-y \times y =) xy - yy = 12 \end{array}} \right\} \text{by quest.}$$

$$\text{By } \begin{cases} \text{first} & - & - & xy = 77 - xx, \\ \text{second} & - & - & xy = 12 + yy; \end{cases}$$

$$\text{Th. } - & - & 77 - xx = 12 + yy.$$

$$\text{Or } - & - & 65 + yy = xx;$$

$$\text{Th. } - & - & \sqrt{65 - yy} = x;$$

$$\text{But } y \sqrt{65 - yy} - yy = 12 \text{ by second,}$$

$$\text{Or } - & - & \sqrt{65 - yy} = \frac{12 + yy}{y};$$

$$\text{Th. } - & - & 65 - yy = \frac{144 + 24yy + y^2}{yy},$$

$$\text{Or } - & - & 65y^2 - y^4 = 144 + 24yy + y^2.$$

$$\text{Or } - & - & -144 = 2y^4 - 41yy;$$

$$\text{Th. } - & - & -\frac{144}{2} = y^4 - \frac{41}{2}yy;$$

$$\text{But } \left(\frac{1681}{16} - \frac{144}{2} = \right) \frac{529}{16} = y^4 - \frac{41}{2}yy + \frac{41^2}{4};$$

$$\text{Th. } - & - & -\frac{23}{4} = y^2 - \frac{41}{4};$$

$$\text{And } - & \left(\frac{41+23}{4} = \right) 16 = y^2;$$

$$\text{Whence } - & - & 4 = y.$$

QUESTION CLXXXIV.

What two numbers are those, whose product is 15 ;
and the sum of their squares is 34 ?

If the greater number $\quad = x ;$

Then the lesser $\quad = \frac{15}{x},$

And $\quad xx + \frac{15 \times 15}{xx} = 34$ by quest.

Or $\quad x^4 + 225 = 34xx ;$

Th. $\quad x^4 - 34x^2 = -225 :$

But $x^4 - 34x^2 + 17 \times 17 = (289 - 225 =) 64 ;$

Th. $x^2 - 17 = 8 ;$ and $x^2 = (17 + 8 =) 25 :$

Whence $\quad x = 5.$

QUESTION CXLXXXV.

What two numbers are those whose product is 35 ; and
the difference of their cubes is 218 ?

If the lesser number be $x ;$

Then the greater is $\frac{35}{x} :$

Now by quest. $\quad 218 = \left(\frac{35}{x} \right)^3 - x^3 = \frac{42875}{x^3} - x^3,$

Or $\quad 218x^3 = 42875 - x^6 ;$

Th. $\quad x^6 + 218x^3 = 42875 :$

But $x^6 + 218x^3 + 109^3 = (42875 + 11881 =) 54756 ;$

Th. $x^3 + 109 = 234 ;$ and $x^3 = (234 - 109 =) 125 :$

Whence $\quad x = (\sqrt[3]{125} =) 5.$

QUES-

QUESTION CLXXXVI.

What two numbers are those, whose sum is 5; and the sum of their 5th powers 275?

If x and y represent the numbers required,

Then (because $x+y=5$) $5x^3y^2+5x^2y^3=25x^2y^2$;

And $x^3+3x^2y+3xy^2+y^3=125$;

Or $5x^4y+15x^3y^2+15x^2y^3+5xy^4=625xy$;

And $x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5=3125$;

But $x^5 + \dots + y^5 = 275$;

Th. (by sub.) $5x^4y+10x^3y^2+10x^2y^3+5xy^4=2850$;

And (by ditto) $5x^3y^2+5x^2y^3=625xy-2850$;

Also by 1st $25x^2y^2=625xy-2850$;

Or $x^2y^2=25xy-114$;

Th. $x^2y^2-25xy=-114$;

But $x^2y^2-25xy+\frac{25}{4}=\frac{625}{4}-114$

$$=\frac{169}{4};$$

Th. $(xy-\frac{25}{2})$ or $\frac{25}{2}-xy=\frac{13}{2}$;

And $(\frac{25-13}{2}=) 6=xy$;

Now (by quest 160) $\begin{cases} x=(5+\sqrt{25-4\times 6\times \frac{1}{2}})=3, \\ y=(5-\sqrt{25-4\times 6\times \frac{1}{2}})=2. \end{cases}$

QUESTION CLXXXVII.

There are three numbers whose differences are equal, their sum is 9; and the sum of their fourth powers 707: What are those numbers?

If $\begin{cases} z-x \\ z \\ z+x \end{cases}$ be the numbers required;

Then $3z =$ (their sum $=$) 9; and $z=3$:

$$\text{Now } \begin{cases} 3-z^4 = 81-108x^2+54x^4-12x^6+x^8, \\ 3^4 = 81, \\ 3+x^4 = 81+108x^2+54x^4+12x^6+x^8; \end{cases}$$

Whence $707 = 243+108x^2+2x^4$ by quest.

Or $464 = 108x^2+2x^4$,

Th. $232 = x^4+54xx$:

But $961 = x^4+54xx+729$;

Th. $31 = x^2+27$; or $4=xx$; and $2=x$.

QUES.

QUESTION CLXXXVIII.

The reckoning of some men and boys at a tavern came to 53 shillings; of which the men paid each as many shillings as there were men in company; and the boys paid each 1 shilling; now if the number of men and boys were interchanged, the expence (regulated as before) would have been but 23 shillings: How many men and boys were there?

Suppose there were x men, and y boys;

Then $xx + y = 53$: Or $xx = 53 - y$, } per qu.

And $yy + x = 23$: Or $x = 23 - yy$.

Whence $529 - 46y^2 + y^4 = 53 - y$;

Th. $y^4 - 46y^2 + y + 476 = 0$.

If $y = -1$; Then $430 = 0$: } of which $\left\{ \begin{array}{l} 1, 2, 5, 10, 43, \\ 1, 2, 4, 7, 14, 17, \\ 1, 2, 4, 6, 9, \end{array} \right\}$ are } divisors

$y = 0$; Then $476 = 0$:

$y = 1$; Then $432 = 0$:

Where 5, 4, and 3, differ by unity;

And $\frac{y^4 - 46y^2 + y + 476}{y - 4} = y^3 + 4y^2 - 30y = 1197$

Th. $y = 4$.

G 5 QUES.

QUESTION CLXXXIX.

There are three numbers which have equal differences ; if the square of the least be added to the product of the two greater, the sum will be 576 ; but if the square of the greatest be added to the product of the two lesser, the sum will be 792 : What are these numbers ?

If $\begin{cases} z-x \\ z \\ z+x \end{cases}$ represent the numbers required ;

$$\text{Then } \frac{z-z^2}{z+x} + \frac{z+x}{z-x} \times z = 576 \quad \text{by quest.}$$

$$\text{And } \frac{z+z^2}{z-x} + \frac{z-x}{z+x} \times z = 792$$

Now } $(4zx - 2zx =) 2z = (792 - 576 =) 216,$
their }
diff. }

$$\text{Or } - \quad zx = 108 ; \text{ and } z = \frac{108}{x} :$$

$$\text{But } \frac{108}{x} - x + \frac{108}{x} + x \times \frac{108}{x} = 576 \text{ by first ;}$$

$$(\text{Since } \frac{11664}{xx} - 216 + xx = \frac{108}{x} - x,$$

$$\text{And } \frac{11664}{xx} + 108 = \frac{108}{x} + x \times \frac{108}{x}) :$$

$$\text{Th. } \frac{23328}{xx} - 108 + xx = 576,$$

$$\text{Or } - \quad \frac{23328}{xx} + xx = (576 + 108 =) 684,$$

$$\text{Or } - \quad 23328 + x^4 = 684xx ;$$

$$\text{Th. } - \quad x^4 - 684xx = -23328 :$$

$$\text{But } x^4 - 684xx + 342^2 = (116964 - 23328 =) 93636 ;$$

$$\text{Th. } (x^2 - 342 ; \text{ Or } 342 - xx = 306,$$

$$\text{And } - \quad (342 - 306 =) 36 = xx ; \text{ also } 6 = x.$$

Q U E S.

QUESTION CXC.

There are two numbers the sum of whose squares is 89; and their sum multiplied by the greater produces 104: What are those numbers?

Suppose x = greater, and y = lesser;

$$\begin{array}{l} \text{Then } \dots \dots \dots xx + yy = 89 \\ \text{And } (x + y)x = xx + xy = 104 \end{array} \left. \vphantom{\begin{array}{l} \text{Then} \\ \text{And} \end{array}} \right\} \text{by quest.}$$

$$\text{By } \left\{ \begin{array}{l} \text{first} \dots \dots \dots xx = 89 - yy \\ \text{second} \dots \dots \dots xx = 104 - xy \end{array} \right.$$

$$\text{Th. } \dots \dots \dots 89 - yy = 104 - xy,$$

$$\text{And } \dots \dots \dots x = \frac{15 + y}{y};$$

$$\text{Also } \dots \dots \dots xx = \frac{225 + 30y + y^2}{yy};$$

$$\text{But } \frac{225 + 30y + y^2}{yy} + yy = 89 \text{ by first,}$$

$$\text{Or } 225 + 30y + y^2 + y^2 = 89 yy,$$

$$\text{Th. } \dots \dots \dots 2y^2 + 59y^2 = -225;$$

$$\text{But } y^2 - \frac{59}{2}y^2 + \frac{59}{4}y^2 = \left(\frac{3481}{16} - 225 \right) = \frac{1681}{16};$$

$$\text{Th. } y^2 - \frac{59}{4}y^2 = \frac{41}{4}; \text{ and } y^2 = \left(\frac{59 + 41}{4} \right) = 25;$$

$$\text{Whence } \dots \dots \dots y = 5.$$

QUES.

QUESTION CXCI.

What two numbers are those, whose sum is 14; and if the sum of their squares be multiplied by the sum of their cubes the product will be 72800?

If the diff. of the 2 num. = $2x$;

$$\text{Then (by qu.) } \left\{ \begin{array}{l} \frac{14+2x}{2} = 7+x = \square \\ \frac{14-2x}{2} = 7-x = \sqcap \end{array} \right\} \text{ number;}$$

$$\text{And } \left\{ \begin{array}{l} 49+14x+x^2 = \square \\ 49-14x+x^2 = \square \end{array} \right\} \text{sq. of } \{ \begin{array}{l} 5 \\ 3 \end{array} \}$$

$$\text{Th. } 98+2x = \text{sum of sqrs.}$$

$$\text{Also } \left\{ \begin{array}{l} 343+147x+21x^2+x^3 = \square \\ 243-147x+21x^2+x^3 = \square \end{array} \right\} \text{cu. of } \{ \begin{array}{l} 5 \\ 3 \end{array} \}$$

$$\text{Th. } 686+42x^2 = \text{sum of cu.}$$

$$\text{But } 98+2xx \times 686+42xx = 72800 \text{ by quest.}$$

$$\text{That is, } 67228+5488x^2+84x^4 = 72800,$$

$$\text{Or } 84x^4+5488x^2+ = 5572,$$

$$\text{Or } 3x^4+196x^2 = 199;$$

$$\text{Th. } x^4 + \frac{196}{3}xx = \frac{199}{3};$$

$$\text{But } x^4 + \frac{196}{3}xx + \frac{9}{3} = \frac{199}{3} + \frac{9604}{3} \\ = \frac{10201}{9};$$

$$\text{Th. } x^2 + \frac{98}{3} = \frac{101}{3}; \text{ and } x^2 = \left(\frac{101-98}{3} = \right) 1;$$

$$\text{Whence } x = 1;$$

$$\text{And } \left\{ \begin{array}{l} (7+1) = 8 \\ (7-1) = 6 \end{array} \right\} \text{the numbers required.}$$

Q U E S.

QUESTION CXCV

What two numbers are to a sum of 35

QUESTION CXCV

'Tis required to divide the number 35, into two such parts; that the square of the lesser, multiplied by the greater, may produce 750?

Let x = the lesser part,

Th. $35 - x$ = the greater part;

And $35 - x \times xx = 35xx - x^3 = 750$ per question;

Th. $x^3 - 35xx + 750 = 0$:

If $x = -1$; Then $714 = 0$;
 If $x = 0$; Then $750 = 0$;
 If $x = 1$; Then $716 = 0$;

of which $\left\{ \begin{array}{l} 1, 2, 3, 6, 7, 14, 17, \\ 1, 2, 3, 5, 6, 10, 25, \\ 1, 2, 4, 170, \end{array} \right\}$ are divisi.

Now 6, 5, and 4 differ by unity;

And $\frac{x^3 - 35xx + 750}{x - 5} = xx - 30x - 150$;

Th. $x = 5$.

QUES.

QUESTION CXCIH.

What two numbers are as 3 to 2; the sum of whose cubes is 280?

If x = the greater of those numbers,

Then $(3 : 2 :: x :) \frac{2x}{3}$ = the lesser;

Whence $(x^3 + \frac{8x^3}{27}) \frac{35x^3}{27} = 280$, by quest.

Or (by Division) $\frac{x^3}{27} = (8 =) 2^3$,

Or (by Multiplication) $x^3 = (2^3 \times 27 =) 2^3 \times 3^3$;

Th. $x = (2 \times 3 =) 6$.

QUESTION CXCIV.

A bought 7 sheep more than B for 15*l*.; now the square of B's sheep is equal to the number of shillings that A paid for each sheep: How many did each buy?

If B bought x sheep, then A bought $x+7$;

And $x+7$ sheep at xx shill. per sheep cost x^2+7xx shill.

But $x^2+7xx = (15 \times 20 =) 300$ per quest.

Th. $x^2+7xx-300=0$.

If $x=-1$; $294=0$;
 If $x=0$; $300=0$;
 If $x=1$; $292=0$;

Where 6, 5 & 4 differ by Unity;

And $\frac{x^2+7xx-30}{x-5} = xx+2x+60$;

Th. $x=5$.

QUES.

QUESTION CXCV.

A man playing at hazard, won the first throw as much as he had in his pocket; at the second throw he won 5 shillings more than the square root of what he then had; at the third throw he won the square of all he then had; and then he had 112 l. 16 s. : What had he at first?

Suppose that at first he had x shillings;

Then after winning x he had $2x$;

And winning $\sqrt{2x} + 5$ he had $2x + \sqrt{2x} + 5$;

Substitute

Then winning y he had $y + y$;

Now

$$y + y = (112 \text{ l. } 16 \text{ s.}) = 2256 \text{ sh. by qu.}$$

But

$$y + y + \frac{1}{2} = (2256 + \frac{1}{2}) = \frac{9025}{4};$$

Th.

$$y + \frac{1}{2} = \frac{95}{2}; \text{ And } y = \frac{95-1}{2} = 47;$$

But (by restitution)

$$2x + \sqrt{2x} + 5 = 47,$$

Or

$$2x + \sqrt{2x} = (47-5) = 42,$$

Th.

$$x + \frac{\sqrt{2x}}{2} = \left(\frac{42}{2}\right) = 21;$$

But

$$x + \frac{\sqrt{2x}}{2} + \frac{2}{16} = \left(21 + \frac{2}{16} = \frac{338}{16}\right) = \frac{169 \times 2}{16};$$

Th.

$$x + \frac{\sqrt{2x}}{2} + \frac{2}{16} = \frac{13\sqrt{2}}{4}; \text{ And } x = \frac{13-1 \times \sqrt{2}}{4} = \frac{12\sqrt{2}}{4} = 3\sqrt{2};$$

Whence

$$x = (3\sqrt{2} \times 3\sqrt{2} = 9 \times 2 =) 18.$$

Q U E S.

QUESTION CXCVI.

What two numbers are as 5 to 6 ; and the sum of their cube roots is 6 ?

Let y = the lesser of the numb. requi.

Th. $(5 : 6 :: y : \frac{6y}{5})$ = the greater number ;

But $\sqrt[3]{\frac{6y}{5}} + y^{\frac{1}{3}} = 6$ by question,

That is $\sqrt[3]{\frac{6}{5}} \times y^{\frac{1}{3}} + y^{\frac{1}{3}} = 6$; put $a = \sqrt[3]{\frac{6}{5}}$

Then $y^{\frac{1}{3}} = \frac{6}{a+1}$:

And $y = \left(\frac{6}{a+1} \right)^3$

But $(a =) \sqrt[3]{\frac{6}{5}} = \sqrt[3]{1,2} = 1,06273$ (see the operation)

Let $\sqrt[3]{1,2} = 1+r$,

Then $1,2 = 1 + 3r + 3r^2 + r^3$,

And $0,2 = 3r + 3r^2 + r^3$,

QUES.

QUESTION CXCI. [Continued.]

Or $0, 2 = 3r + 3r^2$ { because r^3 is small
in comparison of
 r & r^2 ;

Th $\frac{0, 2}{3} = r + r^2,$

And $\left(\frac{1}{4} + \frac{2}{30}\right) \frac{19}{60} = \frac{1}{4} + r + r^2$

Then $\frac{1}{2} \sqrt{\frac{19}{15}} = \frac{1}{2} + r$

And $\frac{\sqrt{\frac{19}{15}} - 1}{2} = r = \frac{\sqrt{1,26} - 1}{2} = \begin{cases} \frac{1,1254 - 1}{2} \\ = 0,06273 \end{cases}$

Whence $\sqrt[3]{1,2} = (1 + r) = 1,06273$

Then $y = \frac{6}{2,0627} \Big| ^3 = 24,612.$

This operation is performed in a manner like that of Dr. Halley's irrational theorem.

QUESTION CXCVII.

'Tis required to divide the number 56, into two such parts that the sum of their cubes may be to the cube of the greater as 245 to 238?

Let x be the greater, and y the lesser part;

Then $x^3 + y^3 : x :: 245 : 238$ by quest.

That is, $238x^3 + 238y^3 = 245x^3$,

Or $- - - - 238y^3 = 7x^3$;

Th. $- - - - 34y^3 = x^3$,

And $- - - - \sqrt[3]{34} \times y = x$;

But $- - - \sqrt[3]{34} \times y + y = 56$ by quest.

Th. $- - - - y = \frac{56}{\sqrt[3]{34} + 1}$;

But $- - - \sqrt[3]{34} = 3,239609$; see the operation,

Let $- - - \sqrt[3]{34} = 3 + r$;

Then $- - - 34 = 27 + 27r + 9r^2 + r^3$,

And $- - - 7 = 27r + 9r^2 + r^3$,

Or $- - - 7 - r^3 = 27r + 9r^2$,

Th. $- - - \frac{7}{9} - \frac{r^3}{9} = r^2 + 3r$;

But $\frac{9}{4} + \frac{7}{9} - \frac{r^3}{9} = r^2 + 3r + \frac{2}{3}$,

Or $- - - \frac{109}{36} - \frac{r^3}{9} = r^2 + 3r + \frac{2}{3}$;

Th. $\sqrt{\frac{109}{36}} - \frac{r}{9} = r + \frac{2}{3}$;

QUES-

QUESTION CXCVII. [Continued.]

Or $\sqrt{\frac{109}{36}} = r + \frac{1}{2}$ (rejecting $\frac{r^3}{9}$ as inconsiderable)

Th. $\sqrt{\frac{109}{36}} = r + \frac{1}{2}$

Or $1,74005, \text{ \&c.} = r + \frac{1}{2}$

Or $0,24005, \text{ \&c.} = r$

Now $0,001536, \text{ \&c.} = \frac{r^3}{9}$

And $\frac{0,001536, \text{ \&c.}}{1,74005 \times 2} = 0,000441 = \sqrt{\frac{109}{36}} - \sqrt{\frac{109}{36} - \frac{r^3}{9}}$

Th. $0,239609, \text{ \&c.} = r = (0,24005, \text{ \&c.} - 0,000441)$

And $3,239609, \text{ \&c.} = (3 + r) \sqrt[3]{34}$

Therefore $3 = \frac{56}{3,239609 + 1} = 13,2088, \text{ \&c.}$

Q U E S.

QUESTION CXCVIII.

What two numbers are those the sum of whose squares is 58; and their product multiplied by the greater gives 147?

If x be the greater, and y the lesser of those numbers;

Then $xx + yy = 58$; Or $xx = 58 - yy$:

And $(xy \times x =) xxy = 147$; Or $xx = \frac{147}{y}$:

Th. $58 - yy = \frac{147}{y}$;

Or $58y - y^3 = 147$.

Or $y^3 - 58y + 147 = 0$.

* If $y = -1$; $204 = 0$: } $\frac{58}{0}$ which { 1, 2, 3, 4, 6, 12, 17, }
 If $y = 0$; $147 = 0$: } { 1, 3, 7, 21, 49, }
 If $y = 1$; $90 = 0$: } { 1, 2, 3, 5, 6, 9, 10, 15, 18, } are divi.

Now 4, 3, 2, (one of which numbers is taken from each of those ranks of divisors.) differ by unity;

Also $y - 3$ will divide $y^3 - 58y + 147$, and leave no remainder: See the work.

$$\begin{array}{r}
 y-3 \mid y^3 - 58y + 147 \quad (y^2 + 3y - 49) \\
 \underline{y^3 - 3y^2} \\
 + 3y^2 - 58y \\
 \underline{3y^2 - 9y} \\
 - 49y + 147 \\
 \underline{- 49y + 147} \\
 0 \quad . \quad 0
 \end{array}$$

Th. $y = 3$; and $xx = \frac{147}{3} = 49$. Or $x = 7$.

• By Sir Isaac Newton's method of finding divisors.

Q U E S -

QUESTION CXCVIX.

'Tis required to divide 16^l. so, among two persons, that the cube of the one's share, may exceed the cube of the others, by 386 ?

If the greater share be x pounds,
Then the lesser will be $16 - x$;

Now $x^3 - 16 - x^3 = 386$ by qu.

And $4096 - 768x + 48x^2 - x^3 = 16 - x^3$,

Th. $2x^3 - 48x^2 + 768x - 4096 = 386$,

Or $x^3 - 24x^2 + 384x - 2048 = 193$;

Th. $x^3 - 24x^2 + 384x - 2241 = 0$:

If $x = -1$; $2650 = 0$:
 $x = 0$; $2241 = 0$:
 $x = 1$; $1880 = 0$:
 of which $\left\{ \begin{array}{l} 1, 2, 5, 10, 25, 53 \\ 1, 3, 9, 27, 83 \\ 1, 2, 4, 5, 8, 10, 20 \end{array} \right\}$ divisions.

Where 10, 9, and 8 differ by unity ;

And $\frac{x^3 - 24x^2 + 384x - 2241}{x - 9} = xx - 15x + 249$:

Th. $x = 9$.

QUES.

QUESTION CC.

What two numbers are those, whose product is 945 ; and if from the square of the greater, the lesser be taken, the remainder will be 1198 ?

Suppose x = the greater, and y the lesser of those numbers ;

Then $xy = 945$; Or $y = \frac{945}{x}$

And $xx - y = 1198$; Or $y = xx - 1198$

} per quest.

Th. $xx - 1198 = \frac{945}{x}$;

Or $x^3 - 1198x - 945 = 0$.

If $x = -1$; Then $252 = 0$

If $x = 0$; Then $945 = 0$

If $x = 1$; Then $2142 = 0$

of which { 1, 2, 3, 4, 6, 7, 9, 12, 18, 21, 28, 36, 1, 3, 5, 7, 9, 15, 21, 27, 35, 45, 105, 1, 2, 3, 6, 7, 9, 12, 14, 17, 18, 21, 24, 28, 34,

} are divisi.

Now 36, 35, 34 differ by unity ;

And $\frac{x^3 - 1198x - 945}{x - 35} = x^2 + 35x + 27$.

Th. $x = 35$.

QUESTION CCI.

A mason had two cubical pieces of marble, the difference of whose solidities was 386; now a solid of equal base with the greater, and as high as the lesser, would contain 567 inches: What were the sides of those cubes?

Suppose x = the side of the greater; y the side of the lesser;

Then by quest. $\begin{cases} x^3 - y^3 = 386, \\ xxy = (567 =) 9 \times 9 \times 7; \end{cases}$

And by second) $y = \left(\frac{9 \times 9 \times 7}{xx} = \right) \frac{9^2 \times 7}{x^2};$

Th. $y^3 = \frac{9^6 \times 7^3}{x^6};$

Now (by first) $x^3 - \frac{9^6 \times 7^3}{x^6} = 386;$

Or $x^9 - 386x^6 - 9^6 \times 7^3 = 0;$

But $\frac{x^9 - 386x^6 - 9^6 \times 7^3}{x^3 - 9^3} = x^6 + 343x^3 + 9^3 \times 7^3;$

Th. $x^3 = 9^3;$

And $x = 9.$

QUESTION CCII.

A mason had 2. cubical pieces of marble, the side of one exceeded the side of the other by 7 inches; and the solid inches in both made 873: What were their dimensions?

If the side of the lesser cube $= x;$

Then the side of the greater $= x + 7;$

And $x^3 + x^3 + x + 7^3 = 873,$ by quest.

Hence $2x^3 + 21x^2 + 147x - 530 = 0;$

If $x = -1;$ $\left| \begin{array}{l} -658 = 0: \\ -530 = 0: \\ -360 = 0: \end{array} \right| \begin{array}{l} 1, 2, 7, 14, 47, \\ 1, 2, 5, 10, 53, \\ 1, 2, 3, 4, 5, 6, 8, \text{ \&c.} \end{array} \left| \begin{array}{l} \text{are divi.} \end{array} \right|$

If $x = 0;$ $\left| \begin{array}{l} -658 = 0: \\ -530 = 0: \\ -360 = 0: \end{array} \right| \begin{array}{l} 1, 2, 7, 14, 47, \\ 1, 2, 5, 10, 53, \\ 1, 2, 3, 4, 5, 6, 8, \text{ \&c.} \end{array} \left| \begin{array}{l} \text{are divi.} \end{array} \right|$

If $x = 1;$ $\left| \begin{array}{l} -658 = 0: \\ -530 = 0: \\ -360 = 0: \end{array} \right| \begin{array}{l} 1, 2, 7, 14, 47, \\ 1, 2, 5, 10, 53, \\ 1, 2, 3, 4, 5, 6, 8, \text{ \&c.} \end{array} \left| \begin{array}{l} \text{are divi.} \end{array} \right|$

Where 7, 5, 3, differ by (2) a divisor of (2) the coefficient of (x^3) the highest power of (x) the unknown quantity;

And $\frac{2x^3 + 21x^2 + 147x - 530}{2x - 5} = x^2 + 13x + 106;$

Th. $2x = 5; \text{ And } x = \frac{5}{2}.$

H

QUES.

QUESTION CCLIII.

Two masons A and B jointly perform a piece of work in 12 days; now if the sum of the days in which they could each have separately performed the same, be multiplied by the days in which A alone (he working quicker than B) could have done it, the product will be 1000: In what time could each do it?

Suppose A could do it alone in x days,

And B in y days;

Then $(x : 1 :: 12 :) \frac{12}{x} =$ the work done by A ,

And $(y : 1 :: 12 :) \frac{12}{y} =$ the work done by B ,

} in 12 days:

$$\text{Th. } \frac{12}{x} + \frac{12}{y} = 1,$$

$$\text{Or } 12x + 12y = xy,$$

$$\text{Or } 12y = xy - 12x;$$

$$\text{Th. } \frac{12x}{x-12} = y.$$

$$\text{And } \frac{12xx}{x-12} = 1000.$$

But $(x+y \times x =) xx + xy = 1000$ by quest.

$$\text{That is } xx + \frac{12xx}{x-12} = 1000,$$

$$\text{Or } x^3 - 12xx + 12x^2 = 1000x - 12000;$$

$$\text{Th. } x^3 - 1000x + 12000 = 0.$$

$$\text{If } x = -1; \quad 12999 = 0;$$

$$\text{If } x = 0; \quad 12000 = 0;$$

$$\text{If } x = 1; \quad 11001 = 0;$$

are divi.

Now 21, 20, 19 differ by unity;

$$\text{And } \frac{x^3 - 1000x + 12000}{x-20} = x^2 + 20x - 600;$$

$$\text{Th. } x = 20;$$

$$\text{And } \left(\frac{12 \times 20}{20-12} \right) 30 = y.$$

QUES-

QUESTION CCIV.

A, *B*, and *C*, who among them had 2000 shillings, went to play, and *B* lost to *A* the square root of what *A* began with, and had 341 shillings left; but if he had lost to *C* the cube root of what *C* began with, he would have had 362 shillings left: What sum had each at first?

Suppose *A* had x ; *B*, y ; and *C*, z shillings;

$$\begin{aligned} \text{Then } x + y + z &= 2000; \text{ or } y = 2000 - x - z, \\ y - x^{\frac{1}{2}} &= 341; \text{ or } y = 341 + x^{\frac{1}{2}}, \\ y - z^{\frac{1}{3}} &= 362; \text{ or } y = 362 + z^{\frac{1}{3}}, \end{aligned} \quad \left. \vphantom{\begin{aligned} y - x^{\frac{1}{2}} &= 341; \text{ or } y = 341 + x^{\frac{1}{2}}, \\ y - z^{\frac{1}{3}} &= 362; \text{ or } y = 362 + z^{\frac{1}{3}}, \end{aligned}} \right\} \text{by qu.}$$

Whence $341 + x^{\frac{1}{2}} = 362 + z^{\frac{1}{3}},$

Or $x^{\frac{1}{2}} = 21 + z^{\frac{1}{3}};$

Th. $x = 441 + 42z^{\frac{1}{3}} + z^{\frac{2}{3}};$

Now (by 1st and 3d) $362 + z^{\frac{1}{3}} = 2000 - 441 - 42z^{\frac{1}{3}} - z^{\frac{2}{3}} - z;$

Or $z + z^{\frac{2}{3}} + 43z^{\frac{1}{3}} - 1197 = 0.$

If $z^{\frac{1}{3}} = -2;$	Then	$-1287 = 0;$	of which	$1, 3, 9, 11, 13, 33, 39,$	are divisors;
If $z^{\frac{1}{3}} = -1;$		$-1240 = 0;$		$1, 2, 4, 5, 8, 10, 20, 31, 40,$	
If $z^{\frac{1}{3}} = 0;$		$-1197 = 0;$		$1, 3, 7, 9, 19, 21, 57,$	
If $z^{\frac{1}{3}} = 1;$		$1152 = 0;$		$1, 2, 3, 4, 6, 8, 12, 16,$	
If $z^{\frac{1}{3}} = 2;$		$1099 = 0;$		$1, 7, 157,$	

Where 11, 10, 9, 8, 7, differ by unity;

And $\frac{z + z^{\frac{2}{3}} + 43z^{\frac{1}{3}} - 1197}{z^{\frac{1}{3}} - 9} = z^{\frac{2}{3}} + 10z^{\frac{1}{3}} + 133;$

Th. $z^{\frac{1}{3}} = 9;$

And $z = (9^3 =) 729.$

QUESTION CCV.

A charitable person gave 20 shillings among some poor persons, men and women, and to each as many pence as there were poor men; now if he had given a like sum to a number of persons equal to the square of the number of poor women it would have cost him 64 shillings: How many men and women did he give to?

Suppose to x men and y women.

Then $(x+y \times x) \quad xx + xy = (20 \times 12 =) 240$ } by quest.

And $yy \times x = 64 \times 12 = 768$

Now $240 - xx = (xy =) \frac{768}{y}$

Th. $240 - \frac{768}{y} = (xx =) \frac{768}{y}$

That is $240 - \frac{768}{y} = \frac{589824}{yy}$

Or $240 - \frac{768}{y} = \frac{12288}{yy}$

Th. $5y^4 - 16y^3 - 12288 = 0$

If $y = -1$; Then $12267 = 0$; $\frac{1}{2} \left\{ \begin{array}{l} 1, 3, 9, 29, 47, \\ 1, 2, 3, 4, 6, 8, 12, 16, 48, \end{array} \right\}$

$y = 0$; Then $12288 = 0$; $\frac{1}{3} \left\{ \begin{array}{l} 1, 2, 3, 4, 6, 8, 12, 16, 48, \\ 1, 7, 49, 251, \end{array} \right\}$

$y = 1$; Then $12299 = 0$; $\frac{1}{5}$

Where 9, 8, 7, differ by (unity) a divisor of (5) the coefficient of
(y^4) the highest power of (y) the unknown quantity;

And $\frac{5y^4 - 16y^3 - 12288}{y - 8} = 5y^3 + 24y^2 + 192y + 1536$;

Th. $y = 8$.

QUESTION CCVI.

If the product of the solidities of two cubes, whose sides differ by 4, be multiplied by the solidity of the greater, it will produce 3176523: What are their sides?

If x = side of the gr. and y = side of the less cube;

Then $x - y = 4$; Or $x = y + 4$ } by quest.
 And $x^3 \times x^3 y^3 = 3176523$; Or $x^6 = \frac{3176523}{y^3}$

Whence $y + 4 = \sqrt[6]{\frac{3176523}{y^3}}$

Or $y + 4 \times y^3 = (3176543 =) 147^3$

Th. $y + 4 \times y = (147 =) 49 \times 3$

And $y + 4 \times y^{\frac{1}{2}} = 7\sqrt{3}$; Th. $y^{\frac{1}{2}} + 4y^{\frac{1}{2}} = 7\sqrt{3}$

Substitute $z - x = y^{\frac{1}{2}}$; and $z^3 - x^3 = 7\sqrt{3}$

Then $z^3 - 3z^2x + 3zx^2 - x^3 - 4x - 4x = z^3 - x^3$

Th. $z^3 - x^3 = 7\sqrt{3}$

But $(z^3 - x^3 =) z^3 - \frac{4^3}{3^3 z^3} = 7\sqrt{3}$

Th. $z^3 - \frac{64}{27 z^3} = 7\sqrt{3}$

But $z^6 - 7\sqrt{3} \times z^3 + \frac{49 \times 3}{4} = (\frac{64}{27} + \frac{147}{4} =) \frac{4225}{36 \times 3}$

Th. $z^3 - \frac{7\sqrt{3}}{2} = \frac{65}{6\sqrt{3}}$; and $z^3 = \frac{65}{6\sqrt{3}} + \frac{7\sqrt{3}}{2}$

$\frac{64}{27} = \frac{64}{27}$

Whence $z = \frac{3\sqrt{3}}{4\sqrt{\frac{1}{3}}}$

But $(z^3 - x^3 =) \frac{64}{3\sqrt{3}} - x^3 = 7\sqrt{3}$

Th. $\frac{1}{\sqrt{3}} = x$

Now $(x - x =) \frac{4}{\sqrt{3}} - \frac{1}{\sqrt{3}} = \frac{3}{\sqrt{3}} = y^{\frac{1}{2}}$

And $(\frac{3}{\sqrt{3}} =) 9 = x$

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QUESTION CCVH.

What two numbers are those, whose sum, and product, being severally multiplied by the lesser will produce 175, and 250?

If $x =$ gr. and $y =$ the less. of those numbers;

Then $(x + y \times y =) xy + y = 175$
And $(xy \times y =) xy = 250$ } per quest.

Th. $\frac{175 - yy}{y} = (x =) \frac{250}{yy}$;

And $175y - y^3 = 270$.

Th. $y = 1,4458$, &c. See the operation*.

Let $y = \frac{1}{2} - r$;

Then $175y = \frac{325}{2} - 175r$,

And $y^3 = \frac{1}{8} - \frac{3}{4}r + \frac{3}{2}r^2 - r^3$;

By sub. $250 = \frac{2073}{4} - \frac{673}{4}r - \frac{3}{2}r^2 + r^3$,

Or $2000 = 2073 - 1346r - 36r^2 + 8r^3$,

(By transp.) $1346r + 36r^2 - 8r^3 = 73$;

Now, 1st. let $1346r = 73$;

Then $r = (\frac{73}{1346} =) 0,0542$,

2d. Let $1346r + 36r^2 = 73$.

That is $1346r + 36 \times 0,0542r = 73$.

Or $1346r + 1,8512r = 73$;

Th. $(\frac{73}{1347,8512} =) 0,054156 = r$;

3d. $1346r + 36r^2 - 7r^3 = 73$,

That is, $1346r + 1,949616r - 0,023456r = 73$;

Th. $r = \frac{73}{1347,92616} = 0,0541572$, &c.

Now $\frac{1}{2} - r = y$;

That is $1,5 - 0,0541572 = 1,4458429$, &c. = y .

* This operation is conformable to Dr. Halley's rational theorem.

Q U E S.

QUESTION CCVIII.

What two numbers are those, the difference of whose squares is 9; and the sum of their cubes 189?

Now $xx - yy = 9$; Or $xx = 9 + yy$, } by quest.

And $x^3 + y^3 = 189$; Or $x^3 = 189 - y^3$, }

Also the 2d equation being } $x = \frac{189 - y^3}{9 + yy}$;
divided by the 1st, gives }

Th. $xx = \frac{35721 - 378y^3 + y^6}{81 + 18yy + y^4}$;

But (by first) $\frac{35721 - 378y^3 + y^6}{81 + 18yy + y^4} = 9 + yy$,

Whence $35721 - 378y^3 = 729 + 243y^2 + 27y^4$;

Th. $y^4 + 14y^3 + 9y^2 - 1296 = 0$.

If $y = -2$;
If $y = -1$;
If $y = 0$;
If $y = 1$;
If $y = 2$;

Then $\left\{ \begin{array}{l} 1356 = 0 \\ 1300 = 0 \\ 1296 = 0 \\ 1272 = 0 \\ 1132 = 0 \end{array} \right.$

Of which $\left\{ \begin{array}{l} 1, 2, 3, 4, 6, 12, 113, \\ 1, 2, 4, 5, 10, 13, 20, 25, \\ 1, 2, 3, 4, 6, 8, 9, 12, \\ 1, 2, 3, 4, 6, 8, 12, 24, \\ 1, 2, 4, 283, \end{array} \right.$

are divisors;

Where 6, 5, 4, 3, 2, differ by unity;

And $\frac{y^4 + 14y^3 + 9y^2 - 1296}{y - 4} = y^3 + 18yy + 81y + 324$;

Th. $y - 4 = 0$. . . $y = 4$.

QUESTION CCIX.

What two numbers are those whose sum is 5; and the sum of their cubes being multiplied by the cube of the lesser will produce 280?

If x be the greater, and y the lesser of those numbers,
Then $x + y = 5$; Or $x = 5 - y$ } per quæ,
And $(x^3 + y^3) \times y^3 = x^3 y^3 + y^6 = 280$

But $(5 - y)^3 = 125 - 75y + 15y^2 - y^3 = x^3$

And $125y^3 - 75y^4 + 15y^5 - y^6 = x^3 y^3$

Th. $125y^3 - 75y^4 + 15y^5 - y^6 + y^6 = 280$ per second.

Or $15y^5 - 75y^4 + 125y^3 - 280 = 0$

Or $3y^5 - 15y^4 + 25y^3 - 560 = 8$

If $y = -1$; Then $-99 = 0$; and 1, 3, 9, 11 }
 $y = 0$; Then $-56 = 0$; and 1, 2, 4, 7, 8 } are
 $y = 0$; Then $-49 = 0$; and 1, 3, 5, 9 } divisors.

Where 3, 2, and 1, differ by unity,

And $\frac{3y^5 - 15y^4 + 25y^3 - 56}{y - 2} = 3y^4 - 9y^3 + 7y^2 + 14y + 28$

Th. $y = 2$; and $x = (5 - 2) = 3$.

QUES.

QUESTION CCX.

The value of x , y , and z , in the following equations, is required ;

Viz.
$$\begin{cases} x^3 + y^3 + z^3 = 50 \\ y^3 + x = 18 \\ y^3 + z = 17 \end{cases} ?$$

Now (the diff two last) $x - z = 1$;

Th. $x = 1 + z$;

And $x^3 = 1 + 3z + 3z^2 + z^3$;

But from third $y^3 = 17 - z$;

Th. (by 1st.) $1 + 3z + 3z^2 + z^3 + 17 - z + z^3 = 50$;

That is $2z^3 + 3z^2 + 2z = 32$;

Th. $2z^3 + 3z^2 + 2z - 32 = 0$.

If $z = -1$; Then $33 = 0$:
 If $z = 0$; Then $32 = 0$:
 If $z = 1$; Then $25 = 0$:
 of which $\begin{cases} 1, 3, 11, \\ 1, 2, 4, 8, 16, \\ 1, 5, 25, \end{cases}$ are divisors

Where 3, 2, 1, differ by (unity) a divisor of (2) the coefficient of (z^3) the highest power of (z) the unknown quantity ;

And
$$\frac{2z^3 + 3z^2 + 2z - 32}{z - 2} = 2z^2 + 7z + 16 ;$$

Th. $z = 2$.

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What are the values of x and y in the following equations?

$$\text{Viz. } \begin{cases} xxxxy + xyxxx = 156 \\ xxyyy + xxxxy = 234 \end{cases} ?$$

Let $xy = x$;

Then $x^3y^3 + xy^3 = 156$

And $x^3y^3 + x^4y^3 = 234$ } by qu.

Th. $y^3 = \frac{156}{x^3 + x}$

And $y^3 = \frac{234}{x^2 + x^4}$;

Whence $\frac{156}{x^3 + x} = \frac{234}{x^2 + x^4}$; Or $\frac{x^2}{x^3 + 1} = \frac{3}{x + x^3}$;

Or $2x + 2x^3 = 3x^2 + 3$;

Th. $2x^3 - 3x^2 + 2x - 3 = 0$;

If $x = -1$; Then $10 = 0$;
 If $x = 0$; Then $3 = 0$;
 If $x = 1$; Then $2 = 0$;

which $\begin{cases} 1, 2, 5, 10, \\ 1, 3, \\ 1, 2, \end{cases}$ are divisors

Where 5, 3, 1, differ by two;

And $\frac{2x^3 - 3x^2 + 2x - 3}{2x - 3} = x^2 + 1$;

Th. $2x = 3$;

And $x = \frac{3}{2}$;

But (by first) $\frac{1}{2}x^3 + \frac{3}{2}y^3 = 156$,

That is $\frac{3}{2}y^3 + y^3 = \left(\frac{156 \times 2}{3}\right) 104$,

Hence $y^3 = \left(\frac{104 \times 4}{13}\right) 32$;

Th. $y = 2$;

And $x = \left(\frac{3}{2} \times 2\right) 3$.

QUEST-

QUESTION COXIL

What two numbers are those, whose products multiplied by the greater will produce 405; and their difference multiplied by the lesser, 20?

If x represent the greater, and y the lesser;

$$\begin{array}{l} \text{Then } (xy \times x =) x^2 y = 405, \\ \text{And } (x - y \times y =) xy - yy = 20, \end{array} \quad \left. \vphantom{\begin{array}{l} \text{Then } (xy \times x =) x^2 y = 405, \\ \text{And } (x - y \times y =) xy - yy = 20, \end{array}} \right\} \text{by quest.}$$

$$\text{Now by } \begin{cases} \text{first} & - & xy = \frac{405}{x}, \\ \text{second} & - & xy = 20 + yy; \end{cases}$$

$$\text{Th. } - & - & - & - & \frac{405}{x} = 20 + yy,$$

$$\text{And } - & - & - & 2 \frac{405}{20 + yy} = x,$$

$$\text{Also } - & - & - & \frac{405y}{20 + yy} = xy:$$

$$\text{But (by second) } \frac{405y}{20 + yy} - yy = 20;$$

$$\text{Th. } - & 405y - 20y^2 - y^4 = 400 + 20y^2,$$

$$\text{Or } y^4 + 40y^2 - 405y + 400 = 0.$$

$$\begin{array}{l} \text{If } y = -1; \\ \text{If } y = 0; \\ \text{If } y = 1; \end{array} \quad \left. \vphantom{\begin{array}{l} \text{If } y = -1; \\ \text{If } y = 0; \\ \text{If } y = 1; \end{array}} \right\} \text{then } \begin{cases} 846 = 0; \\ 400 = 0; \\ 36 = 0; \end{cases} \quad \left. \vphantom{\begin{cases} 846 = 0; \\ 400 = 0; \\ 36 = 0; \end{cases}} \right\} \begin{cases} 1, 2, 3, 6, 9, 47, \\ 1, 2, 4, 5, 8, 10, 20, \\ 1, 2, 3, 4, 9, 12, 18, \end{cases} \quad \left. \vphantom{\begin{cases} 1, 2, 3, 6, 9, 47, \\ 1, 2, 4, 5, 8, 10, 20, \\ 1, 2, 3, 4, 9, 12, 18, \end{cases}} \right\} \text{are all}$$

Where 6, 5, and 4, differ by unity;

$$\text{And } \frac{y^4 + 40y^2 - 405y + 400}{y^2 - 5} = y^2 + 5y + 6y - 80,$$

$$\text{Th. } - & - & - & - & y = 5.$$

QUESTION CCXIII.

What two numbers are those, whose product multiplied by the lesser will produce 225; but if their difference be multiplied by the greater, 36?

If $x =$ the greater, and $y =$ the lesser of those numbers;

Then $(xy \times y =) xy^2 = 225,$ } by quest.

And $(x - y \times x =) xx - xy = 36,$ }

Now (by first) $xy = \frac{225}{y},$

And (by second) $xx - 36 = xy;$

Whence $xx - 36 = \frac{225}{y},$

Th. $xx = \frac{225}{y} + 36;$

But (by first) $x = \frac{225}{y},$

And $xx = \left(\frac{225 \times 225}{y^2} = \right) \frac{50625}{y^2};$

Whence $\frac{225}{y} + 36 = \frac{50625}{y^2},$

Or $\frac{25}{y} + 4 = \frac{5625}{y^2},$

Or $25y^2 + y^4 = 5625;$

Th. $4y^4 + 25y^2 - 5625 = 0.$

If $y = -1;$	$5604 = 0;$	$\frac{5}{1}$	$1, 2, 3, 4, 6, 12, 467,$	are div.
If $y = 0;$	$5625 = 0;$	$\frac{5}{0}$	$1, 3, 5, 9, 15, 25, 45,$	
If $y = 1;$	$5596 = 0;$	$\frac{5}{1}$	$1, 2, 4, 1399,$	

Where 6, 5, 4, differ by (1) a divisor of (4) the coefficient of (y^4) the highest power of $y;$

And $4y^4 + 25y^2 - 5625$

$\frac{y-5}{y-5}$
 $= 4y^3 + 45y^2 + 225y + 1125.$

Th. $y = 5.$

QUES;

QUESTION CCXIV.

There are two numbers, the difference of whose cubes is 604; and if their difference be multiplied by the greater, the product will be 36: What are those numbers?

If x = the gr. and y = the less. of those numbers;

Then $x - y \times x = 36$; Or $x - y = \frac{36}{x}$, } by quest.

Also $x^3 - y^3 = 604$.

And dividing the 2d equ. by the 1st. $\left\{ \begin{array}{l} x^2 + xy + y^2 = \left(\frac{604x}{36} \right) = \frac{151x}{9} \end{array} \right.$

Also by 2d. $x^2 - 2xy + y^2 = \left(\frac{36 \times 36}{xx} \right) = \frac{1296}{xx}$;

The diff. of 2 last is $3xy = \frac{151x}{9} - \frac{1296}{xx}$;

Th. $\frac{11664}{151 - 27y} = x^3$;

But by 2d. $\frac{11664}{151 - 27y} - y^3 = 604$;

Th. $27y^4 - 151y^3 + 16308y - 79540 = 0$.

If $y = -2$;	110516 = 0;	1, 2, 4, 7, 14, 28, 3947,	are divisors;
If $y = -1$;	95670 = 0;	1, 2, 3, 5, 6, 9, 10, 15,	
If $y = 0$;	79540 = 0;	1, 2, 4, 5, 10, 20, 41, 97,	
If $y = 1$;	63356 = 0;	1, 2, 4, 15839,	
If $y = 2$;	47700 = 0;	1, 2, 3, 4, 5, 6, 9, 10,	

Where 7, 6, 5, 4, 3, differ by (1) a divisor of (27) the coefficient of (y^4) the highest power of y ;

And $\frac{27y^4 - 151y^3 + 16308y - 79540}{y - 5}$
 $= 27y^3 - 16y^2 - 80y + 15908$;

Th. $y = 5$.

QUES

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QUESTION CCXV.

What two numbers are those, the sum of whose cubes is 468; and the square of the greater added to their product, makes 84?

If x = the gr. and y = the kff. of those numbers;

$$\left. \begin{array}{l} \text{Then } x + y = 84; \text{ Or } x + y = \frac{84}{x}, \\ \text{And } x^3 + y^3 = 468, \end{array} \right\} \text{by quest.}$$

$$\left. \begin{array}{l} \text{Now 1d equ.} \\ \text{divided by} \\ \text{1st gives} \end{array} \right\} x^2 - xy + y^2 = \left(\frac{468x}{84} = \right) \frac{39x}{7}$$

$$\text{And (add. } 3xy) x^2 + 2xy + y^2 = \frac{39x}{7} + 3xy;$$

$$\text{Now (from 1st) } x^2 + 2xy + y^2 = \left(\frac{84}{x} \times \frac{84}{x} = \right) \frac{7056}{xx};$$

$$\text{Th. } \frac{39x}{7} + 3xy = \frac{7056}{xx};$$

$$\text{Or } 39x^3 + 21xy^2 = (7056 \times 7 =) 49392;$$

$$\text{Th. } x^3 = \left(\frac{49392}{39 + 21y} = \right) \frac{16464}{13 + 7y};$$

$$\text{Now (by 2d) } \frac{16464}{13 + 7y} + y^3 = 468;$$

$$\text{Th. } 7y^4 + 13y^3 - 3276y + 10380 = 0.$$

$$\left. \begin{array}{l} \text{If } y = -1; \quad 13650 = 0 : \frac{1}{5} \quad | 1, 2, 3, 5, 6, 10, 15, \\ \text{If } y = 0; \quad 10380 = 0 : \frac{2}{3} \quad | 1, 2, 3, 4, 5, 6, 9, \\ \text{If } y = 1; \quad 7126 = 0 : \frac{1}{6} \quad | 1, 2, 4, 13, \text{ \&c.} \end{array} \right\} \text{are div.}$$

Where 6, 5, 4, differ by unity.

$$\text{And } \frac{7y^4 + 13y^3 - 3276y + 10380}{y - 5}$$

$$= 7y^3 + 48y^2 + 240y - 2076;$$

$$\text{Th. } y = 5.$$

Q U E S -

What two numbers are those, the difference of whose squares is 16; and the sum of their cubes 152?

If x = the gr. and y = the less of those numbers;

Then $x^2 - y^2 = 16$; Or $x^2 = 16 + y^2$, } by qu.

And $x^3 + y^3 = 152$; Or $x^3 = 152 - y^3$,

But $(x^2)^3 = x^6 = 4096 + 768y^2 + 48y^4 + y^6$,

And $(x^3)^2 = x^6 = 23104 - 304y^3 + y^6$,

Th. $4096 + 768y^2 + 48y^4 + y^6 = 23104 - 304y^3 + y^6$,

Or $4096 + 768y^2 + 48y^4 = 23104 - 304y^3$,

Or $48y^4 + 304y^3 + 768y^2 = 19008$;

Th. $3y^4 + 19y^3 + 48y^2 - 1188 = 0$.

If $y = -1$; then $-1156 = 0$; $\left| \begin{array}{c} 1, 2, 4, 17, \\ \end{array} \right|$

If $y = 0$; then $-1188 = 0$; $\left| \begin{array}{c} 1, 2, 3, 4, 6, 9, 11, 12, \\ \end{array} \right|$

If $y = 1$; then $-1118 = 0$; $\left| \begin{array}{c} 1, 2, 13, 43, \\ \end{array} \right|$

Where 4, 3, and 2, differ by unity;

And $3y^4 + 19y^3 + 48y^2 - 1188$

$$= 3y^3 + 28y^2 + 13zy + 396;$$

Th. $y = 3$

QUES-

QUESTION CCXVII.

What two numbers are those, the sum of whose square roots is 4; and the difference of their cube roots is 1?

Suppose $x =$ the gr. and $y =$ the left. of those numbers ;

Then $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 4$; Or $(x^{\frac{1}{2}} =) x^{\frac{1}{2}} = 4 - y^{\frac{1}{2}}$; (transp)

And $x^{\frac{1}{3}} - y^{\frac{1}{3}} = 1$; Or $(x^{\frac{1}{3}})^3 - x^{\frac{2}{3}} = 1 + y^{\frac{2}{3}}$, by q,

$$\text{Th. } \sqrt[4]{-y \frac{1}{8}} = (x =) \sqrt{1 + y \frac{1}{8}}; \quad \text{q. d.}$$

That is $16 - 8\frac{1}{2} + 7 = 1 + 3\frac{1}{2} + 3\frac{1}{2} + 7$, and

$$\text{Th. } 15 = 3y\frac{1}{2} + 8y\frac{1}{2} + 3y\frac{1}{2}:$$

Let $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} = 1 + x$; $x =$ _____

Then $\dots 37\frac{1}{2} = 3 + 12x + 18x^2 + 12x^3, \&c.$

And $8y^{\frac{1}{3}} = 8 + 24x + 24x^2 + 8x^3$.

$$A16 \quad - \quad - \quad - \quad 3y^{\frac{2}{3}} = 3 + 6x + 3x^2;$$
$$Th: \quad 15 = 14 + 42x + 45x^2 + 20x^3, \text{ etc}$$
$$\frac{1}{20} \approx 0.05 \quad \frac{1}{3} \approx 0.33 \quad \frac{1}{4} \approx 0.25 \quad \frac{1}{5} \approx 0.2$$

Or writing $20x^3$ as small: $\frac{1}{x^2} = x^2 + \frac{14}{15}x$.

$$\text{For } \left(\frac{1}{2} + \frac{49}{2} \right) \frac{54}{2} = 3^2 + 4^2 + \dots + 49^2$$

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$$\text{Th. } \cdot \quad \frac{1}{15} = x + \frac{1}{15} \quad \text{--- (1)}$$

13

And $\frac{\sqrt{54}-7}{15} = \frac{(7,348469-7)}{15}$

$$= 0.232331 = 1\%$$

Now by restoring

$$20x^3 = 20x, 023231$$

and renewing the op-

ration - - -)

And $y^{\frac{1}{5}} = 1.0232106;$

... ..

$$y = 1,1476592,$$

And - - - - - $x = 8,5773504$.

Q U E S -

QUESTION CCXVIII.

When the days were 16 hours long, a person being asked what hour it was, replied; The cube root of the hours to come till night, being added to the square root of those past since morning, their sum will be the hour already past: Required the time of the day?

If yy be the hours past; and x those to come;

Then $- - yy + x = 16$; Or $x = 16 - yy$ } per quo

And $- - y + \sqrt[3]{x} = y^2$; Or $x = yy - y^3$ }

Th. $- - y^6 - 3y^5 + 3y^4 - y^3 = 16 - yy$;

Or $y^6 - 3y^5 + 3y^4 - y^3 + y^2 = 16$.

Let $- - - - - z + z = y$;

Then $64 + 192z + 240z^2 + 160z^3, \&c. = y^6$

$48 + 96z + 72z^2 + 24z^3, \&c. = 3y^4$

$4 + 4z + z^2 = y^2$

$116 + 292z + 313z^2 + 184z^3, \&c. = y^6 + 3y^4 + y^2$

$96 + 240z + 240z^2 + 120z^3, \&c. = 3y^5$

$8 + 12z + 6z^2 + z^3, \&c. = y^3$;

Th. $12 + 40z + 67z^2 + 63z^3, \&c. = 16$,

Or $- - - - - 40z + 67z^2 + 63z^3 = (16 - 12) 4$

1. If $- - - - - 40z' = 4$; $z' = \frac{4}{40} = \frac{1}{10}$,

2. If $40z'' + 67z'z'' = 4$; $z'' = \frac{4}{40 + \frac{67}{10}} = \frac{40}{467}$

3. If $40z + 67z''z + 63z''z''z = 4$;

Then $40z + \frac{67 \times 40}{467} z + \frac{63 \times 40 \times 40}{467 \times 467} z = 4$,

Or $8723560z + 1251560z + 100800z = 872356$,

Th. $z = \frac{872356}{10075920} = 0,086578, \&c.$ And $y = 2,086578$.

QUESTION CCXIX.

A company of men and women spent 4*l.* 16*s.* each man paid the square of the shillings that each woman paid; and by that means, he paid as many shillings more than her, as there were women in company; now if there had been no more women than men, they would have collected among them but 3*l.* How many men and women were there?

Suppose each woman paid x shillings,

Then each man paid xx shillings,

And $xx - x =$ the women in company;

Then $x^2 - x^2 =$ shill. paid by all the women,

And $96 - \frac{x^2 - x^2}{x} =$ ditto paid by all the men;

Th. $\frac{96 + x^2 - x^2}{xx} =$ the number of men:

Now $\left(\frac{96 + x^2 - x^2}{xx} \times x \right) = \frac{96 + x^2 - x^2}{x} =$ the sum

that would have been paid by the women if the number were no greater than the number of men;

But $96x + x^3 - x^2 + \frac{96 + x^2 - x^2}{x} = 60$ per quest.

Or $96x + x^3 - x^2 + 96 + x^2 - x^2 = 60x,$

Or $96x - x^2 + 96 + x^2 = 60x,$

Or $x^4 - xx - 36x - 96 = 0;$

But $\frac{x^4 - xx - 36x - 96}{x - 4} = x^3 + 4x^2 + 15x + 24;$

Th. $x = 4.$

Hence there were 12 women and 3 men.

QUESTION CCXX.

The product of the areas of 2 squares, the sum of whose sides is 15, being multiplied by the area of the lesser, will produce 153664 : What are the areas of those squares ?

Let x = the side of the less, and y = the side of the gr. sq.

Then $(x^2 y^2 \times x^2 =) x^4 y^2 = 153664$.

Or $x^2 y = \sqrt{153664} = 392$.

Or $xx = \frac{392}{y}$.

Th. $x = \left(\frac{\sqrt{392}}{y} \right) \frac{14\sqrt{2}}{\sqrt{y}}$.

But $(x + y = 15 : \text{Th.}) x = 15 - y$.

Th. $15 - y = \frac{14\sqrt{2}}{\sqrt{y}}$.

Or $15y^{\frac{1}{2}} - y^{\frac{3}{2}} = 14\sqrt{2}$.

Th. $y^{\frac{3}{2}} - 15y^{\frac{1}{2}} + 14\sqrt{2} = 0$.

If $y^{\frac{1}{2}} = -\sqrt{2}$;	then $27\sqrt{2} = 0$;	And $\sqrt{2}, 3\sqrt{2}, 9\sqrt{2}$;
If $y^{\frac{1}{2}} = 0$;	then $13\sqrt{2} = 0$;	And $\sqrt{2}, 2\sqrt{2}, 7\sqrt{2}$;
If $y^{\frac{1}{2}} = \sqrt{2}$;	then $1\sqrt{2} = 0$;	And $\sqrt{2}$.

Of which $3\sqrt{2}, 2\sqrt{2}$, and $\sqrt{2}$, differ by $\sqrt{2}$;

Th. $2\sqrt{2} = y^{\frac{1}{2}}$.

Th. $y = (2\sqrt{2} \times 2\sqrt{2} =) 8$; and $yy = 64$.

Also $x = (15 - x =) 7$; and $xx = 49$.

How much wheat at 5s. 4d. per bush, and rye at 3s. 7d. per bush, will compose a mixture, that may be sold for 4s. 7d. per bushel?

If x = the bushels of wheat, and y those of rye,

Then x bush. at 64d. a bush. = $64x$,

And y - 43 - - - = $43y$,

Also $x + y$ - 55 - - - = $55x + 55y$,

But (by quest.) - $64x + 43y = 55x + 55y$,

Or (by transposition) - $9x = 12y$,

Th. - - - - - $3x = 4y$:

Whence as $3 : 4 :: y : x$.

Therefore all numbers that are as 3 to 4 will answer the question.

Or the values of x in whole numbers, will continually encrease by the coefficient of y , and the values of y by the coefficient of x :

That is 4, 8, 12, 16, 20, &c. will be the values of x ,

And 3, 6, 9, 12, 15, &c. will be the values of y .

QUESTION CCXXII.

What are the values of x and y in the equation $9x - 7y = 6$?

Suppose the values of x and y to be equal;

Then $9 - 7 \times x = 6$,

And $x (=y) = \left(\frac{6}{9-7} = \frac{6}{2} = 3 \right)$.

But the values of $\left\{ \begin{matrix} x \\ y \end{matrix} \right\}$ in whole numbers encrease by the coefficients of $\left\{ \begin{matrix} y \\ x \end{matrix} \right\}$ by quest.

Th. $x = 3, 10, 17, 24, 31$, &c.

And $y = 3, 12, 21, 30, 39$, &c.

QUES-

QUESTION CCXXIII.

What quantities of tobacco at $16d.$ and $10d.$ per lb. may be mix'd with 50 lb. of tobacco at $8d.$ per lb. so that the whole may be worth $12d.$ per lb.

Suppose x lb. at $16d.$ and y lb. at $10d.$ be used;

Then $16x + 10y + 50 \times 8 = x + y + 50 \times 12$;

Or $16x + 10y = 12x + 12y + 50 \times 4$;

Th. $4x = 2y + 200$;

Now $100 + y = 2, 4, 6, 8, 10, \&c.$ by quest.

And $x = 1, 2, 3, 4, 5, \&c.$

But the value of y will not be an affirmative number, till the terms of the series $2, 4, 6, \&c.$ exceed 100 ;

Th. $100 + y = 102, 104, 106, 108, 110, \&c.$

$x = 51, 53, 55, \&c.$

And $y = 2, 4, 6, 8, 10, \&c.$

QUESTION CCXXIV.

'Tis required to buy 20 fowles for $20d.$ viz. geese at $4d.$ each; quails at $2d.$ each; and larks at $1d.$ each?

Let x = the number of geese; y of quails; and z , of larks;

Then $x + y + z = 20$;

And $4x + 2y + z = 20$;

But $16x + 2y + z = (20 \times 4 =) 80$; by seconds;

(By subtr.) $15x + y = (80 - 20 =) 60$;

Th. $y = 60 - 15x = 4 - x \times 15$.

Now by quest. $y = 15, 30, 45, \&c.$

$4 - x = 1, 2, 3, \&c.$

But (by first) $y \sqsupset 20$;

Th. the only answer $y = 15$,

in whole numbers is $x = (4 - 1 =) 3$,

$z = 2$.

QUES-

QUESTION CCXXV.

How many ways can 100 pounds be paid by guineas at 21, and pistoles at 17 shillings, each?

If x guineas, and y pistoles, must be used;

Then $21x + 17y = 100 \times 20 = 2000$ per quest.

Assume $x + y = s$;

Then $\begin{cases} 21x + 17y = 21s \\ 17s + 17y = 17s \end{cases}$ by multiplication,

And $\begin{cases} 4y = 21s - 2000 \\ 4s = 2000 - 17y \end{cases}$ by subtraction;

Th. $\begin{cases} s - \left(\frac{2000}{4}\right) = 95 \frac{1}{4} \\ s - \left(\frac{2000}{4}\right) = 117 \frac{1}{4} \end{cases}$

Also when x and y are integers, s must be an integer;

Th. $s = 96, 100, 104, 108, 112, 116$;

When	$\left\{ \begin{array}{l} s = 96; x = (500 - 17 \times 24) = 92; \\ s = 100; x = (500 - 17 \times 25) = 75; \\ s = 104; x = (500 - 17 \times 26) = 58; \\ s = 108; x = (500 - 17 \times 27) = 41; \\ s = 112; x = (500 - 17 \times 28) = 24; \\ s = 116; x = (500 - 17 \times 29) = 7; \end{array} \right.$	And	$\left\{ \begin{array}{l} y = 4, \\ y = 25, \\ y = 46, \\ y = 67, \\ y = 88, \\ y = 109, \end{array} \right.$

QUES.

QUESTION CCXXVI.

'Tis required to divide 24 into three such parts, that if the first be multiplied by 36, the second by 24, and the third by 8, the sum of those products may be 516.

Let x , y , and z , represent the three parts;

$$\begin{array}{l} \text{Then} \quad - \quad x + y + z = 24 \\ \text{And} \quad 36x + 24y + 8z = 516 \end{array} \quad \left. \vphantom{\begin{array}{l} \text{Then} \\ \text{And} \end{array}} \right\} \text{per quest.}$$

$$\begin{array}{l} \text{Or} \quad - \quad - \quad x + y = 24 - z \\ \text{And} \quad - \quad 36x + 24y = 516 - 8z \end{array} \quad \left. \vphantom{\begin{array}{l} \text{Or} \\ \text{And} \end{array}} \right\} \text{by transposition,}$$

$$\text{Or} \quad - \quad - \quad 9x + 6y = 129 - 2z;$$

$$\begin{array}{l} \text{But} \quad - \quad 9x + 9y = 216 - 9z \\ \text{And} \quad - \quad 6x + 6y = 144 - 6z \end{array} \quad \left. \vphantom{\begin{array}{l} \text{But} \\ \text{And} \end{array}} \right\} \text{by multiplication;}$$

$$\begin{array}{l} \text{Th.} \quad - \quad - \quad 3y = 87 - 7z \\ \text{And} \quad - \quad - \quad 3x = 4z - 15 \end{array} \quad \left. \vphantom{\begin{array}{l} \text{Th.} \\ \text{And} \end{array}} \right\} \text{by subtraction;}$$

$$\text{Th.} \quad - \quad x = \left(\frac{1}{3}z\right) 12\frac{1}{3}; \text{ and } y = \left(\frac{1}{3}z\right) 3\frac{2}{3};$$

$$\text{Also} \quad - \quad - \quad y = 29 - \frac{7z}{3}; \text{ and } x = \frac{4z}{3} - 5$$

Th. (when y and x are whole numbers) z must be a multiple of 3;

But 6, 9, and 12, are the only multiples of 3 within the above limits;

$$\begin{array}{l} \text{Th.} \quad x = 6; y = 15; \text{ and } z = 3, \\ \quad \quad x = 9; y = 8; \text{ and } z = 7, \\ \quad \quad x = 12; y = 1; \text{ and } z = 11. \end{array}$$

QUES.

QUESTION CCXXVII.

Asked *A*, who sat with a basket of apples to sell, how many he had? *A* reply'd, That he could not tell; but remembered that when he told them into his basket by two's, three's, four's, five's, and six's, there always remained 1 apple over, but that telling them by sevens none remained: How many had he?

The number 60 may be divided by 2, 3, 4, 5, and 6, without a remainder;

The question therefore is to find a number that divided by 60 will leave 1, and by 7 no remainder.

Suppose x the number, y and z any whole number;

$$\text{Then } \frac{x}{60} = y + \frac{1}{60}; \text{ Or } x = 60y + 1,$$

$$\frac{x}{7} = z; \text{ Or } x = 7z;$$

$$\text{Th. } 60y + 1 = 7z; \text{ Or } 1 = 7z - 60y;$$

$$\text{Let } z = x' = 4 + 8y, \text{ (} = 7 \text{)} \quad 7x' = 28 + 56y;$$

$$\text{Then } 1 = 7x' - 14 + 56y - 60y,$$

$$\text{That is } 15 = 7x' - 4y; \text{ Th. } (15 =) 5 = (x' =) 7,$$

$$\text{And } x = (60y + 1 = 60 \times 5 + 1 =) 301.$$

QUESTION CCXXVIII.

A higgler's servant, who was sent with a basket of eggs, had the misfortune to break them, and being called upon to pay for them, his master forgot the number; but the mistress remembered, that when she told them by two's, 1 remained; by three's, 2 remained; by four's, 3; by fives, 4; by six's, 5; and by seven's, none remained? What number of eggs were there?

Thi,

This question differs from the former only in this ;
that the number required divided by 60 will leave - 1.

That is $\frac{1}{60}x = y - \frac{1}{60}$; Or $x = 60y - 1$,

And $\frac{2}{7}x = z$; Or $x = 7z$;

Th. $60y - 1 = 7z$; And $60y - 7z = 1$;

Put $z = 9y + 1 - z'$

Then $60y - 63y - 7 + 7z' = 1$; Or $7z' - 3y = 8$

Th. $(z' =) y = (\frac{8}{7} =) 2$;

And $x = (60y - 1 = 120 - 1 =) 119$.

QUESTION CCXXIX.

Required the values of x and y , in the equation $71x + 17y = 1005$?

Substitute $z = 4x = y$;

Then $71x + 17z = 68x = 1005$, by quest.

Th. $17z = 1005 - 3x$

Or $17z = 335 - x \times 3$

Now $\left\{ \begin{array}{l} z = 3, 6, 9, 12, \&c. \\ 335 - x = 17, 34, 51, 68, \&c. \end{array} \right\}$ by qu. 221.

Or $\left\{ \begin{array}{l} z = 3, 3 + 3, 3 + 2 \times 3, \&c. \\ x = 318, 318 - 17, 318 - 2 \times 17, \&c. \end{array} \right\}$ $3 + 3z$

But $(z = 4x = y$ that is) $3 + 3z = 1272 + 68z = y$,

Th. $z = \left(\frac{1269}{71} = \right) 17 \frac{62}{71}$;

And (because $x = 318 - 17z$) $x = \left(\frac{318}{17} = \right) 18 \frac{12}{17}$;

Th. $z = 18$; $x = 12$; and $y = 9$.

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QUESTION CCXXX.

What are the values of x and y in the equations,
 $xy + 666 = x^3$; And $xx + yy - 95 = 5y$?

Now (from 1st) $xy = x^3 - 666$; And $y = \frac{x^3 - 666}{x}$;

Th. $yy = \frac{x^3 - 666}{x}$; And $5y = \frac{5x^3 - 3330}{x}$;

Whence (by 2d.) $\frac{x^3 - 666}{x} + xx - 95 = \frac{5x^3 - 3330}{x}$;

Or $x^6 - 4x^4 - 1332x^3 - 95x^2 + 3330x + 443556 = 0$.

And by the method of divisors, there will be found the
 divisors 10, 9, 8, differing by unity;

And as $x - 9$ will divide the eq. Th. $x = 9$; And $y = 7$.

QUESTION CCXXXI.

A mason had two cubical pieces of marble, containing
 between them 1072 solid inches, and being severally
 placed in his yard, they stand upon 130 square inches of
 the ground: What are the sides of those cubes?

Suppose $x =$ side of $\square$; and $y =$ side of $\square$, cube;

Then $x^3 + y^3 = 1072$; Or $x^3 = 1072 - y^3$;

And $xx + yy = 130$; And $x^2 = 130 - yy$;

But $x^6 = \overline{1072 - y^3}^2 = \overline{130 - yy}^2$;

Hence $y^6 - 195y^4 - 1072y^3 + 25350y^2 - 523908 = 0$.

And by the method of divisors, there will be found the
 divisors 9, 8, 7, 6, 5, differing by unity.

But $y - 7$ will divide the eq. Th. $y = 7$; And $x = 9$.

Q U E S.

QUESTION CCXXXII.

What are the values of x and y in the eq. $5x + 8y = 198$?

Let $x + y = s$. Then
$$\begin{cases} 5x + 5y = 5s; \\ 8x + 8y = 8s; \end{cases}$$

The diff. between these eq. $\begin{cases} 3y = 198 - 5s; \\ 3x = 8s - 198; \end{cases}$
and the given one, are

Hence, when
$$\begin{cases} y \sqsubset 0; \text{ Then } s \sqsupset \left(\frac{198}{5} =\right) 39 \frac{3}{5}; \\ x \sqsubset 0; \text{ Then } s \sqsubset \left(\frac{198}{8} =\right) 24 \frac{6}{8}; \end{cases}$$

Also, when x and y are integers, Then $\frac{1}{3}s$ must be int.
Th. betw. $24 \frac{6}{8}$ and $30 \frac{3}{5}$; $s = 27, 30, 33, 36, 39$.

Now when $s = 27, y = (198 - 135 \times \frac{1}{5}) = 21$; and $x = 6$.

If $s = 30, 33, 36, 39$, Then $y = 16, 11, 6, 1$.

And $x = 14, 22, 30, 38$.

Hence in the eq. of this form, when x increases by the
coeffi. of y , y decreases by the coeffi. of x .

QUESTION CCXXXIII.

What are the values of x and y in the eq. $5x + 8y = 104$?

Suppose the values of x and y to be equal;

Then $5 + 8 \times x = 104$;

Th. $x (=y) = (\frac{104}{13} =) 8$;

Now by quest. last, $x = 8, 16$; and $y = 8, 3$.

QUESTION CCXXXIV.

'Tis required to mix brandy at 15 s. 10 d. per gallon, with rum at 21 s. 7 d. per gallon, and arrack at 22 s. 10 d. per gallon, to make 429 gallons worth 16 s. 8 d. per gallon?

Suppose x gallons of brandy, y of rum, and z of arrack;

Then $x + y + z = 429$

And $190x + 259y + 274z = (429 \times 200 =) 85800$ } by qu.

But $190x + 190y + 190z = (429 \times 190 =) 81510$

(By subtract.) $69y + 84z = 4290$

Or $23y + 28z = 1430$

Substitute $y + z = n$

Then $\begin{cases} 23y + 23z = 23n \\ 28y + 28z = 28n \end{cases}$ } by multipl.

And by subtr. $\begin{cases} 5y = 28n - 1430 \\ 5z = 1430 - 23n \end{cases}$

Th. $\begin{cases} n - \frac{1430}{5} = 51\frac{2}{5} \\ n - \frac{23n}{5} = 62\frac{4}{5} \end{cases}$

And when y and z are integers $\frac{n}{5}$ must be an integer,

Whence $n = 55, 60,$

$y = \left(\frac{28n}{5} - 286 = \right) 22, 50$

$z = \left(286 - \frac{23n}{5} = \right) 33, 10$

$x = (429 - y - z =) 374, 369$

QUES-

QUESTION CCXXXV.

A clock has 2 indices *A* and *B*; *A* goes round the circumference once in 12 hours, and *B* goes round once in 1 hour: How often, and at what times, are they together in every 12 hours?

Suppose x = the time when the two indices will be together;

H. Cir. H. Cir.

Then $12 : 1 :: x : \frac{1}{12}x$ = the space *A* moves thro'.

And $1 : 1 :: x : x$ = the space *B* moves thro'.

But in x time, *B* has moved 1 circumference more than *A*;

Th. $x - \frac{1}{12}x = 1$. Hence $x = 1 \frac{1}{11}$ h. 5 27, &c.

Therefore the two indices will be together 11 times in 12 hours; and at the distance of $1 \frac{1}{11}$ hours.

QUESTION CCXXXVI.

A clock has three indices *A*, *B*, and *C*, *A* goes round the circumference once in 12 hours, *B* goes round the circumference once in 1 hour, and *C* goes round the circumference once in 1 minute: How often and at what times are they together in every 12 hours?

Suppose x the time when *A* and *B* are together,

And y = the time when *B* and *C* are together;

Then $x = \frac{12}{11}$, $\frac{2 \times 12}{11}$, $\frac{3 \times 12}{11}$, &c. to $\frac{12n}{11}$, by quest. last.

But $1 : 1 : y : y$ = space *B* moves thro'.

And $\frac{1}{60} : 1 : y : 60y$ = space *C* moves thro'.

And because *C* has moved 1 circumference more than *B*;

Th. $60y - y = 1$. Th. $y = (\frac{1}{59} \text{ h.} =) \frac{m}{59}$, &c.

Or $y = \frac{1}{59}$, $\frac{2}{59}$, $\frac{3}{59}$, $\frac{m}{59}$, &c.

Now find the val. of m and n in $\frac{m}{59} = \frac{12n}{11}$ That $x = y$.

Th. $11m = 708n$; Hence $n = 11, 22$, &c.

And $m = 708, 1416$, &c.

But $\frac{12 \times 11}{11} = 12$; And $\frac{208}{11} = 12$.

Therefore the 3 hands are together only at 12 o'clock.

QUESTION CCXXXVII.

Required the least number, that being divided by 28, will leave n ; and by 19, will leave m , for remainders?

Let $x = \text{N}^{\circ}$ sought, and y, z , any whole N° .

$$\begin{aligned} \text{Then } \frac{x}{28} &= y + \frac{n}{28}; \text{ Or } x = 28y + n \\ \text{And } \frac{x}{19} &= z + \frac{m}{19}; \text{ Or } x = 19z + m \end{aligned} \left. \vphantom{\begin{aligned} \frac{x}{28} &= y + \frac{n}{28} \\ \frac{x}{19} &= z + \frac{m}{19} \end{aligned}} \right\} \text{ by quest.}$$

$$\text{Assume } \begin{cases} A = 19a = 28b + 1 \\ B = 28c = 19d + 1 \end{cases} \text{ where } a, b, c, \text{ and } d, \text{ are unknown,}$$

$$\text{Or } \begin{cases} 19a - 28b = 1, \\ 28c - 19d = 1: \end{cases}$$

$$\text{Now 1st. let } b' - 1 = b;$$

$$\text{Then } 19a - 28b' + 28 = 1; \text{ Or } 19a - 28b' = -27,$$

$$\text{Where if } a = b'; a = (b') = \frac{-27}{-9} = 3;$$

$$\text{Th. } (3 - 1) \times 2 = b; \text{ And } A = (19 \times 3) = 57;$$

$$\text{Secondly, let } d' + 8 = d;$$

$$\text{Then } 28c - 19d' - 152 = 1; \text{ Or } 28c - 19d' = 153.$$

$$\text{Where if } c = d'; c = (d') = \frac{153}{9} = 17;$$

$$\text{Th. } (17 + 8) \times 25 = d; \text{ And } B = (17 \times 28) = 476;$$

$$\text{But be-cause } \begin{cases} A = 28b + 1 \\ B = 19d + 1 \end{cases}; \text{ Th. } \begin{cases} nA = 28nb + n, \\ mB = 19md + m; \end{cases}$$

From which equations, compared with the 2 first, it will appear,

$$\text{That } \begin{cases} nA \\ mB \end{cases} \text{ will di- vide by } \begin{cases} 28 \\ 19 \end{cases} \text{ and leave } \begin{cases} n \\ m \end{cases} \text{ for remaind.}$$

$$\text{And because } \begin{cases} A = 19a, \\ B = 28c; \end{cases}$$

QUES.

QUESTION CCXXXVII. [Continued.]

Th. $\begin{cases} nA \\ mB \end{cases}$ will di- $\begin{cases} 19 \\ 28 \end{cases}$ and leave no remainder:

Th. $nA + mB$ being divided by 28, will leave n ; and by 19, will leave m for remainders:

But the numbers, which being divided by 28, and 19, will leave m and n for remainders, differ by $(28 \times 19 =)$

532.
Th. $\frac{nA + mB}{532} = \frac{57n + 476m}{532}$ will leave x for a remainder.

EXAMPLE.

The cycle of the sun 17; and the cycle of the moon 13, being given; to find the year of the Dionysian period?

Now $57 \times 17 = 969$,

And $476 \times 13 = 8188$;

532)7152(13

532
1837
1596
241

ANSWER. The 241st year thereof.

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QUESTION CCXXXVIII.

Required the least number (x) which being severally divided by 28, 19, and 15, will leave the remainders, a , m , and p ?

$$\text{Assume } \left\{ \begin{array}{l} A = (19 \times 15a) = 285a = 28b + 1, \\ B = (28 \times 15c) = 420c = 19d + 1, \\ C = (28 \times 19e) = 532e = 15f + 1; \end{array} \right\} \text{ or } \left\{ \begin{array}{l} 285a - 28b = 1, \\ 420c - 19d = 1, \\ 532e - 15f = 1; \end{array} \right.$$

First; Let $9a + 3 + b' = b$;

Then $33a - 28b' = 85$;

Th. (if $a = b'$) $a = \frac{85}{3} = 17$; And $A = 285 \times 17 = 4845$.

Secondly; Let $21c + 1 + d' = d$;

Then $21c - 19d' = 20$;

Th. (if $c = d'$) $c = \frac{20}{2} = 10$; And $B = 420 \times 10 = 4200$.

Thirdly; Let $34e + 6 + f' = f$;

Then $22e - 15f' = 91$;

Th. (if $e = f'$) $e = \frac{91}{7} = 13$; And $C = (532 \times 13) = 6916$.

Now because $\left\{ \begin{array}{l} A = 28b + 1 \\ B = 19d + 1 \\ C = 15f + 1 \end{array} \right\}$ Therefore $\left\{ \begin{array}{l} nA = 28nb + n, \\ mB = 19md + m, \\ pC = 15pf + p; \end{array} \right.$

That is $\left| \begin{array}{l} nA \\ mB \\ pC \end{array} \right|$ will divide by $\left| \begin{array}{l} 28 \\ 19 \\ 15 \end{array} \right|$ and leave the rem. $\left| \begin{array}{l} n \\ m \\ p \end{array} \right|$:

And because $\left| \begin{array}{l} A = 19 \times 15a \\ B = 28 \times 15b \\ C = 28 \times 19c \end{array} \right|$ Therefore $\left| \begin{array}{l} nA \\ mB \\ pC \end{array} \right|$ will divide by $\left| \begin{array}{l} 19 \text{ and } 15 \\ 28 \text{ and } 15 \\ 28 \text{ and } 19 \end{array} \right|$ without rem.

Th;

QUESTION CCXXXVIII. [Continued.]

Th. $nA + mB + pC$ will divide by 28, 19 and 15, and leave the remainders n , m , and p ;

Th. $\left(\frac{nA + mB + pC}{28 \times 19 \times 15} = \right) \frac{nA + mB + pC}{7980}$ will leave x for a remainder;

That is $\frac{4845n + 4200m + 6916p}{7980}$ will leave x for a remainder?

EXAMPLE.

In what year of the Julian period, was the cycle of the sun 12, the cycle of the moon 13, and the Roman indiction 14?

Now $4845 \times 12 = 58140$
 $4200 \times 13 = 54600$
 $6916 \times 14 = 96824$ } their sum is 209564;

Then $\frac{209564}{7980} = 26$; And remainder 2084 is the answer.

COROL. If it be required to find the least number (x) that being divided, by M , will leave m ; by N , n ; by P , p ; by Q , q ; by R , r ; &c. as remainders (M , N , O , P , Q , R , &c. being prime to each other)?

Assume $A = \overline{NPQR}$, &c. $\times a = MB + 1$,

$B = \overline{MPQR}$, &c. $\times c = Nd + 1$,

$C = \overline{MNQR}$, &c. $\times e = Pf + 1$,

$D = \overline{MNPQ}$, &c. $\times g = Qb + 1$;

QUESTION CCXXXVIII. [Continued.]

And having severally found the values of A, B, C, D , &c. as above;

Then $\frac{Ax + Bx + Cx + Dx + Ex, \&c.}{A \times B \times C \times D \times E, \&c.}$ will leave x for a remainder.

EXAMPLE.

The least number that can be divided by 2, 3, 5, 7, and 11, severally; and leave 1, 2, 3, 4, and 5, for remainders, is required?

$$\text{Here } \left\{ \begin{array}{l} A = (3 \cdot 5 \cdot 7 \cdot 11a =) 1155a = 2b + 1, \\ B = (2 \cdot 5 \cdot 7 \cdot 11c =) 770c = 3d + 1, \\ C = (2 \cdot 3 \cdot 7 \cdot 11e =) 462e = 5f + 1, \\ D = (2 \cdot 3 \cdot 5 \cdot 11g =) 330g = 7h + 1, \\ E = (2 \cdot 3 \cdot 5 \cdot 7i =) 210i = 11k + 1; \end{array} \right.$$

$$\text{Or } \left\{ \begin{array}{l} 1155a - 2b = 1, \\ 770c - 3d = 1, \\ 462e - 5f = 1, \\ 330g - 7h = 1, \\ 210i - 11k = 1; \end{array} \right.$$

First; Let $578a - b' = b$;

Then $1155a - 1156a + 2b' = 1$; Or $2b' - a = 1$,

Th. $(b' =) a = \frac{1}{2} = 1$; And $A = (1155 \times 1 =) 1155$:

Secondly;

QUESTION CCXXXVIII. [Continued.]

Secondly ; Let $255e + 1 + d' = d$;

Then $770e - 765e - 3 - 3d' = 1$; Or $5e - 3d' = 4$;

Th. $e = (d' =) \frac{4}{3} = 2$; And $B = (770 \times 2 =) 1540$;

Thirdly ; Let $91e + 1 + f' = f$;

Then $462e - 455e - 5 - 5f' = 1$; Or $7e - 5f' = 6$;

Th. $e = (f' =) \frac{6}{2} = 3$; And $C = (462 \times 3 =) 1386$;

Fourthly ; Let $46g + b' = b$;

Then $330g - 322g - 7b' = 1$; Or $8g - 7b' = 1$;

Th. $g = (b' =) \frac{1}{1} = 1$; And $D = (330 \times 1 =) 330$;

Fifthly ; Let $18i + k' = k$;

Then $210i - 198i - 11k' = 1$; Or $12i - 11k' = 1$;

Th. $i = (k' =) \frac{1}{1} = 1$; And $E = (210 \times 1 =) 210$;

Then $\frac{1155 \times 1 + 1540 \times 2 + 1386 \times 3 + 330 \times 4 + 210 \times 5}{2 \times 3 \times 5 \times 7 \times 11}$

$$= \frac{1155 + 3080 + 4158 + 1320 + 1050}{2310}$$

$$= \frac{10763}{2310} = 4 + \frac{1523}{2310} ;$$

Then 1523 is the number required.

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QUESTION CCXXXIX.

Forty-one persons, men, women, and children, spent 40*l.* whereof each man paid 4*s.* each woman 3*s.* and each child 4*d.* How many of each were there?

Suppose there were x , men; y , women; and z children;

$$\begin{aligned} \text{Then } x + y + z &= 41, \\ \text{And } 4x + 3y + \frac{1}{4}z &= 40, \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{Then } x + y + z &= 41, \\ \text{And } 4x + 3y + \frac{1}{4}z &= 40, \end{aligned}} \right\} \text{per quest.}$$

Now $12x + 9y + z = 120$, by second,

$$\begin{aligned} \text{Or } x + y &= 41 - z, \\ \text{And } 12x + 9y &= 120 - z, \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{Or } x + y &= 41 - z, \\ \text{And } 12x + 9y &= 120 - z, \end{aligned}} \right\} \text{by transposition;}$$

$$\text{Also } \begin{cases} 12x + 12y = 492 - 12z \\ 9x + 9y = 369 - 9z \end{cases} \quad \text{by multipl.}$$

$$\text{Again } \begin{cases} 3x = 8z - 249 \\ 3y = 372 - 11z \end{cases} \quad \text{by subtraction;}$$

$$\text{Th. } \begin{cases} z - \left(\frac{249}{8}\right) = 31\frac{3}{8}, \\ z - \left(\frac{372}{11}\right) = 33\frac{2}{11}; \end{cases}$$

Also, when x and y are whole numbers, z must be a multiplier of 3:

But the only multiplier of 3, between $31\frac{3}{8}$, and $33\frac{2}{11}$, is 33:

$$\text{Th. } z = 33; x = \left(\frac{8 \times 33 - 249}{3}\right) = 5; \text{ And } y = 3.$$

Q U E S

QUESTION CCXL

If the product of the solidities of two cubes, the sum of whose sides is 10, be multiplied by the solidity of the greater, it will produce 3176523: What are their sides?

If x be the side of the cube,

And y the side of the lesser;

Then $(x^3 \times y^3) = x^6 y^3 = 3176523$.

And $x^3 = \frac{3176523}{y^3}$

But $(x + y = 10$: Th.) $x = 10 - y$

And $x^6 = (10 - y)^6$

Th. $\frac{(10 - y)^6}{y^3} = \frac{3176523}{y^3}$

Or $(10 - y)^6 \times y^3 = 3176523$

Th. $(10 - y) \times y^{\frac{1}{3}} = \sqrt[6]{3176523}$

Or $10 - y = \frac{\sqrt[6]{3176523}}{y^{\frac{1}{3}}} = (\sqrt[3]{49 \times 3}) = 7\sqrt{3}$

Th. $y^{\frac{1}{3}} = 10\sqrt{\frac{1}{3}} + 7\sqrt{3} = 9$

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QUESTION CCXLI.

How many ways may 4 sorts of wine, whose prices are 16, 10, 8, and 6 *d.* per quart, be mixed; so as to make 100 quarts in all, and be worth 12 *d.* per quart?

Let the required quantities of the four several sorts of wine be represented by $x, y, u,$ and z ;

$$\text{Then } \left\{ \begin{array}{l} x + y + u + z = 100 \\ 16x + 10y + 8u + 6z = (100 \times 12 =) 1200 \end{array} \right\} \begin{array}{l} \text{Eq. 1} \\ \text{Eq. 2} \end{array}$$

$$\text{Or } \left\{ \begin{array}{l} y + u + z = 100 - x \\ 10y + 8u + 6z = 1200 - 16x \end{array} \right\} \text{ by trans.}$$

$$\text{And } \left\{ \begin{array}{l} 10y + 8u + 10z = 1000 - 10x \\ 6y + 5u + 6z = 600 - 6x \end{array} \right\} \text{ by multi.}$$

$$\text{Also by sub. } \left\{ \begin{array}{l} 2u + 4z = 6x - 200 \\ 4y + 2u = 600 - 10x \end{array} \right.$$

$$\text{Th. } \left\{ \begin{array}{l} x - \left(\frac{200}{6}\right) = 33 \frac{1}{3} \\ x - \left(\frac{600}{10}\right) = 60 \end{array} \right.$$

$$\text{But by transp. } \left\{ \begin{array}{l} u + z = 100 - x - y \\ 8u + 6z = 1200 - 16x - 10y \end{array} \right.$$

$$\text{And by multip. } \left\{ \begin{array}{l} 8u + 8z = 800 - 8x - 8y \\ 6u + 6z = 600 - 6x - 6y \end{array} \right.$$

$$\text{Whence } \left\{ \begin{array}{l} z = 4x + y - 200 \\ u = 300 - 5x - 2y \end{array} \right.$$

$$\text{Th. } \left\{ \begin{array}{l} y - \frac{200 - 4x}{2} \\ y - \frac{300 - 5x}{2} \end{array} \right.$$

From

QUESTION CCXLI. [Continued.]

From which limits it will appear, that

If $x = 59; y \sqsupset 2\frac{1}{2}; y \sqsubset -36$; Th. y has 2 values,

If $x = 58; y \sqsupset 5; y \sqsubset -32$; Th. y has 4 values,

If $x = 57; y \sqsupset 7\frac{1}{2}; y \sqsubset -28$; Th. y has 7 values,

&c.

&c.

&c.

&c.

If $x = 51; y \sqsupset 22\frac{1}{2}; y \sqsubset -4$; Th. y has 22 values,

If $x = 50; y \sqsupset 25; y \sqsubset 6$; Th. y has 24 values,

If $x = 49; y \sqsupset 27\frac{1}{2}; y \sqsubset 4$; Th. y has 23 values,

If $x = 48; y \sqsupset 30; y \sqsubset 8$; Th. y has 21 values,

&c.

&c.

&c.

&c.

If $x = 34; y \sqsupset 65; y \sqsubset 64$; Th. y has 0 values.

Hence the number of the values of y may be obtained by summing two arithmetical progressions;

In the first; The least term is 2, greatest term 22, and number of terms 9:

Th. the sum is $(22 + 2 \times \frac{9}{2}) = 108$.

In the second; the least term is 0, greatest term 24; and number of terms 17;

Th. the sum is $(24 + 0 \times \frac{17}{2}) = 204$:

Th. $108 + 204 = 312$ is the required number of answers.

See QUEST. 6. PART II.

QUES.

MATHEMATICAL QUESTION CCXLII.

To find a number, the products of which by two given numbers 32 and 8, may be the square numbers?

If x be the number required;

Then $32x$ and $8x$ are square numbers,

And their roots are $4\sqrt{2x}$ and $2\sqrt{2x}$;

Th. $x = \frac{1}{2}$ a square number:

But 4, 16, 36, 64, 100, 144, &c. are square numbers;

Th. 2, 8, 18, 32, 50, 72, &c. $= x$.

QUESTION CCXLIII.

Required two such square numbers, that their difference may be a square number?

Let $x =$ root of $\square$ squ. $x + d =$ root of $\square$ squ.

And $z =$ root of their difference;

Then $(x + d)^2 - xx = 2dx + dd = zz$, by quest.

That is $2x + d \times d = zz \times 1$;

Then by $\left\{ \begin{array}{l} 2x + d = 1, 2, 3, 4, 5, 6, \&c. \text{ to } n, \\ \end{array} \right.$

qu. 221 $\left\{ \begin{array}{l} \end{array} \right. \quad \begin{array}{l} x = d, 2d, 3d, 4d, 5d, 6d, \&c. \text{ to } nd; \end{array}$

Now $(zz =) d^2$ is a rational square, when x and d are square numbers;

And because $2x + d = n$; Th. $x = \frac{1}{2} \times n - d$;

And $x + d = \frac{1}{2} \times n + d$;

Now let $m^2 = n$; And $r^2 = d$;

Then $\frac{1}{2} \times m^2 + r^2 - \frac{1}{2} \times m^2 - r^2 = r^2 m^2$,

Th. $m^2 + r^2 - m^2 - r^2 = 4r^2 m^2$;

Where m and r , may be any numbers whatsoever; only $\square \square r$.

COROL. I. $m^2 - r^2$, and $2rm$, are the roots of two sq. n^{os} whose sum will be a sq. n^o viz. $m^2 + r^2$.

COROL. II. The sq^s of all n^{os} that are in proportion, as $m^2 - r^2$, $2rm$, and $m^2 + r^2$, will have the property required in the question,

REPOSITORY.

EXAMPLES.

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If Then For

m	r	Then		For	
		$2rm$	$m^2 - r^2$	$m^2 + r^2$	$2rm^2 + m^2 - r^2 = m^2 + r^2$
2	1	4	3	5	$16 + 9 = 25$
3	1	6	8	10	$36 + 64 = 100$
4	1	8	15	17	$64 + 225 = 289$
5	1	10	24	26	$100 + 576 = 676$
6	1	12	35	37	$144 + 1225 = 1369$
7	1	14	48	50	$196 + 2304 = 2500$
8	1	16	63	65	$256 + 3969 = 4225$
9	1	18	80	82	$324 + 6400 = 6724$
3	2	12	5	13	$144 + 25 = 169$
4	2	16	12	20	$256 + 144 = 400$
5	2	20	21	29	$400 + 441 = 841$
6	2	24	32	40	$576 + 1024 = 1600$
7	2	28	45	53	$784 + 2025 = 2809$
8	2	32	60	68	$1024 + 3600 = 4624$
9	2	36	77	85	$1296 + 5929 = 7225$
4	3	24	7	25	$576 + 49 = 625$
5	3	30	16	34	$900 + 256 = 1156$
6	3	36	27	45	$1296 + 729 = 2025$
7	3	42	40	58	$1764 + 1600 = 3364$
8	3	48	55	73	$2304 + 3025 = 5329$
9	3	54	72	90	$2916 + 5184 = 8100$
5	4	40	9	41	$1600 + 81 = 1681$
6	4	48	20	52	$2304 + 400 = 2704$
7	4	56	33	65	$3136 + 1089 = 4225$
8	4	64	48	80	$4096 + 2304 = 6400$
9	4	72	65	97	$5184 + 4225 = 9409$
6	5	60	11	61	$3600 + 121 = 3721$
7	5	70	24	74	$4900 + 576 = 5476$
8	5	80	39	89	$6400 + 1521 = 7921$
9	5	90	56	106	$8100 + 3136 = 11236$
7	6	84	13	85	$7056 + 169 = 7225$
8	6	96	28	100	$9216 + 784 = 10000$
9	6	108	45	117	$11664 + 2025 = 13689$
8	7	112	15	113	$12544 + 225 = 12769$
9	7	126	32	130	$15876 + 1024 = 16900$
9	8	144	17	145	$20736 + 289 = 21025$

QUES.

QUESTION CXXIV.

Two numbers in proportion as n to p are required, so that the sum of their squares, may be a square number?

By COROL. I. QUEST. 243. The sum of the squares of $m^2 - r^2$, and $2rm$; is a square number:

But by quest. $n : p :: m^2 - r^2 : 2rm$;

Th. $2rm = \frac{pm^2 - pr^2}{n}$;

Or $pr^2 + 2rm = pm^2$;

Th. $r^2 + \frac{2nm}{p}r = m^2$;

But $r^2 + \frac{2nm}{p}r + \frac{nm^2}{p^2} = \frac{pp + nm \times nm}{p^2}$;

Th. $r + \frac{nm}{p} = \sqrt{pp + nm \times \frac{m}{p}}$;

And $r = \sqrt{pp + nm} - n \times \frac{m}{p}$;

Therefore for m , some multiple of p must be assumed.

EXAMPLE.

To find two numbers in proportion as 8 to 15, the sum of whose squares may be a square number?

Let $n = (2 \times 15 =) 30$; Then $r = (\sqrt{225 + 64 - 8 \times \frac{15}{2}} =) 18$;

Then $(30^2 - 18^2 =) 576$, and $(2 \times 30 \times 18 =) 1080$, are the numbers required.

For $(576^2 + 1080^2 =) 331776 + 1166400 = 1498176$
 $= 1224^2$.

QUES.

QUESTION CCXLV.

'Tis required to divide a given square number into two square numbers ?

Let xx be the square to be divided :

$$\text{Now } \overline{m^2 + r^2 \times x}^2 = \overline{m^2 - r^2 \times x}^2 + \overline{2rmx}^2$$

COROL. II. QUEST. 243.

$$\text{Or } \overline{m^2 + r^2}^2 \times xx = \overline{m^2 - r^2 \times x}^2 + \overline{2rmx}^2$$

$$\text{Th. } - - - xx = \frac{\overline{m^2 - r^2}^2}{\overline{m^2 + r^2}^2} x + \frac{\overline{2rm}^2}{\overline{m^2 + r^2}^2} x$$

Where m and r may be taken at pleasure ; so $m \square r$

EXAMPLE.

Divide 25, into 2 square numbers ?

Assume $m = 2$; and $r = 1$;

$$\text{Then } 25 = \left(\frac{4-1}{4+1} \times 5 \right)^2 + \left(\frac{2 \times 2}{4+1} \times 5 \right)^2 = 9 + 16 :$$

Assume $m = 3$; and $r = 1$;

$$\text{Then } 25 = \left(\frac{9-1}{9+1} \times 5 \right)^2 + \left(\frac{2 \times 3}{9+1} \times 5 \right)^2 = 16 + 9 :$$

Assume $m = 3$; and $r = 2$;

$$\begin{aligned} \text{Then } 25 &= \left(\frac{9-4}{9+4} \times 5 \right)^2 + \left(\frac{2 \times 2 \times 3}{9+4} \times 5 \right)^2 \\ &= \left(\frac{5}{13} \right)^2 + \left(\frac{30}{13} \right)^2. \end{aligned}$$

QUEST.

QUESTION CCXLVI.

*Tis required to divide a given number, which is the sum of two square numbers, into two other square numbers?

If the roots of the two squares that compose the given the given number be a , and d ; also $a \supset d$;

Then $aa + dd =$ the given number :

Let $\begin{cases} mx + a = \\ nx - d = \end{cases}$ the root $\begin{cases} \text{greater} \\ \text{of the} \end{cases}$ $\begin{cases} \text{square required;} \\ \text{lesser} \end{cases}$

Then $aa + dd = \begin{cases} mnx + 2amx + aa + \\ nxx - 2dnx + dd, \end{cases}$

Or $- 2dnx = mnx + 2amx + nxx,$

Or $- 2dn = mnx + 2am + nxx;$

Th. $\frac{dn - am \times 2}{mn + nn} = x:$

Where $n \supset m$; but n must not be in proportion to m , as $d + a$ to $d - a$; because then $dn = am$.

EXAMPLE.

Let it be required to divide the number 1450, which is the sum of 35^2 and 15^2 , into two other square numbers?

Assume $n = 2$, and $m = 1$; Then

$$\left(\frac{35 \times 2 - 15 \times 1 \times 2}{1 \times 1 + 2 \times 2} = \frac{70 - 15 \times 2}{1 + 4} = \frac{55 \times 2}{5} = \right) 22 = x,$$

And $\begin{cases} (1 \times 22 + 15 = 22 + 15 =) 37 \\ (2 \times 22 - 35 = 44 - 35 =) 9 \end{cases}$ are the roots of the required squares.

QUES.

QUESTION CCXLVII

'Tis required to find 2 such square numbers, that if any given number be added to their product, the sum may be a square number?

Let x and m , be the roots of the required squares;

And n the given number;

Then $mxx + n$, must be a square number:

Let $mx + d$, be the root thereof;

Then $mxx + 2mdx + dd = mxx + n$

Or $2mdx + dd = n$,

Or $2mdx = n - dd$;

Th. $x = \frac{n - dd}{2md}$

Where dd may be any square number less than n ; and n any number whatsoever.

EXAMPLE.

If the given number n be 28;

Then $dd = 25, 16, 9, 4, 1$;

$n - dd = 3, 12, 19, 24, 27$;

If $m = 1$; $x = \frac{3}{10}, \frac{12}{8}, \frac{19}{6}, \frac{24}{4}, \frac{27}{2}$;

If $m = 2$; $x = \frac{3}{20}, \frac{12}{16}, \frac{19}{12}, \frac{24}{8}, \frac{27}{4}$;

If $m = 3$; $x = \frac{3}{30}, \frac{12}{24}, \frac{19}{18}, \frac{24}{12}, \frac{27}{6}$;

&c.

&c.

&c.

QUEST

QUESTION CCXLVIII.

Required two square numbers, that shall have a given difference?

If x and $n+d$ be the roots of the required squares;

And n be the given difference:

Then $(x+d)^2 - xx = 2xd + dd = n$;

Th. $x = \frac{n-dd}{2d}$;

Where dd may be any square number less than n .

EXAMPLE.

If the given difference n be 60;

Then $dd = 49, 36, 25, 16, 9, 4, 1$;

And $n-dd = 11, 24, 35, 44, 51, 56, 59$;

Th. $x = \frac{11}{2}, \frac{24}{2}, \frac{35}{2}, \frac{44}{2}, \frac{51}{2}, \frac{56}{2}, \frac{59}{2}$;

COROL. Hence two numbers, m and n , being given, a third z , may be found, which being added to, or taken from, each of the given numbers, the sums, or remainders, shall be square numbers?

For $m \pm z = xx + xd + dd$,

And $n \pm z = xx$;

Th. $m - n = 2xd + dd$; whence $x = \frac{m-n-dd}{2d}$

And $z = xx = n$.

Q U E S T

QUESTION CCXLIX.

To find a perfect number (that is a number equal to all its aliquot parts) ?

Let $y^n x =$ the number required;

Then its aliquot parts will be

$1 \cdot y \cdot y^2 \cdot y^3$, &c. to y^n . $x \cdot xy \cdot xy^2$, &c. to xy^{n-1} ;

But $1+y+y^2+y^3$, &c. $+x+xy+xy^2$, &c. $= y^n x$;

Th. $1+y+y^2+y^3$, &c. $= y^n x - x - xy - xy^2$, &c.

And $\frac{1+y+y^2+y^3}{y^n-1-y-y^2-y^3}$, &c. $= \frac{x}{y^n-1-y-y^2-y^3}$, &c.

Now if $y^n-1-y-y^2-y^3$, &c. $= 1$; $x = 1+y+y^2$, &c.

And $y^n-1-y-y^2$, &c. $= 1$, when $y = 2$.

For $2^2-1-2=(4-3)=1$; $2^3-1-2-4=(8-7)=1$ &c.

But x in $y^n x$ is a prime number;

Th. $1+2+4+8$, &c. $(=x)$ must be a prime number;

But $(1+2=)3$ is a prime No. Th. $3 \times 2 = 6$

And $(1+2+4=)7$ is ditto Th. $7 \times 4 = 28$

Also $1+2+4+8+16=31$ do. Th. $31 \times 16 = 496$

&c.

For $(\frac{6}{2} + \frac{6}{3} + \frac{6}{6} =) 3 + 2 + 1 = 6$;

And $\frac{28}{2} + \frac{28}{4} + \frac{28}{7} + \frac{28}{14} + \frac{28}{28} = 28$, &c.

QUES:

QUESTION CCL.

What are the values of x , y , z , and u , in the equation, $ax + ay = az + au$?

By division $x + y = z + u$,

And by transp. $x - z = u - y$,

Then (q. 221.) $x : 1 :: x - z : u - y$.

Whence (the value of z being fixed) x , y , z and u , have innumerable values,

But if $z = x$; Then $y = u$.

QUESTION CCLI.

What are the values of x , y , z , u , v and i , in the equation, $ax + a^2y + a^3z = au + a^2v + a^3i$?

By division $x + ay + a^2z = u + av + a^2i$;

Where if $x = u$; $ay + a^2z = av + a^2i$;

In which (by quest. last) If $y = v$,

Then $z = i$.

COROL. Hence if there be never so many terms in an equation of the above form (the number of terms on each side being equal) viz.

$$ax + by^2 + cz^3 + du^4, \&c. = ax + \beta y^2 + \gamma z^3 + \delta u^4, \&c.$$

One value of each of the unknown coefficients, a , β , γ , δ , &c. will be found by making,

$$a = a,$$

$$\beta = b,$$

$$\gamma = c, \&c.$$

The

Mathematical Repository,

PART II.

QUESTION I.

'T IS required to find the sum (A) of n terms of the series of numbers 0, 1, 2, 3, 4 etc. to $n-1$?

Since this is required by the given number n only; the sum A must be equal to n , or its powers, multiplied by some, yet unknown, coefficients;

And because n is greater than $n-1$ the last term, of that part of the series, whose sum is required;

Th. (n taken n times $\equiv$) nn must be greater than A :
Let therefore $xnn - yn = A$;

Now if n , the next greater term of the series be added thereto, the number of terms will be $n+1$; and the sum of them $A+n$:

Then (because x and y (tho' unknown) are constant quantities)

$$x \times n \times 1 - y \times n + 1 = A + n,$$

That is $xn^2 + 2xn + x - yn - y = A + n$;

And $2xn + x - y = n$ } by subtracting
the value of A

Or $2xn + x = n + y$;

Now (by Quest. 250) if $x = y$,

Then $2x = 1$;

Th. $x (=y) = \frac{1}{2}$;

And $\frac{nn}{2} - \frac{n}{2} = A (= \frac{n \times n - 1}{2})$

K

Corol.

QUESTION I. continued.

Corol. The sum of n terms of $1 + 2 + 3 + 4$, etc.
 ($= n+1$ terms of $0 + 1 + 2 + 3$ etc.) will be found to be
 $\frac{n+1 \times n}{2}$, by writing $n+1$ for n , in the above expression.

In an Arithmetical progression (i. e. a rank or series of numbers whose differences are equal) if,

a } represent { the least (Number or) term,
 x } { the greatest term,
 d } { the difference of any two adjacent terms,
 n } { the number of terms,
 s } { the sum of all the terms:

Then, QUESTIONS II and III.

a , d , and n , are given; to find x , and s ?

Now a = the least term
 $a + d$ = the second
 $a + 2d$ = the third
 $a + 3d$ = the fourth
 $a + 4d$ = the fifth
 etc. etc.

} because d is the difference of any two adjacent terms.

Then $a + n-1 \times d$ = (the n^{th} term =) x ,

And s = $\left\{ \begin{array}{l} n \text{ terms of } 1 + 1 + 1 + 1 \text{ etc.} \times a + \\ n \text{ terms of } 0 + 1 + 2 + 3 \text{ etc.} \times d: \end{array} \right.$

But na = n terms of $1 + 1 + 1 + 1$ etc.

And, $\frac{n \times n - 1}{2} = n$ terms of $0 + 1 + 2 + 3$ etc. by Qu. 1.

Th. $s = na + \frac{n \times n - 1}{2} d$.

EXAMPLE.

'Tis required to find the sum of n terms of, the series of odd numbers, 1, 3, 5, 7, etc?

Here $a = 1$; and $d = 2$;

Th. $(n + \frac{n \times n - 1}{2} \times 2 = n + nn - n =) nn$ is the sum required.

Corol.

QUESTION III. continued

Corol. Hence every square number (nn) is the sum of n terms of the series of odd numbers, beginning with unity?

QUESTION IV.

How many terms of the series of odd numb. 1. 3. 5 etc. must be added together, to produce the 6th power of 12?

If $6 = 2m$, $12 = r$; and $n =$ the number of terms required;

Then $1+3+5$ etc. to n terms $= r^{2m}$;

But n terms of $1+3+5$ etc. $= nn$ (by corol. quest. last)

Th. $nn = r^{2m}$; or $n = r^m$;

In this example $n = (12^3) = 1728$.

QUESTION V.

'Tis required to find 13 terms of the indefinite series, 3. 5. 7. 9 etc; whose sum, may be the 3^d power of 13?

Let $x =$ the least term of that part of the series required; $3 = m$ and $13 = n$:

Then $(nx + \frac{nn-n}{2} 2 =) nx + nn - n$ will be the sum of n terms by question 2^d:

But $nx + nn - n = n^3$ by question 3;

And $x + n - 1 = n^2 - 1$ by division;

Th. $x = n^2 - 1 - n + 1$.

That is $x = (13^2 - 12 =) 157$.

QUESTIONS VI and VII.

In an arithmetical progression are given a , z , and n , to find d , and s ?

By question 2; $a + \frac{n-1}{1} \times d = z$,

Or $\frac{n-1}{1} \times d = z - a$

Th. $d = \frac{z-a}{n-1}$

By quest. 3; $na + \frac{n \times n - 1}{2} d = s$

And (from above) $na + \frac{n \times n - 1}{2} \times \frac{z-a}{n-1} = s$

That is $na + \frac{n \times z - a}{2} = s$

Or $\left(\frac{2na + nz - na}{2} \right) = \frac{na + nz}{2} = s$

Th. $\frac{n \times z + z}{2} = s$

QUESTIONS VIII and IX.

In an arithmetical progression, are given a , d , and z , to find n and s ?

By quest. 2; $a + \frac{n-1}{1} \times d = z$,

Or $nd - d = z - a$

Th. $n = \left(\frac{z-a+d}{d} \right) = \frac{z-a}{d} + 1$ (By

QUESTION IX.

(By quest. 7.) $\frac{n \times a + z}{2} = s;$

Then $z = \frac{2s}{n+a};$

And by qu. 8; $\frac{z-a}{d} + 1 = \frac{2s}{z+a};$

Th. $\frac{z-a}{d} + 1 \times \frac{z+a}{2} = s.$

QUESTIONS X and XI.

In an arithmetical progression are given a , n and s to find z , and d .

By quest. 7; $\frac{na + nz}{2} = s;$

Or $na + nz = 2s;$

Th. $z = \frac{2s - na}{n};$

Or $z = \frac{2s}{n} - a.$

By qu. 3^d; $na + \frac{n \times n - 1}{2} d = s;$

Or $\frac{n \times n - 1}{2} d = s - na;$

Th. $d = \frac{s - na \times 2}{n \times n - 1}.$

QUESTIONS XII and XIII.

In an arithmetical progression are given a , x , and s ;
to find n , and d ?

By quest. 7; $\frac{n \times a + x}{2} = s$;

Th. $n = \frac{2s}{a+x}$.

By quest. 9; $\frac{x-a}{d} + 1 \times \frac{x+a}{2} = s$,

Or $\frac{x-a}{d} + 1 = \frac{2s}{x+a}$

Or $\frac{x-a}{d} = \frac{2s}{x+a} - 1$,

Or $\frac{x+a \times x-a}{2} = 2ds - d \times x + a$;

Th. $\frac{x+a \times x-a}{2s - x + a} = d$.

QUESTIONS XIV and XV.

In an arithmetical progression are given a , d , and s ;
to find x , and n ?

By quest. 9. $\frac{x-a}{d} + 1 \times \frac{x+a}{2} = s$,

Or $\frac{xx - aa}{2d} + \frac{x+a}{2} = s$,

Or $xx - aa + xd + ad = 2ds$;

Th. $x^2 + xd = 2ds + aa - ad$;

But $x^2 + xd + \frac{dd}{4} = 2ds + aa - ad + \frac{dd}{4}$,

Or

QUESTION XIV. continued.

Or $4z^2 + 4zd + dd = 8ds + 4a^2 - 4ad + dd,$

Or $2z + d = \sqrt{8ds + 2a + d^2};$

Th. $2z + d = \sqrt{8ds + 2a + d^2};$

And $z = \sqrt{8ds + 2a + d^2} - d.$

By qu. 3^d; $na + \frac{n \times n - 1}{2} d = s,$

That is $2na + nnd - nd = 2s,$

Or $dn + 2a - d \times n = 2s;$

Then $nn + \frac{2a - d}{d} n = \frac{2s}{d};$

But $nn + \frac{2a - d}{d} n + \frac{2a - d}{2d}^2 = \left(\frac{2s}{d} + \frac{2a - d}{2d} \right)^2 = \frac{8ds + 2a - d^2}{4dd},$

Th. $n + \frac{2a - d}{2d} = \sqrt{\frac{8ds + 2a - d^2}{4dd}};$

And $n = \sqrt{\frac{8ds + 2a - d^2}{4dd}} - \frac{2a - d}{2d}$

QUESTIONS XVI and XVII.

In an arithmetical progression are given z , d , and n ; to find a , and s ?

By question 2^d; $a + n - 1 \times d = z;$

Th. $a = z - n - 1 \times d.$

By question 7; $\frac{n \times a + z}{2} = s,$

But above $a = z - n - 1 \times d,$

Th. $s + z = 2z - n - 1 \times d;$

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QUESTION XVII, continued.

Th. $\frac{2nx - nxn - 1 \times d}{2} = s,$

That is $nx - \frac{nxn - 1}{2} d = s.$

QUESTIONS XVIII and XIX.

In an arithmetical progression are given, d , n , and s ; to find a , and x ?

By quest. 3^d; $na + \frac{nxn - 1}{2} d = s,$

Or $na = s - \frac{nxn - 1}{2} d;$

Th. $a = \frac{s}{n} - \frac{n-1}{2} d.$

By quest. 17; $nx - \frac{nxn - 1}{2} d = s;$

Th. $x = \frac{s}{n} + \frac{n-1}{2} d.$

QUESTIONS XX and XXI.

In an arithmetical progression are given, x , n , and s ; to find a , and d ?

By quest. 7. $\frac{n \times a + x}{2} = s,$

Or $a + x = \frac{2s}{n};$

Th. $a = \frac{2s}{n} - x.$

By

QUESTION XXI.

By quest. 17. $nx - \frac{n \times n - 1}{2} d = s,$

Or $nx - s = \frac{n \times n - 1}{2} d;$

Th. $\frac{nx - s \times 2}{n \times n - 1} = d.$

QUESTIONS XXII and XXIII.

In an arithmetical progression are given, $x, d,$ and $s;$ to find $a,$ and $n?$

By quest. 9. $\frac{x-a}{d} + 1 \times \frac{x+a}{2} = \frac{s}{d},$

Or $xx - aa + xd + ad = 2ds;$

Th. $na - da = xx + xd - 2ds;$

But $aa - da + \frac{dd}{4} = xx + xd + \frac{dd}{4} - 2ds;$

Or $2a - d = \sqrt{xx + xd - 8ds};$

Th. $2a - d = \sqrt{xx + xd - 8ds};$

And $a = \frac{\sqrt{xx + xd - 8ds} + d}{2};$

By quest. 17. $nx - \frac{n \times n - 1}{2} d = s,$

Or $2nx - nnd + nd = 2s,$

Or $dnn - 2x + d \times n = 2s,$

Th. $nn - \frac{2x + d}{d} n = -\frac{2s}{d};$

But $nn - \frac{2x + d}{d} n + \left[\frac{2x + d}{2a} \right]^2 = \left[\frac{2x + d}{2d} \right]^2 - \frac{2s}{d};$

That is $n - \frac{2x + d}{2d} = \frac{2x + d^2 - 8sd}{4dd};$

QUESTION XXIII. continued.

$$\text{Th. } n - \frac{2x+d}{2d} = \frac{\sqrt{2x+d^2-8sd}}{2d},$$

$$\text{And } n = \frac{2x+d}{2d} \pm \frac{\sqrt{2x+d^2-8sd}}{2d}.$$

QUESTION XXIV.

Two men A and B set out at the same time; A travels 8 miles a day; and B travels the first day 1 mile, the second day 2 miles, the third day 3 miles, etc. In how many days will B overtake A?

If x be the number days required;

Then A will travel $8x$ miles;

Also the sum of an arithmetical progression whose first term is 1, common difference 1, and N^o. of terms x is

$$\left(\frac{x+1 \times x}{2} = \right) \frac{xx+x}{2} \text{ by corol. to quest. 1st;}$$

$$\text{Th. B will travel } \frac{xx+x}{2} \text{ miles:}$$

$$\text{But } \frac{xx+x}{2} = 8x \text{ by quest.}$$

$$\text{Th. } \frac{x+1}{2} = 8;$$

$$\text{And } x = 15.$$

QUES-

QUESTION XXV.

A Stationer sold 7 reams of paper, the particular prices whereof were certain numbers of shillings in arithmetical progression; the price of the second ream, that is, of that next above the cheapest, was 8 shillings; and the price of the dearest ream was 23 shillings: What was the price of each ream?

If z = the price of the cheapest ream,

And x = the difference of their prices;

Then $z + x = 8$, } by quest. 2^d;

And $z + 6x = 23$, }

Th. $8 - x = (z =) 23 - 6x$;

Whence $x = \left(\frac{23-8}{5} = \right) 3$.

QUESTION XXVI.

Two post boys A, and B, set out, at the same time, from two cities, which are 360 miles asunder, in order to meet each other: A rides 40 miles the first day, 38 the second, 36 the third, and so on, decreasing 2 miles every day; but B goes 20 miles the first day, 22 the second, 24 the third, etc. increasing 2 miles every day; In what number of days will they meet?

If x = the time of their meeting;

Then x terms of $40 + 38 + 36$ etc. $= 40x - \frac{xx-x}{2} 2$ by q. 17.

And x terms of $20 + 22 + 24$ etc. $= 20x + \frac{xx-x}{2} 2$ by q. 3.

Now $40x - \frac{xx-x}{2} 2 + 20x + \frac{xx-x}{2} 2 = 360$ by question,

That is $(40x + 20x =) 60x = 360$;

Th. $x = 6$.

QUESTION XXVII.

To find three numbers in arithmetical progression, whereof the sum of the squares shall be 1232, and the square of the mean shall exceed the product of the extremes by 16?

If x = the mean number; and z = the common difference;

Then $x - z$; x ; and $x + z$, are the numbers;

But $xx = x - z \times x + z + 16$ by quest.

That is $xx = xx - zz + 16$;

Th. $zz = 16$; and $z = 4$;

Now $x - 4^2 = xx - 8x + 16$,

$$xx = xx,$$

And $x + 4^2 = xx + 8x + 16$;

Th. $1232 = 3xx + 32$,

And $400 = xx$;

Th. $20 = x$.

QUESTION XXVIII.

One being asked what were the several ages of his five children, answered, that the age of the eldest exceeded that of the second by 2 years; and by the same excess the second exceeded the third, the third the fourth, etc; and if the age of the eldest child were multiplied by the age of the youngest it would produce 128. The age of each child is required?

If x = the age of the eldest;

Then $x - (5 - 1 \times 2) = 8$ = age of the youngest, by q. 16.

Now $(x - 8 \times x) = xx - 8x = 128$ by quest.

But $xx - 8x + 16 = (128 + 16) = 144$;

Th. $x = (12 + 4) = 16$.

QUES.

QUESTION XXIX.

There is a number consisting of three places, whose digits are in arithmetical progression; if this number be divided by the sum of its digits the quotient will be 26; and if to the number you add 198, the digits will be inverted: What is the number?

If x be the middle digit and d the common difference of the digits?

$$\begin{array}{l} \text{Then} \left\{ \begin{array}{l} x-d \\ x \\ x+d \end{array} \right\} \text{ are the digits and } \left\{ \begin{array}{l} 100x-100d \\ 10x \\ x+d \end{array} \right\} \text{ digits valued accord. to their places and } \left\{ \begin{array}{l} 100x+100d \\ 10x \\ x-d \end{array} \right\} \end{array}$$

Th. $3x = \text{sum digits}; 111x - 99d = \text{numb.}; 111x + 99d = \text{digits inverted}$

Now $111x - 99d + 198 = 111x + 99d$ by quest.

Or $198 = 198d;$

Th. $1 = d;$

Also $\frac{111x - 99}{3} = 26$ by quest.

Or $111x - 99 = 78x;$

Th. $x = \left(\frac{99}{111-78} = \frac{99}{33} = \right) 3.$

And the number is 234.

QUESTION XXX.

If the sum of 6 numbers in arithmetical progression be 48, and the product of the common difference multiplied into the least term be equal to the numbers of terms: What are the numbers of that progression?

If $6 = n; 48 = s; a = \text{least term and } d = \text{common difference};$

Then $da = (n-1)6; \text{ or } a = \frac{6}{d} \text{ (per question);}$

Also

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QUESTION XXX. continued.

Also $s = na + \frac{n \times n - 1}{1 \times 2} d$ by quest 3;

That is $48 = 6a + \left(\frac{6 \times 5}{2} d =\right) 15d$,

Or $16 = 2a + 5d$;

But $16 = \frac{12}{d} + 5d$. (by writing $\frac{6}{d}$ for a)

Th. $16d = 12 + 5dd$,

Or $dd - \frac{16}{5}d = -\frac{12}{5}$;

But $d - \frac{8}{5} = \left(\frac{64 - 12}{25} =\right) \frac{4}{25}$;

Th. $d - \frac{8}{5} = \frac{2}{5}$; and $d = \left(\frac{8+2}{5} =\right) 2$.

QUESTION XXXI.

There are 4 numbers in arithmetical progression, whereof the product of the extremes is 3250; and that of the means 3300: What are the numbers?

If z = the first number and x = the common difference; Then z ; $z+x$; $z+2x$; and $z+3x$; will be the numbers;

And $(z \times z + 3x =) zz + 3xz = 3250$ } by qu.

Also $(z+x \times z+2x =) zz + 3xz + 2xx = 3300$ }

Th. (by subtraction) $2xx = 50$,

And $x = 5$.

Now $zz + 3 \times 5z = 3250$ (by first);

But $zz + 15z + \frac{15^2}{2} = \left(3250 + \frac{225}{4} =\right) \frac{13225}{4}$;

Th. $z + \frac{15}{2} = \frac{115}{2}$;

And $z = \left(\frac{115-15}{2} =\right) 50$.

QUES-

QUESTION XXXII.

The continual product of four numbers in arithmetical progression is 945; and their common difference 2: What are those numbers?

If x = the least number required;

Then x , $x+2$, $x+4$, and $x+6$ will be the numbers;

But $x \times x+2 \times x+4 \times x+6 = x^4 + 12x^3 + 44x^2 + 48x$;

Th. $x^4 + 12x^3 + 44x^2 + 48x = 945$ by quest.

Where $x = 3$.

QUESTION XXXIII.

'Tis required to find (B) the sum of n terms of, the series of square numbers, 0, 1, 4, 9, 16 etc. to $\frac{n-1}{2}$?

Since n^3 is greater than B (See quest 1st.)

Let $xn^3 - yn^2 + zn = B$;

Then (because the $n+1^{\text{th}}$ term of the series is nn)

$$x \times n+1^3 - y \times n+1^2 + z \times n+1 = B + nn;$$

Now $xn^3 + 3xn^2 + 3xn + x = x \times n+1^3$,

$$yn^2 + 2yn + y = y \times n+1^2,$$

$$zn + z = z \times n+1;$$

Th. $xn^3 + 3xn^2 + 3xn + x - yn^2 - 2yn - y + zn + z = B + nn$;

And $3xn^2 + 3xn + x - yn^2 - 2yn - y + zn = nn$ { by subtracting
the value of B }

Or $3xn^2 + 3xn + x - yn^2 - 2yn - y + zn = nn + 2yn + y$;

Whence by corol. qu. 251. $\left\{ \begin{array}{l} 3x = 1, \\ 3x = 2y, \\ x + z = y; \end{array} \right.$

Th. $\left\{ \begin{array}{l} x = \frac{1}{3}, \\ \frac{1}{3} = y, \\ z = (\frac{1}{3} - \frac{1}{3}) = \frac{0}{3}; \end{array} \right.$

Th.

QUESTION XXXIII. continued.

$$\text{Th. } \left(\frac{n^3}{3} - \frac{n^2}{2} + \frac{n}{6} \right) = \frac{n, n-1, 2n-1}{1, 2, 3} = B.$$

Corol. the sum of n terms of 1, 4, 9, 16. etc. will be found $= \frac{n, n+1, 2n+1}{1, 2, 3}$; by writing $n+1$ for n in the value of B .

$$\text{Note } \frac{n, n-1, 2n-1}{1, 2, 3} \text{ signifies } \frac{n \times n-1 \times 2n-1}{1 \times 2 \times 3}$$

QUESTION XXXIV.

Two travellers A and B set out together, from the same place; A goes 8 miles the first day, 12 the second, 16 the third, and so on, increasing 4 miles every day: But B goes 1 mile the first day, 4 the second, 9 the third, and so on, according to the squares of the number of days: How many days must they travel before B overtakes A?

Suppose B will overtake A in x days,

$$\text{Th. } x \text{ terms of } 8+12+16 \text{ etc.} = 8x + \frac{x \times x - 1}{2} \times 4 \text{ by q. 3.}$$

$$\text{And } x \text{ terms of } 1+4+9 \text{ etc.} = \frac{x \times x + 1 \times 2x + 1}{1 \times 2 \times 3} \quad (\text{by cor. qu. 33.})$$

$$\text{Now by quest. } \frac{x \times x + 1 \times 2x + 1}{1 \times 2 \times 3} = 8x + \frac{x \times x - 1}{2} \times 4,$$

$$\text{Or by division } \frac{x+1 \times 2x+1}{2 \times 3} = 8 + x - 1 \times \frac{4}{2},$$

$$\text{Or } 2x^2 + 3x + 1 = 48 + 12x - 12,$$

$$\text{Th. } x^2 - \frac{9}{2}x = \frac{35}{2}$$

$$\text{But } \left(x^2 - \frac{9}{2}x + \frac{9}{4} \right)^2 = \left(\frac{81}{16} + \frac{35}{2} \right) \frac{361}{16};$$

$$\text{Th. } x - \frac{9}{4} = \frac{19}{4}; \text{ and } x = \left(\frac{19+9}{4} \right) = 7.$$

QUEST.

QUESTION XXXV.

The sum (1225) of the series 0, 1, 2, 3, 4 etc. continued to an unknown number of terms, being given, to find the sum of the squares of these terms?

If $1225 = A$; $0 + 1 + 4 + 9$ etc. $= B$; No. terms $= n$:

Then (by quest. 3) $\frac{n \cdot n - 1}{1 \cdot 2} = A$;

And (by qu. 33) $\frac{n \cdot n - 1 \cdot 2n - 1}{1 \cdot 2 \cdot 3} = B$:

The quotient arising from the division of the second by the first $\left\{ \frac{2n - 1}{3} = \frac{B}{A} \right.$;

Th. $n = \frac{3B + A}{2A}$;

And $n - 1 = \frac{3B - A}{2A}$;

But $\left(\frac{n}{1} \times \frac{n - 1}{2} \right) = \frac{3B + A}{2A} \times \frac{3B - A}{2A} = A$, by first,

Or $\frac{9B^2 - A^2}{8A^2} = A$;

Or $9B^2 - A^2 = 8A^2$;

Or $9B^2 = 8A^2 + A^2$;

Th. $B^2 = \frac{8A^2 + A^2}{9}$;

And $B = \frac{A}{3} \times \sqrt{8A + 1}$.

In this example $B = \left(\frac{1225}{3} \times \sqrt{8 \times 1225 + 1} \right) 40425$

QUES.

QUESTION XXXVI.

What is the sum of 10 square numbers, whose roots are in an arithmetical progression, the least term of which is 3, and the common difference 2?

Let $10 = n$; $3 = a$; $2 = d$; then it is required to find (S) the sum of n terms of a^2 , $a+d^2$, $a+2d^2$ etc.

Now $a^2 = aa$,

$$\overline{a+d^2} = aa + 2 \times 1ad + dd,$$

$$\overline{a+2d^2} = aa + 2 \times 2ad + 4dd,$$

$$\overline{a+3d^2} = aa + 2 \times 3ad + 9dd,$$

$$\overline{a+4d^2} = aa + 2 \times 4ad + 16dd;$$

etc.

etc.

etc.

$$\text{Th. S.} = \begin{cases} \text{Sum of } n \text{ terms of } \overline{1+1+1} \text{ etc.} & \times aa \\ + D^o. & \text{of } \overline{0+1+2+3} \text{ etc.} & \times 2ad \\ + D^o. & \text{of } \overline{0+1+4+9} \text{ etc.} & \times dd \end{cases}$$

But n terms of $\overline{1+1+1} \text{ etc.} = n$,

$$D^o. \text{ of } \overline{0+1+2+3} \text{ etc.} = \frac{n \times n - 1}{1 \times 1} \text{ by quest. I;}$$

$$\text{And } D^o. \text{ of } \overline{0+1+4+9} \text{ etc.} = \frac{n \times n - 1 \times 2n - 1}{1 \times 2 \times 3} \text{ by q. 33}$$

$$\text{Th. S.} = naa + n \times n - 1 \times ad + \frac{n \times n - 1 \times 2n - 1}{1 \times 2 \times 3} dd.$$

$$\text{In this exa. S.} = \left(10 \cdot 3^2 + 10 \cdot 9 \cdot 3 \cdot 2 + \frac{10 \cdot 9 \cdot 19}{1 \cdot 2 \cdot 3} \times 2^2 = \right) (1770.)$$

QUES.

QUESTION XXXVII.

If the sum of 10 numbers in arithmetical progression be 120; and the sum of their squares 1770: What are those numbers?

If a = least number; d = common difference;
 n = (10 =) number of terms; s = 120; and S = 1770.

Then (quest. 3) $s = na + \frac{n \cdot n - 1}{1 \cdot 2} d$;

And (quest. 36) $S = na^2 + n \cdot n - 1 \cdot ad + \frac{n \cdot n - 1 \cdot 2n - 1}{1 \cdot 2 \cdot 3} d^2$;

Now $ss = n^2 a^2 + n^2 \cdot n - 1 \cdot ad + \frac{n^2 \cdot n - 1}{2 \cdot 2} dd$;

And $\frac{ss}{n} = na^2 + n \cdot n - 1 \cdot ad + \frac{n \cdot n - 1}{2 \cdot 2} dd$;

By subtraction $\left\{ \begin{array}{l} S - \frac{ss}{n} = \frac{n \cdot n - 1 \cdot 2n - 1}{1 \cdot 2 \cdot 3} d^2 - \frac{n \cdot n - 1}{2 \cdot 2} d^2 \end{array} \right.$

Or $S - \frac{ss}{n} = \frac{2n - 1}{3} \cdot \frac{n - 1}{2} \times \frac{n \cdot n - 1}{1 \cdot 2} d^2$;

But $\frac{n + 1}{6} = \left(\frac{2n - 1}{3} \cdot \frac{n - 1}{2} \right) \frac{4n - 2 - 3n + 3}{2 \cdot 3}$;

Th. $S - \frac{ss}{n} = \frac{n + 1}{6} \times \frac{n \cdot n - 1}{1 \cdot 2} d^2$;

That is $\frac{nS - ss}{n} = \frac{n - 1 \cdot n \cdot n + 1}{1 \cdot 3 \cdot 4} dd$;

Th. $\frac{nS - ss \times 3 \cdot 4}{n - 1 \cdot nn \cdot n + 1} = dd$;

Or $\frac{nS - ss \times 3}{n - 1 \cdot n + 1} \times \frac{4}{nn} = dd$;

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QUESTION XXXVII continued.

$$\text{Th. } \frac{2}{x} \times \sqrt{\frac{n^3 - 55 \times 3}{n-1 \cdot n+1}} = d.$$

$$\text{In this examp. } \left(\frac{2}{10} \sqrt{\frac{10 \cdot 1770 - 120 \times 3}{9 \cdot 11}} \right)^2 = d$$

Having found d , x may be found by qu. 18.

QUESTION XXXVIII.

The sum ($\mathfrak{B}$) of x terms of the series of triangular numbers, 0, 1, 3, 6, 10, etc. (the terms of which series are [0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190, 210, 231, 253, 276, 301, 327, 354, 383, 413, 444, 476, 509, 543, 579, 616, 655, 695, 737, 780, 825, 871, 919, 969, 1020, 1072, 1125, 1180, 1237, 1295, 1355, 1416, 1479, 1543, 1609, 1677, 1747, 1818, 1891, 1965, 2041, 2119, 2199, 2280, 2363, 2447, 2533, 2620, 2709, 2799, 2891, 2984, 3079, 3175, 3273, 3373, 3474, 3576, 3679, 3783, 3889, 3996, 4105, 4215, 4326, 4438, 4551, 4665, 4781, 4898, 5016, 5136, 5257, 5379, 5502, 5626, 5751, 5877, 6004, 6132, 6261, 6391, 6522, 6654, 6787, 6921, 7057, 7194, 7332, 7471, 7611, 7752, 7894, 8037, 8181, 8326, 8472, 8619, 8767, 8916, 9066, 9217, 9369, 9522, 9676, 9831, 9987, 10144, 10302, 10461, 10621, 10782, 10944, 11107, 11271, 11436, 11602, 11769, 11937, 12106, 12276, 12447, 12619, 12792, 12966, 13141, 13317, 13494, 13672, 13851, 14031, 14212, 14394, 14577, 14761, 14946, 15132, 15319, 15507, 15696, 15886, 16077, 16269, 16462, 16656, 16851, 17047, 17244, 17442, 17641, 17841, 18042, 18244, 18447, 18651, 18856, 19062, 19269, 19477, 19686, 19896, 20107, 20319, 20532, 20746, 20961, 21177, 21394, 21612, 21831, 22051, 22272, 22494, 22717, 22941, 23166, 23392, 23619, 23847, 24076, 24306, 24537, 24769, 24992, 25216, 25441, 25667, 25894, 26122, 26351, 26581, 26812, 27044, 27277, 27511, 27746, 27982, 28219, 28457, 28696, 28936, 29177, 29419, 29662, 29906, 30151, 30397, 30644, 30892, 31141, 31391, 31642, 31894, 32147, 32401, 32656, 32912, 33169, 33427, 33686, 33946, 34207, 34469, 34732, 35006, 35281, 35557, 35834, 36112, 36391, 36671, 36952, 37234, 37517, 37801, 38086, 38372, 38659, 38947, 39236, 39526, 39817, 40109, 40402, 40696, 40991, 41287, 41584, 41882, 42181, 42481, 42782, 43084, 43387, 43691, 43996, 44302, 44609, 44917, 45226, 45536, 45847, 46159, 46472, 46786, 47101, 47417, 47734, 48052, 48371, 48691, 49012, 49334, 49657, 49981, 50306, 50632, 50959, 51287, 51616, 51946, 52277, 52609, 52942, 53276, 53611, 53947, 54284, 54622, 54961, 55301, 55642, 55984, 56327, 56671, 57016, 57362, 57709, 58057, 58406, 58756, 59107, 59459, 59812, 60166, 60521, 60877, 61234, 61592, 61951, 62311, 62672, 63034, 63397, 63761, 64126, 64492, 64859, 65227, 65596, 65966, 66337, 66709, 67082, 67456, 67831, 68207, 68584, 68962, 69341, 69721, 70102, 70484, 70867, 71251, 71636, 72022, 72409, 72797, 73186, 73576, 73967, 74359, 74752, 75146, 75541, 75937, 76334, 76732, 77131, 77531, 77932, 78334, 78737, 79141, 79546, 79952, 80359, 80767, 81176, 81586, 81997, 82409, 82822, 83236, 83651, 84067, 84484, 84902, 85321, 85741, 86162, 86584, 87007, 87431, 87856, 88282, 88709, 89137, 89566, 90006, 90447, 90889, 91332, 91776, 92221, 92667, 93114, 93562, 94011, 94461, 94912, 95364, 95817, 96271, 96726, 97182, 97639, 98097, 98556, 99016, 99477, 99939, 100402, 100866, 101331, 101797, 102264, 102732, 103201, 103671, 104142, 104614, 105087, 105561, 106036, 106512, 106989, 107467, 107946, 108426, 108907, 109389, 109872, 110356, 110841, 111327, 111814, 112302, 112791, 113281, 113772, 114264, 114757, 115251, 115746, 116242, 116739, 117237, 117736, 118236, 118737, 119239, 119742, 120246, 120751, 121257, 121764, 122272, 122781, 123291, 123802, 124314, 124827, 125341, 125856, 126372, 126889, 127407, 127926, 128446, 128967, 129489, 130012, 130536, 131061, 131587, 132114, 132642, 133171, 133701, 134232, 134764, 135297, 135831, 136366, 136902, 137439, 137977, 138516, 139056, 139597, 140139, 140682, 141226, 141771, 142317, 142864, 143412, 143961, 144511, 145062, 145614, 146167, 146721, 147276, 147832, 148389, 148947, 149506, 150066, 150627, 151189, 151752, 152316, 152881, 153447, 154014, 154582, 155151, 155721, 156292, 156864, 157437, 158011, 158586, 159162, 159739, 160317, 160896, 161476, 162057, 162639, 163221, 163804, 164388, 164973, 165559, 166146, 166734, 167322, 167911, 168501, 169092, 169684, 170276, 170870, 171465, 172061, 172658, 173256, 173855, 174455, 175056, 175657, 176259, 176862, 177466, 178071, 178677, 179284, 179892, 180501, 181111, 181722, 182334, 182947, 183561, 184176, 184792, 185409, 186027, 186646, 187266, 187887, 188509, 189132, 189756, 190381, 191007, 191634, 192262, 192891, 193521, 194152, 194784, 195417, 196051, 196686, 197322, 197959, 198597, 199236, 199876, 200517, 201159, 201802, 202446, 203091, 203737, 204384, 205032, 205681, 206331, 206982, 207634, 208287, 208941, 209596, 210252, 210909, 211567, 212226, 212886, 213547, 214209, 214872, 215536, 216201, 216867, 217534, 218202, 218871, 219541, 220212, 220884, 221557, 222231, 222906, 223582, 224259, 224937, 225616, 226296, 226977, 227659, 228342, 229026, 229711, 230397, 231084, 231772, 232461, 233151, 233842, 234534, 235227, 235921, 236616, 237312, 238009, 238707, 239406, 240106, 240807, 241509, 242212, 242916, 243621, 244327, 245034, 245742, 246451, 247161, 247872, 248584, 249297, 250011, 250726, 251442, 252159, 252877, 253596, 254316, 255037, 255759, 256482, 257206, 257931, 258657, 259384, 260112, 260841, 261571, 262302, 263034, 263767, 264501, 265236, 265972, 266709, 267447, 268186, 268926, 269667, 270409, 271152, 271896, 272641, 273387, 274134, 274882, 275631, 276381, 277132, 277884, 278637, 279391, 280146, 280902, 281659, 282417, 283176, 283936, 284697, 285459, 286222, 286986, 287751, 288517, 289284, 290052, 290821, 291591, 292362, 293134, 293907, 294681, 295456, 296232, 297009, 297787, 298566, 299346, 300127, 300909, 301692, 302476, 303261, 304047, 304834, 305622, 306411, 307201, 307992, 308784, 309577, 310371, 311166, 311962, 312759, 313557, 314356, 315156, 315957, 316759, 317562, 318366, 319171, 319977, 320784, 321592, 322401, 323211, 324022, 324834, 325647, 326461, 327276, 328092, 328909, 329727, 330546, 331366, 332187, 333009, 333832, 334656, 335481, 336307, 337134, 337962, 338791, 339621, 340452, 341284, 342117, 342951, 343786, 344622, 345459, 346297, 347136, 347976, 348817, 349659, 350502, 351346, 352191, 353037, 353884, 354732, 355581, 356431, 357282, 358134, 358987, 359841, 360696, 361552, 362409, 363267, 364126, 364986, 365847, 366709, 367572, 368436, 369301, 370167, 371034, 371902, 372771, 373641, 374512, 375384, 376257, 377131, 378006, 378882, 379759, 380637, 381516, 382396, 383277, 384159, 385042, 385926, 386811, 387707, 388604, 389502, 390401, 391301, 392202, 393104, 394007, 394911, 395816, 396722, 397629, 398537, 399446, 400356, 401267, 402179, 403092, 404006, 404921, 405837, 406754, 407672, 408591, 409511, 410432, 411354, 412277, 413201, 414126, 415052, 415979, 416907, 417836, 418766, 419697, 420629, 421562, 422496, 423431, 424367, 425304, 426242, 427181, 428121, 429062, 430004, 430947, 431891, 432836, 433782, 434729, 435677, 436626, 437576, 438527, 439479, 440432, 441386, 442341, 443297, 444254, 445212, 446171, 447131, 448092, 449054, 450017, 450981, 451946, 452912, 453879, 454847, 455816, 456786, 457757, 458729, 459702, 460676, 461651, 462627, 463604, 464582, 465561, 466541, 467522, 468504, 469487, 470471, 471456, 472442, 473429, 474417, 475406, 476396, 477387, 478379, 479372, 480366, 481361, 482357, 483354, 484352, 485351, 486351, 487352, 488354, 489357, 490361, 491366, 492371, 493377, 494384, 495391, 496400, 497409, 498419, 499430, 500441, 501452, 502464, 503476, 504489, 505502, 506516, 507531, 508546, 509562, 510579, 511596, 512614, 513632, 514651, 515671, 516691, 517712, 518733, 519754, 520776, 521798, 522821, 523844, 524867, 525891, 526915, 527940, 528965, 529990, 531016, 532042, 533068, 534094, 535121, 536148, 537175, 538202, 539230, 540258, 541286, 542314, 543342, 544371, 545400, 546430, 547460, 548490, 549520, 550550, 551580, 552610, 553640, 554670, 555700, 556730, 557760, 558790, 559820, 560850, 561880, 562910, 563940, 564970, 566000, 567030, 568060, 569090, 570120, 571150, 572180, 573210, 574240, 575270, 576300, 577330, 578360, 579390, 580420, 581450, 582480, 583510, 584540, 585570, 586600, 587630, 588660, 589690, 590720, 591750, 592780, 593810, 594840, 595870, 596900, 597930, 598960, 599990, 601020, 602050, 603080, 604110, 605140, 606170, 607200, 608230, 609260, 610290, 611320, 612350, 613380, 614410, 615440, 616470, 617500, 618530, 619560, 620590, 621620, 622650, 623680, 624710, 625740, 626770, 627800, 628830, 629860, 630890, 631920, 632950, 633980, 635010, 636040, 637070, 638100, 639130, 640160, 641190, 642220, 643250, 644280, 645310, 646340, 647370, 648400, 649430, 650460, 651490, 652520, 653550, 654580, 655610, 656640, 657670, 658700, 659730, 660760, 661790, 662820, 663850, 664880, 665910, 666940, 667970, 669000, 670030, 671060, 672090, 673120, 674150, 675180, 676210, 677240, 678270, 679300, 680330, 681360, 682390, 683420, 684450, 685480, 686510, 687540, 688570, 689600, 690630, 691660, 692690, 693720, 694750, 695780, 696810, 697840, 698870, 699900, 700930, 701960, 702990, 704020, 705050, 706080, 707110, 708140, 709170, 710200, 711230, 712260, 713290, 714320, 715350, 716380, 717410, 718440, 719470, 720500, 721530, 722560, 723590, 724620, 725650, 726680, 727710, 728740, 729770, 730800, 731830, 732860, 733890, 734920, 735950, 736980, 738010, 739040, 740070, 741100, 742130, 743160, 744190, 745220, 746250, 747280, 748310, 749340, 750370, 751400, 752430, 753460, 754490, 755520, 756550, 757580, 758610, 759640, 760670, 761700, 762730, 763760, 764790, 765820, 766850, 767880, 768910, 769940, 770970, 772000, 773030, 774060, 775090, 776120, 777150, 778180, 779210, 780240, 781270, 782300, 783330, 784360, 785390, 786420, 787450, 788480, 789510, 790540, 791570, 792600, 793630, 794660, 795690, 796720, 797750, 798780, 799810, 800840, 801870, 802900, 803930, 804960, 805990, 807020, 808050, 809080, 810110, 811140, 812170, 813200, 814230, 815260, 816290, 817320, 818350, 819380, 820410, 821440, 822470, 823500, 824530, 825560, 826590, 827620, 828650, 829680, 830710, 831740, 832770, 833800, 834830, 835860, 836890, 837920, 838950, 839980, 841010, 842040, 843070, 844100, 845130, 846160, 847190, 848220, 849250, 850280, 851310, 852340, 853370, 854400, 855430, 856460, 857490, 858520, 859550, 860580, 861610, 862640, 863670, 864700, 865730, 866760, 867790, 868820, 869850, 870880, 871910, 872940, 873970, 875000, 876030, 877060, 878090, 879120, 880150, 881180, 882210, 883240, 884270, 885300, 886330, 887360, 888390, 889420, 890450, 891480, 892510, 893540, 894570, 895600, 896630, 897660, 898690, 899720, 900750, 901780, 902810, 903840, 904870, 905900, 906930, 907960, 908990, 910020, 911050, 912080, 913110, 914140, 915170, 916200, 917230, 918260, 919290, 920320, 921350, 922380, 923410, 924440, 925470, 926500, 927530, 928560, 929590, 930620, 931650, 932680, 933710, 934740, 935770, 936800, 937830, 938860, 939890, 940920, 941950, 942980, 944010, 945040, 946070, 947100, 948130, 949160, 950190, 951220, 952250, 953280, 954310, 955340, 956370, 957400, 958430, 959460, 960490, 961520, 962550, 963580, 964610, 965640, 966670, 967700, 968730, 969760, 970790, 971820, 972850, 973880, 974910, 975940, 976970, 978000, 979030, 980060, 981090, 982120, 983150, 984180, 985210, 986240, 987270, 988300, 989330, 990360, 991390, 992420, 993450, 994480, 995510, 996540, 997570, 998600, 999630, 1000660, 1001690, 1002720, 1003750, 1004780, 1005810, 1006840, 1007870, 1008900, 1009930, 1010960, 1011990, 1013020, 1014050, 1015080, 1016110, 1017140, 1018170, 1019200, 1020230, 1021260, 1022290, 1023320, 1024350, 1025380, 1026410, 1027440, 1028470, 1029500, 1030530, 1031560, 1032590, 1033620, 1034650, 1035680, 1036710, 1037740, 1038770, 1039800, 1040830, 1041860, 1042890, 1043920, 1044950, 1045980, 1047010, 1048040, 1049070, 1050100, 1051130, 1052160, 1053190, 1054220, 1055250, 1056280, 1057310, 1058340, 1059370, 1060400, 1061430, 1062460, 1063490, 1064520, 1065550, 1066580, 1067610, 1068640, 1069670, 1070700, 1071730, 1072760, 1073790, 1074820, 1075850, 1076880, 1077910, 1078940, 1079970, 1081000, 1082030, 1083060, 1084090, 1085120, 1086150, 1087180, 1088210, 1089240, 1090270, 1091300, 1092330, 1093360, 1094390, 1095420, 1096450, 1097480, 1098510, 1099540, 1100570, 1101600, 1102630, 1103660, 1

QUESTION XXXIX.

The sum (C) of n terms of the series of cube numbers (0, 1, 8, 27, 64 etc. to $n-1^3$) is required?

Let $\frac{xn^4}{4} - \frac{yn^3}{3} + \frac{nu^2}{2} - zn = C;$

Then $x \times n+1 - y \times n+1 + u \times n+1 - z \times n+1 = C + n^3;$

But $4xn^3 + 6xn^2 + 4xn + x = x \times n+1^4;$

$3yn^2 + 3yn + y = y \times n+1^3;$

$2un + u = u \times n+1^2;$

$zn + z = z \times n+1;$

Th. $\left. \begin{array}{r} 4xn^3 + 6xn^2 + 4xn + x \\ -y \\ +u \\ -z \end{array} \right\} = C + n^3$

Then by subtract. $\left\{ \begin{array}{r} 4xn^3 + 6xn^2 + 4xn + x \\ -3y \\ +2u \\ -z \end{array} \right\} = n^3$

Or $\left\{ \begin{array}{r} 4xn^3 + 6xn^2 + 4xn + x \\ +2u \\ +u \end{array} \right\} = \left\{ \begin{array}{r} n^3 + 3yn^2 + 3yn + y \\ +z \end{array} \right\}$

Now by corol to quest. 251. $\left\{ \begin{array}{r} 4x = 1, \\ 6x = 3y, \\ 4x + 2u = 3y, \\ x + u = y + z; \end{array} \right\}$ And th. $\left\{ \begin{array}{r} x = \frac{1}{4}, \\ y = \frac{1}{2}, \\ u = \frac{1}{4}, \\ o = z; \end{array} \right\}$

Th. $\left(\frac{n^4}{4} - \frac{n^3}{2} + \frac{n^2}{4} \right) \frac{n^2 \times n - 1^2}{2 \times 2} = C.$

Corol. By writing $n+1$ for n , in the value of C , it will appear that the sum of n terms of the series 1, 8,

27, 64, 125 etc. $= \frac{n^2 \times n+1^2}{2 \times 2}.$

QUES.

QUESTION XL.

'Tis required to find the sum of 10 cube numbers, whose roots are in an arithmetical progression; the least term of which is 3, and the common difference 2?

Let $10=n$; $3=a$; and $2=d$; then it is required to find (Σ) the sum of n terms of a^3 , $a+d^3$, $a+2d^3$ etc. Now $a^3 = a^3$,

$$\overline{a+d^3} = a^3 + 3 \times 1a^2d + 3 \times 1ad^2 + d^3,$$

$$\overline{a+2d^3} = a^3 + 3 \times 2a^2d + 3 \times 4ad^2 + 8d^3,$$

$$\overline{a+3d^3} = a^3 + 3 \times 3a^2d + 3 \times 9ad^2 + 27d^3,$$

$$\overline{a+4d^3} = a^3 + 3 \times 4a^2d + 3 \times 16ad^2 + 64d^3;$$

etc.

etc.

$$\text{Th. } \Sigma = \left\{ \begin{array}{l} \text{Sum of } n \text{ terms of } \overline{1+1+1 \text{ etc } \times a^3}, \\ + D^0 \text{ of } \overline{0+1+2+3 \text{ etc } \times 3a^2d}, \\ + D^0 \text{ of } \overline{0+1+4+9 \text{ etc } \times 3ad^2}, \\ + D^0 \text{ of } \overline{0+1+8+27 \text{ etc } \times d^3} \end{array} \right\} \begin{array}{l} \text{See question} \\ \left\{ \begin{array}{l} 1, \\ 33 \\ 39 \end{array} \right. \end{array}$$

$$\text{That is } \Sigma = na^3 + \frac{n \cdot n-1}{1 \cdot 2} 3a^2d + \frac{n \cdot n-1 \cdot 2n-1}{1 \cdot 2 \cdot 3} 3ad^2 + \frac{n^2 \cdot n-1^2}{2 \cdot 2} d^3.$$

$$\text{In this exam. } \left\{ \Sigma = \left(10 \times 3^3 + \frac{10 \cdot 9}{1 \cdot 2} \times 3 \cdot 3^2 \cdot 2 + \frac{10 \cdot 9 \cdot 19}{1 \cdot 2 \cdot 3} \times 3 \cdot 3 \cdot 2^2 + \frac{10^2 \cdot 9^2}{2 \cdot 2} \times 2^3 \right) = 29160. \right.$$

QUES.

QUESTION XLI.

The sum of 10 numbers in arithmetical progression is 120; and the sum of their cubes 29160: What are those numbers?

Let a = least number; d = common difference; n = (10 =) number of terms; s = 120; and Σ = 29160:

Then by quest.

$$\left\{ \begin{array}{l} 3) s = na + \frac{n(n-1)}{2}d, \\ 4) \Sigma = na^3 + \frac{n(n-1)}{2} \cdot 3ad^2 + \frac{n(n-1)(2n-1)}{6}ad^2 + \frac{n^3(n-1)^2}{2 \cdot 2 \cdot 2}d^3; \end{array} \right.$$

Now $s^3 = n^3a^3 + \frac{n(n-1)}{2} \cdot 3n^2ad + \frac{n^2(n-1)^2}{2 \cdot 2} \cdot 3nad^2 + \frac{n^3(n-1)^3}{2 \cdot 2 \cdot 2}d^3;$

And $\frac{s^3}{n^3} = na^3 + \frac{n(n-1)}{2} \cdot 3ad + \frac{n(n-1)^2}{2 \cdot 2} \cdot 3ad^2 + \frac{n(n-1)^3}{2 \cdot 2 \cdot 2}d^3;$

Th. $\Sigma = \frac{s^3}{n^3} = \frac{n^3}{3} \cdot \frac{3}{2n-1} \cdot \frac{n-1}{2} \times \frac{n(n-1)}{1 \cdot 2} \cdot 3ad^2 + \frac{n(n-1)}{2 \cdot 2} \times \frac{n(n-1)^2}{2 \cdot 2}d^3;$

But $\frac{n+1}{6} = \frac{3}{2n-1} \cdot \frac{n-1}{2};$ and $\frac{n+1}{2} = \frac{n}{2} \cdot \frac{n-1}{2};$

Th. $\Sigma = \frac{s^3}{n^3} = \frac{3}{1 \cdot 2 \cdot 2} \cdot \frac{n(n-1)}{2} \cdot ad^2 + \frac{n(n-1)^2}{2 \cdot 2 \cdot 2}d^3;$

But $\frac{n-1}{2} \cdot \frac{n+1}{2} = \frac{n-1}{1 \cdot 2} \cdot \frac{n(n-1)}{2} \cdot ad^2 + \frac{n(n-1)^2}{2 \cdot 2 \cdot 2}d^3$ from first;

Th.

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QUESTION XLI. continued.

Th. $\frac{n-1}{2} \cdot \frac{n+1}{2} \text{ add } = \left(\sum \frac{s^3}{nn} \right) \frac{nn \sum - s^3}{nn}$

And $\text{did} = \frac{nn \sum - s^3}{s \cdot n - 1 \cdot n + 1 \cdot nn}$

Th. $s = \frac{2}{n} \sqrt{\frac{nn \sum - s^3}{s \cdot n - 1 \cdot n + 1 \cdot nn}}$

In this exam. $d = \left(\frac{2}{10} \sqrt{\frac{10^2 \cdot 29160 - 120^3}{120 \cdot 9 \cdot 11}} \right)^2$

Having found d , a may be found by quest. 18.

QUESTION XLII.

There are two places 462 miles asunder, from which two persons A and B set out at the same time to meet each other. A goes 1 mile the first day and encreases each succeeding days journey by 1 mile; and B travels each day the cube of the miles that A travels: In what time will they meet?

Suppose x = the time of their meeting;

Then x terms of $1+2+3+4$ etc. $= \frac{x+1 \times x}{2}$ } by co. qu. 1,

And x terms of $1+8+27+64$ etc. $= \frac{x+1 \times x^2}{2 \times 2}$ } by cor qu. 39.

Now by quest. $\frac{x+1^2 \times x^2}{2 \times 2} + \frac{x+1 \times x}{2} = 462$;

And (putting $\frac{x+1 \times x}{2} = y$) $xy + y = 462$;

But $xy + y + \frac{1}{4} = (462 + \frac{1}{4}) = \frac{1849}{4}$;

Th. $y + \frac{1}{2} = \frac{43}{2}$; And $y = \left(\frac{43-1}{2} \right) = 21$;

QUESTION XLII. continued.

But $\left(y = \frac{x+1xx}{2} = \right) \frac{xx+x}{2} = 21\frac{1}{2}$

Th. $xx+x = 42$

But $xx+x+\frac{1}{4} = (42+\frac{1}{4}) = \frac{169}{4}$

Th. $x+\frac{1}{2} = \frac{13}{2}$; And $x = \left(\frac{13-1}{2}\right) = \frac{4}{6}$.

QUESTION XLIII.

Tis required to find the sum C of n terms of the series 0, 1, 4, 10, 20, &c. The terms of which (0, 0+1, 0+1+3, 0+1+3+6, etc.) are the successive sum of the series 0, 1, 3, 6, 10, etc.?

Let $xn^4 - yn^3 + un^2 - zn = C$; as in quest. 39.

Now by cor. quest. 38, the $n+1^{\text{th}}$ term of this series

is $\left(\frac{n \cdot n+1 \cdot n+2}{1 \cdot 2 \cdot 3} = \right) \frac{n^3}{6} + \frac{3n^2}{6} + \frac{2n}{6}$;

Therefore by process in quest. 39.

$$\left. \begin{array}{r} 4xn^3 + 6xn^2 + 4xn + x \\ - 3y \\ + 2u + u \\ - z \end{array} \right\} = \frac{n^3}{6} + \frac{3n^2}{6} + \frac{2n}{6}$$

Or $\left\{ \begin{array}{l} 4xn^3 + 6xn^2 + 4xn + x \\ + 2u + u \end{array} \right\} = \left\{ \begin{array}{l} \frac{1}{6}n^3 + \frac{3}{6}n^2 + \frac{2}{6}n \\ + 2y + 3y + y + z \end{array} \right\}$

Now by corol quest. 251. $\left\{ \begin{array}{l} 4x = \frac{1}{6} \\ 6x = 3y + \frac{3}{6} \\ 4x + 2u = 3y + \frac{2}{6} \\ x + u = y + z \end{array} \right\}$ Therefore $\left\{ \begin{array}{l} x = \frac{1}{24} \\ -\frac{1}{12} = y \\ u = -\frac{1}{24} \\ \frac{1}{12} = z \end{array} \right\}$

L

And

QUESTION XLIII, continued.

$$\text{And } \frac{n^4}{24} + \frac{n^3}{12} - \frac{n^2}{24} - \frac{n}{12} = C = \frac{n-1}{1} \cdot \frac{n}{2} \cdot \frac{n+1}{3} \cdot \frac{n+2}{4}$$

Corol. 1. By writing $n+1$, for n , in the value of C , it will appear that n terms of 1, 4, 10, 20, etc.

$$= \frac{n \cdot n+1 \cdot n+2 \cdot n+3}{1 \cdot 2 \cdot 3 \cdot 4}$$

Corol. 2. If	$\left\{ \begin{array}{l} A \\ B \\ C \\ D \\ \text{etc.} \end{array} \right\}$	represent the sums of n terms of the series.	$\left\{ \begin{array}{l} 1, 2, 3, 4, 5, \text{etc.} \\ 1, 3, 6, 10, 15, \text{etc.} \\ 1, 4, 10, 20, 35, \text{etc.} \\ 1, 5, 15, 35, 70, \text{etc.} \\ \text{etc.} \end{array} \right\}$	whose terms are the suc- cessive sums of the series	$\left\{ \begin{array}{l} 1, 1, 1, 1, 1, \text{etc.} \\ 1, 2, 3, 4, 5, \text{etc.} \\ 1, 3, 6, 10, 15, \text{etc.} \\ 1, 4, 10, 20, 35, \text{etc.} \\ \text{etc.} \end{array} \right\}$

Then from the several corollaries to questions 1, 3, 8, and 43;

$$A = \frac{n \cdot n+1}{1 \cdot 2},$$

$$B = \frac{n \cdot n+1 \cdot n+2}{1 \cdot 2 \cdot 3},$$

$$C = \frac{n \cdot n+1 \cdot n+2 \cdot n+3}{1 \cdot 2 \cdot 3 \cdot 4},$$

$$D = \frac{n \cdot n+1 \cdot n+2 \cdot n+3 \cdot n+4}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5},$$

$$\text{etc.} \qquad \text{etc.}$$

* The numbers of which these series consist are, in general, called figurate numbers.

QUES.

QUESTION XLIV.

Three persons A , B , and C , agreed to play as many games at Piquet, as could be done without the same two persons, playing against each other, twice: The number of games is required?

Now $\left\{ \begin{array}{l} A \text{ and } B \\ A \text{ and } C \\ B \text{ and } C \end{array} \right\}$ are all the variations that can be made, of two persons, out of three;

Th. they must play three games.

Corol. 1. If the number of persons were four, A , B , C , and D ; then,

$\left. \begin{array}{l} AB, \\ AC, BC, \\ AD, BD, CD, \end{array} \right\}$ are the required variations;

And the number of games will be 6:

Corol. 2. If there were five persons, A , B , C , D , E ; Then,

$\left\{ \begin{array}{l} AB, \\ AC, BC, \\ AD, BD, CD, \\ AE, BE, CE, DE, \end{array} \right\}$ will be the different players;

Corol. 3. The number of different pairs or combinations of 2 in n things will be the n^{th} term of the series 0, 1, 3, 6, 10, etc. $= \frac{n \cdot n - 1}{1 \cdot 2}$ by quest. 1.

QUESTION XLV.

Four persons A , B , C , and D , engaged to play at Ombre, as often as they could make a different sett: How many times did they play?

Now $\left\{ \begin{array}{l} AB \text{ and } C, \\ AB \text{ and } D, \\ AC \text{ and } D, \text{ } BC \text{ and } D, \end{array} \right\}$ will be the players, at all the possible different sets of 3, in 4;

Th. They must play 4 times.

Corol. 1. If the number of persons were five, A , B , C , D , and E ; Then,

$\left. \begin{array}{l} ABC, \\ ABD, \\ ABE, \\ ACD, \text{ } BC \text{ } D, \\ ACE, \text{ } BC \text{ } E, \\ ADE, \text{ } BDE, \text{ } CDE, \end{array} \right\}$ will be the different setts;

And they must play 10 times.

Corol. 2. The different combinations of 3, in n things will be the $n-1^{\text{th}}$ term of the series 0, 1, 4, 10, 20, etc.

$$= \frac{n \cdot n-1 \cdot n-2}{1 \cdot 2 \cdot 3} \text{ by quest. 38.}$$

QUES-

QUESTION XLVI.

How often can a different set at Whist, be made by five persons, *A, B, C, D, and E*?

$$\left. \begin{array}{l} ABCD, \\ ABC E, \\ ABDE, \\ ACDE, BCDE \end{array} \right\} \text{ will be the setts;}$$

Th. there will be five setts.

Corol. 1. If there were six persons *A, B, C, D E, and F*. Then,

$$\left. \begin{array}{l} ABCD, \\ ABCE, \\ ABCF, \\ ABDE, \\ ABDF, \\ ABEF, \\ ACDE, BCDE, \\ ACDF, BCDF, \\ ACEF, BCEF, \\ ADEF, BDEF, CDEF, \end{array} \right\} \text{ are the setts;}$$

And, the number of setts is 15.

Corol. 2. The number of combinations of 4 in n things will be the $n-2^{\text{th}}$ term of the series 0, 1, 5, 15,

35, etc. $= \frac{n \cdot n-1 \cdot n-2 \cdot n-3}{1 \cdot 2 \cdot 3 \cdot 4}$ by quest. 43.

QUESTION XLVII.

The number of the combinations of m , in n things is required?

$$\begin{array}{c} \text{By quest.} \left\{ \begin{array}{l} 44 \\ 45 \\ 46 \end{array} \right\} \text{The Combinations of} \left\{ \begin{array}{l} 2 \\ 3 \\ 4 \end{array} \right\} \text{ in } n \text{ things} = \left\{ \begin{array}{l} \frac{n \cdot n-1}{1 \cdot 2}; \\ \frac{n \cdot n-1 \cdot n-2}{1 \cdot 2 \cdot 3}; \\ \frac{n \cdot n-1 \cdot n-2 \cdot n-3}{1 \cdot 2 \cdot 3 \cdot 4}; \end{array} \right.$$

$$\text{Th. the combinations of } m, \text{ in } n \text{ things.} \left\{ \begin{array}{l} \frac{n \cdot n-1 \cdot n-2 \cdot n-3 \text{ etc. to } n-m+1}{1 \cdot 2 \cdot 3 \cdot 4 \text{ etc. to } m} \end{array} \right.$$

QUESTION XLVIII.

A Gentleman who had seven children, each a year older than the next younger, determined to new cloath them; and to bestow as many yards of lace on the trimming of the yongest's suit as he was years old; as many on the second, as the sum of his, and the yongest's age; as many on the third as the sum of their three ages; etc. and agreed with the taylor to pay him (in pence) the product of each child's age, by the number of yards of lace on his cloathes, for the making each suit.

Now the work being done, the taylor's bill amounted to 7*l.* 10*s.* 6*d.* Required the age of each child?

Let

QUESTION XLVIII. continued.

Let $7 = n$; ($7 l. 10 s. 6 d. = 1806 d.$) m ; and $x =$ the youngest child's age:

Then $x, x+1, x+2$, etc. to $x+n-1$ are the children's ages,

And $x, 2x+1, 3x+3$, etc. to $nx + \frac{n \cdot n-1}{2}$ } are the yards of lace on their cloaths;

Whence $\left\{ \begin{array}{l} 2xx+3x+1 \\ 3xx+9x+6 \\ \text{etc. to} \\ nxx + \frac{3n \cdot n-1}{2}x + \frac{n \cdot n-1}{2} \end{array} \right\}$ are the prices of making each suit.

Th. the tax-
for a bill will be $\left\{ \begin{array}{l} \text{terms of } 1, 2, 3, \text{ etc. to } n + ax \\ + n \text{ terms of } 0, 3, 9, \text{ etc. to } \frac{3n \cdot n-1}{2}x \\ + n \text{ terms of } 0, 1, 6, \text{ etc. to } \frac{n \cdot n-1}{2} \end{array} \right\}$

Now if, n term of $1, 3, 3, \text{ etc. to } n = \frac{n \cdot n+1}{2}$ by Corol. quest 43.

QUESTION XLVIII. continued.

2. Let $xn^3 - yn^2 + zn = n$ terms of 0, 3, 9, etc. to $\frac{3n \cdot n-1}{2}$.

Then by writing $\overline{n+1}$ for n , the $\overline{n+1}$ term will be $\frac{3n \cdot \overline{n+1}}{2}$,

And from process in quest. 33.

$$\left. \begin{array}{l} 3xn^2 + 3xn + n \\ -2y - y \\ +z \end{array} \right\} = \left(\frac{3n \cdot \overline{n+1}}{2} \right) = \frac{3n^2}{2} + \frac{3n}{2},$$

$$\text{Or } 3xn^2 + 3xn + x + z = \frac{3n^2}{2} + 2y + \frac{3}{2}xn + y;$$

$$\begin{array}{l} \text{By} \\ \text{Cor.} \end{array} \left\{ \begin{array}{l} 3x = \frac{3}{2}, \\ 3x = 2y + \frac{3}{2}, \\ x + z = y, \end{array} \right\} \text{ therefore } \left\{ \begin{array}{l} x = \frac{1}{2}, \\ 0 = y, \\ z = -\frac{1}{2}. \end{array} \right\}$$

$$\text{And } \left(\frac{1}{2}n^3 - \frac{1}{2}n \right) \frac{\overline{n+1} \cdot n \cdot n-1}{2} = n \text{ terms of } 0, 3, 9, \text{ etc. to } \frac{3n \cdot n-1}{2}.$$

QUESTION XLVIII continued.

3. Let $xn^4 - yn^3 + un^2 - zn = n$ terms of 0, 1, 6, etc. to $\frac{n \cdot n - 1}{2}$;
Then by writing $n+1$ for n , the $n+1^{\text{th}}$ term will be $\left(\frac{n+1 \times nn}{2} = \frac{n^3}{2} + \frac{nn}{2}\right)$;

And from procefs in quest. 39.

$$\left. \begin{array}{l} 4xn^3 + 6xn^2 + 4xn + x \\ - 3y - 3y - y \\ + 2u + u \\ - z \end{array} \right\} = \frac{n^3}{2} + \frac{nn}{2},$$

Or $\left\{ 4xn^3 + 6xn^2 + 4xn + x \right\} = \left\{ \frac{1}{2}n^3 + \frac{1}{2}nn + 3y + 3y + y + z \right\}$;

By Corol. 251. $\left\{ \begin{array}{l} 4x = \frac{1}{2}, \\ 6x = \frac{1}{2} + 3y, \\ 4x + 2u = 3y, \\ x + u = y + z; \end{array} \right\}$ therefore $\left\{ \begin{array}{l} x = \frac{1}{8}, \\ \frac{1}{2} = y, \\ u = -\frac{1}{8}, \\ -\frac{1}{2} = z; \end{array} \right.$

And $\left\{ \frac{1}{2}n^4 - \frac{1}{2}n^3 - \frac{1}{2}n^2 + \frac{1}{2}nn \right\} = n$ terms of 0, 1, 6, etc. $\frac{n \cdot n - 1}{2}$.
Or $\frac{1 \cdot 2 \cdot 3 \cdot 4}{1 \cdot 2 \cdot 3 \cdot 4}$

QUESTION XLVIII. continued.

$$\text{Th. } \frac{n \cdot n+1}{1 \cdot 2} x^2 + \frac{n+1 \cdot n \cdot n-1}{1 \cdot 1 \cdot 2} x + \frac{n+1 \cdot n \cdot n-1 \cdot 3n-2}{1 \cdot 2 \cdot 3 \cdot 4} = m,$$

$$\text{And } x^2 + n-1 \cdot x + \frac{n-1 \cdot 3n-2}{3 \cdot 4} = \frac{2m}{n \cdot n+1},$$

$$\text{Or } x^2 + n-1 \cdot x = \frac{n \cdot n+1}{n-1 \cdot 3n-2} \cdot \frac{2m}{3 \cdot 4};$$

$$\text{But } x^2 + n-1 \cdot x + \frac{n-1}{2} = \frac{2m}{n \cdot n+1} + \frac{n-1 \cdot n-1}{1 \cdot 4} = \frac{n-1 \cdot 3n-2}{3 \cdot 4};$$

$$(\text{Now } \frac{n-1 \cdot n-1}{1 \cdot 4} - \frac{n-1 \cdot 3n-2}{3 \cdot 4} = \frac{1}{3} - \frac{3n-2}{4} \times \frac{3}{4},$$

$$\text{And } \frac{n-1}{1} - \frac{3n-2}{3} = \left(\frac{3n-3}{3} - \frac{3n-2}{3} \right) = \frac{-1}{3};$$

QUESTION XLVIII. continued.

Whence $\frac{n-1 \cdot n-1}{1 \cdot 4} - \frac{n-1 \cdot 3n-2}{3 \cdot 4} = \left(-\frac{1}{3} \times \frac{n-1}{4} = \right) -\frac{n-1}{12}$

Th. $x^2 + n-1 \times x + \frac{n-1}{2} = \frac{2m}{n \cdot n+1} - \frac{n-1}{12}$;

And $x + \frac{n-1}{2} = \sqrt{\frac{2m}{n \cdot n+1} - \frac{n-1}{12}}$;

And $x = \sqrt{\frac{2m}{n \cdot n+1} - \frac{n-1}{12} - \frac{n-1}{2}}$.

In this Example $x = \left(\sqrt{\frac{2 \cdot 1806}{7 \cdot 8} - \frac{6}{12} - \frac{6}{2}} \right) = 5$.

QUESTION XLIX.

The sum (Z) of n terms of the series of m^{th} powers ($1^m, 2^m, 3^m, 4^m$, etc. to n^m) is required?

Assume $a n^m + 1 - b n^m + c n^{m-1} - d n^{m-2} + e n^{m-3}$ etc. to $z n^{m-3}$ $= Z$;

Then $a x n + 1^m + 1 - b x n + 1^m + c x n + 1^{m-1} - d x n + 1^{m-2} + e x n + 1^{m-3}$ etc. to $z x n + 1 = Z + n^m$;

And by proceeding as in questions 1. 33. 38. 39, and 43.)

$$\begin{array}{r}
 \frac{m+1}{1} a n^m + \frac{m+1}{1} \cdot \frac{m}{2} a n^{m-1} + \frac{m+1}{1} \cdot \frac{m}{2} \cdot \frac{m-1}{3} a n^{m-2} + \frac{m+1}{1} \cdot \frac{m}{2} \cdot \frac{m-1}{3} \cdot \frac{m-2}{4} a n^{m-3} + \text{etc. to } + a \\
 - \frac{m}{1} b - \frac{m}{1} \cdot \frac{m-1}{2} b - \frac{m}{1} \cdot \frac{m-1}{2} \cdot \frac{m-2}{3} b - \frac{m}{1} \cdot \frac{m-1}{2} \cdot \frac{m-2}{3} \cdot \frac{m-3}{4} b - \text{etc. to } - b \\
 + \frac{m-1}{1} c + \frac{m-1}{1} \cdot \frac{m-2}{2} c + \frac{m-1}{1} \cdot \frac{m-2}{2} \cdot \frac{m-3}{3} c + \frac{m-1}{1} \cdot \frac{m-2}{2} \cdot \frac{m-3}{3} \cdot \frac{m-4}{4} c + \text{etc. to } + c \\
 - \frac{m-2}{1} d - \frac{m-2}{1} \cdot \frac{m-3}{2} d - \frac{m-2}{1} \cdot \frac{m-3}{2} \cdot \frac{m-4}{3} d - \frac{m-2}{1} \cdot \frac{m-3}{2} \cdot \frac{m-4}{3} \cdot \frac{m-5}{4} d - \text{etc. to } - d
 \end{array}$$

QUESTION XLIX continued.

$$\left\{ \begin{array}{l} \frac{m+1}{1} a_n + \frac{m}{1 \cdot 2} a_n^{m-1} + \frac{m+1}{1 \cdot 2 \cdot 3} a_n^{m-2} + \frac{m+1 \cdot m \cdot m-1 \cdot m-2}{1 \cdot 2 \cdot 3 \cdot 4} a_n^{m-3} \text{ etc.} + a \end{array} \right\} =$$

$$\left\{ \begin{array}{l} \frac{m-1}{1} c + \frac{m-1}{1 \cdot 2} c^2 \text{ etc.} + c \end{array} \right\}$$

$$\left\{ \begin{array}{l} n + \frac{m}{1} b_n^{m-1} + \frac{m \cdot m-1}{1 \cdot 2} b_n^{m-2} + \frac{m \cdot m-1 \cdot m-2}{1 \cdot 2 \cdot 3} b_n^{m-3} \text{ etc.} + b \end{array} \right\} :$$

$$\left\{ \begin{array}{l} \frac{m-2}{1} d \text{ etc.} + e \end{array} \right\}$$

QUESTION XLIX. continued.

Now 1 From Cor. to quest. 251

$$\frac{m+1}{1} a = 1;$$

Th.

2 D°. (by writing $\frac{1}{m+1}$ for a)

Th.

3 D° (and writing $\frac{1}{2}$ for b)

$$\frac{m \cdot m-1}{2 \cdot 3} + \frac{m-1}{1} c = \frac{m \cdot m-1}{1 \cdot 2}$$

Or by division

$$c = \frac{m}{2 \cdot 2}$$

Th.

4 D°

$$\frac{m \cdot m-1 \cdot m-2}{2 \cdot 3 \cdot 4} + \frac{m-1 \cdot m-2}{1 \cdot 2} \times \frac{m}{3 \cdot 4} = \frac{m \cdot m-1 \cdot m-2}{1 \cdot 2 \cdot 3} \times \frac{1}{2} + \frac{m-2}{1}, d$$

$$c = \left(\frac{m}{4} - \frac{m}{6} = \frac{m}{12} \right) \frac{m}{2 \times 2 \cdot 3}$$

QUESTION XLIX continued.

$$\begin{aligned}
 &\text{Or } \frac{m \cdot m - 1}{2 \cdot 3 \cdot 4} + \frac{m \cdot m - 1}{2 \cdot 3 \cdot 4} = \frac{m \cdot m - 1}{2 \cdot 3 \cdot 4} + d_1 \\
 &\text{That is } \frac{m \cdot m - 1}{3 \cdot 4} = \frac{m \cdot m - 1}{3 \cdot 4} + d_2 \\
 &\text{Th. } \frac{m \cdot m - 1 \cdot m - 2 \cdot m - 3}{2 \cdot 3 \cdot 4 \cdot 5} + \frac{m \cdot m - 1 \cdot m - 2 \cdot m - 3}{1 \cdot 2 \cdot 3} \times \frac{m}{3 \cdot 4} = \frac{m \cdot m - 1 \cdot m - 2 \cdot m - 3}{1 \cdot 2 \cdot 3} \times \frac{1}{2} + 0 \\
 &\text{Or } \frac{m \cdot m - 1 \cdot m - 2}{2 \cdot 3 \cdot 4 \cdot 5} + \frac{m \cdot m - 1 \cdot m - 2}{2 \cdot 3 \cdot 3 \cdot 4} + e = \frac{m \cdot m - 1 \cdot m - 2}{2 \cdot 2 \cdot 3 \cdot 4} \\
 &\text{That is } e = \frac{m \cdot m - 1 \cdot m - 2}{2 \cdot 3 \cdot 4} \times \frac{1}{2} - \frac{1}{3} \\
 &\text{Th. } \frac{m \cdot m - 1 \cdot m - 2}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} = \frac{m \cdot m - 1 \cdot m - 2}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6}
 \end{aligned}$$

QUESTION XLIX continued.

And by a similar process may the rest be discovered, as follows, viz.

$$a = \frac{1}{m+1},$$

$$b = \frac{1}{2},$$

$$c = \frac{m}{2 \times 6},$$

$$d = 0,$$

$$e = \frac{m \cdot m - 1 \cdot m - 2}{2 \cdot 3 \cdot 4 \times 30},$$

$$f = 0,$$

$$g = \frac{m \cdot m - 1 \cdot m - 2 \cdot m - 3 \cdot m - 4}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \times 42};$$

etc.

etc.

Now if

$$a = \left(\frac{1}{6}\right)$$

$$\beta = \left(\frac{4}{2} - a \frac{1}{2} + \frac{1}{5}\right)$$

$$\gamma = \left(\frac{6 \cdot 5 \cdot 4}{2 \cdot 3 \cdot 4} \beta - \frac{1}{2} a + \frac{1}{2} - \frac{1}{7}\right)$$

$$\delta = \left(\frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} \gamma - \frac{8 \cdot 7 \cdot 6}{2 \cdot 3 \cdot 4} \beta + \frac{8}{2} a - \frac{1}{2} + \frac{1}{9} - \frac{1}{30}\right);$$

&c.

&c.

QUESTION XLIX continued.

$$\text{Then } Z = \frac{1}{m+1} n^m + \frac{1}{2} n^{m-1} + \frac{m}{2} \frac{n^{m-2}}{2} + \frac{m(m-1)}{2 \cdot 3} \frac{n^{m-3}}{4} + \frac{m(m-1)(m-2)}{2 \cdot 3 \cdot 4} \frac{n^{m-4}}{5} + \frac{m(m-1)(m-2)(m-3)}{2 \cdot 3 \cdot 4 \cdot 5} \frac{n^{m-5}}{6} + \text{etc.}$$

Also if A B C D etc. represent the sum of n terms of the series of $\left\{ \begin{array}{l} \text{1st. powers 0, 1, 2, 3, etc,} \\ \text{2d. powers 0, 1, 4, 9, etc,} \\ \text{3d. powers 0, 1, 8, 27, etc,} \\ \text{4th. powers 0, 1, 16, 81, etc;} \\ \text{etc.} \end{array} \right.$

$$\begin{aligned} \text{Then } A &= \frac{n^2}{2} - \frac{n}{2} \\ B &= \frac{n^3}{3} - \frac{n^2}{2} + \frac{n}{6} \\ C &= \frac{n^4}{4} - \frac{n^3}{2} + \frac{n^2}{2} - \frac{n}{3} \\ D &= \frac{n^5}{5} - \frac{n^4}{2} + \frac{n^3}{3} - \frac{n^2}{2} + \frac{n}{30} \end{aligned}$$

QUESTION XLIX continued.

$$\begin{aligned}
 D &= \frac{1}{2} - \frac{1}{2} + \frac{1}{2} = \frac{1}{2} \\
 E &= \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} = \frac{1}{2} \\
 F &= \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} = \frac{1}{2} \\
 G &= \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} = \frac{1}{2} \\
 H &= \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} = \frac{1}{2} \\
 I &= \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} = \frac{1}{2} \\
 J &= \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} = \frac{1}{2} \\
 K &= \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} = \frac{1}{2} \\
 L &= \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} = \frac{1}{2}
 \end{aligned}$$

etc. QUESTION XLIX continued.

QUESTION L.

The sum Σ of n terms of the m^{th} powers of an arithmetical progression, whose least term is a , and common difference d , is required?

Let A, B, C , etc. be as in quest. last.

$$\begin{aligned} \text{Then } a^m &= a^m \\ \frac{a^m}{a+d} &= a^m + ma^{m-1} \times d + \frac{m \cdot m-1}{1 \cdot 2} a^{m-2} \times dd + \text{etc.} \\ \frac{a^m}{a+2d} &= a^m + ma^{m-1} \times 2d + \frac{m \cdot m-1}{1 \cdot 2} a^{m-2} \times 4d^2 + \text{etc.} \\ \frac{a^m}{a+3d} &= a^m + ma^{m-1} \times 3d + \frac{m \cdot m-1}{1 \cdot 2} a^{m-2} \times 9d^2 + \text{etc.} \end{aligned}$$

$$\text{Th. } \Sigma = na^m + Ana^{m-1}d + B \frac{m \cdot m-1}{1 \cdot 2} a^{m-2} + \dots$$

QUESTIONS LI and LII.

The n^{th} term (x) and the sum of n Terms (s) of a rank or series of numbers, the difference of whose differences (or whose second differences) are equal, are required?

Put a for the least, and x for the greatest term; D for the difference of the two first terms (or for the first of the first differences; and d for the difference of the first differences (or the second difference:)

Then if it be an ascending series,

$$\begin{array}{l}
 a, \\
 a + D, \\
 a + 2D + d, \\
 a + 3D + 3d, \\
 a + 4D + 6d, \\
 a + 5D + 10d,
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{will be the terms} \\ \text{of the series; for} \end{array}
 \left\{ \begin{array}{l} D, \\ D + d, \\ D + 2d, \\ D + 3d, \\ D + 4d, \end{array} \right\} \begin{array}{l} \text{the differences of} \\ \text{those terms, dif. by} \end{array}
 \left\{ \begin{array}{l} d, \\ d, \\ d, \\ d, \end{array} \right\} \begin{array}{l} \text{the above substitut-} \\ \text{ed second difference.} \end{array}$$

Or, if it be a descending series,

$$\begin{array}{l}
 x, \\
 x - D, \\
 x - 2D + d, \\
 x - 3D + 3d, \\
 x - 4D + 6d, \\
 x - 5D + 10d,
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{will be the terms} \\ \text{of the series; for} \end{array}
 \left\{ \begin{array}{l} D, \\ D - d, \\ D - 2d, \\ D - 3d, \\ D - 4d, \end{array} \right\} \begin{array}{l} \text{the differences of} \\ \text{those ter. differ by} \end{array}
 \left\{ \begin{array}{l} d, \\ d, \\ d, \\ d, \end{array} \right\} \begin{array}{l} \text{the above substitut-} \\ \text{ed second difference.} \end{array}$$

QUESTIONS LI and LII. continued.

$$\text{Th. } x = \left\{ \frac{a + \overline{n-1} \times D}{\overline{2-n-1} \times D} \right\} + \frac{d \times \overline{n-2}^{\text{th}} \text{ term of}}{1, 3, 6, 10, \text{etc.}}$$

But the $\overline{n-2}^{\text{th}}$ term of 1, 3, 6, 10, etc. is $\frac{\overline{n-2} \cdot \overline{n-1}}{2 \cdot 2}$ by
 Corol. quest. 1.

$$\text{Th. } x = \left\{ \frac{a + \overline{n-1} \times D}{\overline{2-n-1} \times D} \right\} + \frac{\overline{n-1} \cdot \overline{n-2}}{1 \cdot 2} d.$$

$$\text{Also } s = \left\{ \frac{na + \overline{n \text{ terms of } 0, 1, 2, \&c.} \times D}{\overline{nz - n \text{ terms of } 0, 1, 2, \&c.} \times D} \right\} + \overline{n-1} \text{ terms of } 0, 1, 3, \&c. \times d.$$

But, $\overline{n \text{ terms of } 0, 1, 2, 3, \text{etc.}} = \frac{\overline{n \cdot n-1}}{1 \cdot 2}$ by quest. 1.

And $\overline{n-1 \text{ terms of } 0, 1, 3, 6, \text{etc.}} = \frac{\overline{n \cdot n-1 \cdot n-2}}{1 \cdot 2 \cdot 3}$ by q. 38.

$$\text{Th. } s = \left\{ \frac{na + \frac{\overline{n \cdot n-1}}{1 \cdot 2} D}{\overline{nz - \frac{n \cdot n-1}{1 \cdot 2} D}} \right\} + \frac{\overline{n \cdot n-1 \cdot n-2}}{1 \cdot 2 \cdot 3} d.$$

QUES.

QUESTIONS LI and LII continued.

Example. What is the sum of n terms, of the series of square numbers, 0, 1, 4, 9, 16, etc.?

Square numbers.	First differences.	Second differences.
0	1	
1		2
4	3	
9		2
16	5	
	7	2

It appears above, that in this example, $a = 0$; $D = 1$; and $d = 2$:

$$\begin{array}{l}
 \text{Th. } axn + \frac{n \cdot n - 1}{2} \cdot 1 + \frac{n \cdot n - 1 \cdot n - 2}{1 \cdot 2 \cdot 3} \cdot 2, \\
 \text{Or } \frac{n \cdot n - 1}{2} + \frac{n \cdot n - 1}{2} \times \frac{2n - 4}{3}, \\
 \text{Or } \frac{n \cdot n - 1}{2} \times 1 + \frac{2n - 4}{3}, \\
 \text{Or } \frac{n \cdot n - 1}{2} \times \frac{3 + 2n - 4}{3}, \\
 \text{Th. } \frac{n \cdot n - 1 \cdot 2n - 1}{1 \cdot 2 \cdot 3},
 \end{array}
 \left. \vphantom{\begin{array}{l} \text{Th. } axn + \frac{n \cdot n - 1}{2} \cdot 1 + \frac{n \cdot n - 1 \cdot n - 2}{1 \cdot 2 \cdot 3} \cdot 2, \\ \text{Or } \frac{n \cdot n - 1}{2} + \frac{n \cdot n - 1}{2} \times \frac{2n - 4}{3}, \\ \text{Or } \frac{n \cdot n - 1}{2} \times 1 + \frac{2n - 4}{3}, \\ \text{Or } \frac{n \cdot n - 1}{2} \times \frac{3 + 2n - 4}{3}, \end{array}} \right\} = \text{sum req.}$$

See quest. 33.

QUES-

QUESTIONS LIII and LIV.

The n^{th} term (x) and the sum of n terms (s) of a series of numbers whose third differences are equal, are required?

Putting a = first term; D = first of the first differences; Δ = first of the second differences; and d = third differences: the terms of the series with their first, second, and third differences may be expressed as below.

Terms of the series.	First differences.	second differences	third differences
a	D	Δ	d
$a + D$	$D + \Delta$	$\Delta + d$	d
$a + 2D + \Delta$	$D + 2\Delta$	$\Delta + 2d$	d
$a + 3D + 3\Delta + d$	$D + 3\Delta + 3d$	$\Delta + 3d$	d
$a + 4D + 6\Delta + 4d$	$D + 4\Delta + 6d$	$\Delta + 4d$	d
$a + 5D + 10\Delta + 10d$	$D + 5\Delta + 10d$	$\Delta + 5d$	d
$a + 6D + 15\Delta + 20d$	$D + 6\Delta + 15d$	$\Delta + 6d$	d

QUESTIONS LIII and LIV

QUESTION LIII and LIV continued.

$$\text{Th. } x = \left\{ \begin{array}{l} a + \frac{n-1}{n} \times D + \frac{\Delta \times n-2}{n} \text{ term of } 1, 3, 6, \text{ etc.} \\ + \frac{d \times n-3}{n} \text{ term of } 1, 4, 10, \text{ etc.} \end{array} \right\}$$

$$\text{And } s = \left\{ \begin{array}{l} na + \frac{n}{n} \text{ terms of } 0, 1, 2, 3, \text{ etc.} \times D \\ + \frac{n-1}{n} \text{ terms of } 0, 1, 4, 6, \text{ etc.} \times \Delta \\ + \frac{n-2}{n} \text{ terms of } 0, 1, 4, 10, \text{ etc.} \times d \end{array} \right\}$$

$$\text{Or } x = a + \frac{n-1}{n} \times D + \frac{n-1 \cdot n-2}{1 \cdot 2} \Delta + \frac{n-1 \cdot n-2 \cdot n-3}{1 \cdot 2 \cdot 3} d$$

$$\text{And } s = na + \frac{n \cdot n-1}{1 \cdot 2} D + \frac{n \cdot n-1 \cdot n-2}{1 \cdot 2 \cdot 3} \Delta + \frac{n \cdot n-1 \cdot n-2 \cdot n-3}{1 \cdot 2 \cdot 3 \cdot 4} d$$

See questions 1, 38, and 43.

QUESTION LIV. continued.

Example. What is the sum of n terms of the series of cube numbers
 $0, 1, 8, 27, 64$, etc.?

Numbers.	First Differences.	Second differences.	third differences.
0	1	6	6
1	7	12	6
8	19	18	6
27	37	24	
64	61		
125			

It appears from above, that in this example $d=0$; $D=1$; $\Delta=6$; and

QUESTION LIV continued.

= sum required.

Th. $\frac{n \cdot n-1}{2} + \frac{n \cdot n-1}{2} + \frac{n \cdot n-1 \cdot n-2}{1 \cdot 2 \cdot 3} 6 + \frac{n \cdot n-1 \cdot n-2 \cdot n-3}{1 \cdot 2 \cdot 3 \cdot 4} 6,$

Or $\frac{n \cdot n-1}{2} + \frac{n \cdot n-1}{2} + \frac{6 \cdot n \cdot n-2}{1 \cdot 2 \cdot 3} + \frac{n \cdot n-1}{1 \cdot 2 \cdot 3 \cdot 4} 6 \times n \cdot n-2 \cdot n-3$

Or $\frac{n \cdot n-1}{2} \times 1 + \frac{6 \cdot n \cdot n-2}{3 \cdot 1 \cdot 2} + \frac{6 \cdot n \cdot n-2 \cdot n-3}{3 \cdot 1 \cdot 2 \cdot 3} 4,$

Or $\frac{n \cdot n-1}{2} \times 1 + \frac{6 \cdot n \cdot n-2}{3} \times 1 + \frac{n \cdot n-1}{4} 4,$

Or $\frac{n \cdot n-1}{2} \times 1 + \frac{6 \cdot n \cdot n-2}{3} \times 1 + \frac{n \cdot n-1}{4} 4,$

Or $\frac{n \cdot n-1}{2} \times 1 + \frac{n \cdot n-1}{2} 2,$

Or $\frac{n \cdot n-1}{2} \times 2 + \frac{n \cdot n-1}{2} 2,$

Th. $\left(\frac{n \cdot n-1}{2} \times \frac{n \cdot n-1}{2} \right) \frac{n \cdot n-1}{2 \cdot 2},$

See quest. 39.

QUESTIONS LV and LVI.

The n^{th} term (x), and the sum of the terms (s), of a series of numbers whose m^{th} differences are equal, are required;

Then (by questions 2, 3, 50, 51; and 52, 53)

In a series whose

$$\left\{ \begin{array}{l} 1^{\text{st}} \\ 2^{\text{d}} \\ 3^{\text{d}} \end{array} \right\} \left\{ \begin{array}{l} a \pm \frac{n-1}{1} \cdot d \\ a \pm \frac{n-1}{2} \cdot d \\ a \pm \frac{n-1}{3} \cdot d \end{array} \right\} \left\{ \begin{array}{l} \text{diff. are} \\ \text{equal} \end{array} \right\}$$

n^{th} terms.

And

$$\left\{ \begin{array}{l} na + \frac{n \cdot n-1}{1 \cdot 2} \\ na + \frac{n \cdot n-1}{1 \cdot 2} \cdot d \\ na + \frac{n \cdot n-1}{1 \cdot 2} \cdot d^2 \end{array} \right\} \left\{ \begin{array}{l} \text{sum} \\ \text{of the series} \\ \text{is equal to} \end{array} \right\}$$

QUESTION LV. and LVI.

Therefore, putting d^i = the first of the first differences; d^{ii} = the first of the second differences; d^{iii} = the first of the third differences, etc.

$$\begin{aligned}
 z &= a \pm \frac{n-1}{1} d^i + \frac{n-1 \cdot n-2}{1 \cdot 2} d^{ii} \pm \frac{n-1 \cdot n-2 \cdot n-3}{1 \cdot 2 \cdot 3} d^{iii} \\
 &\quad + \frac{n-1 \cdot n-2 \cdot n-3 \cdot n-4}{1 \cdot 2 \cdot 3 \cdot 4} d^{iv} \text{ etc. to } m+1 \text{ terms} \\
 s &= na \pm \frac{n \cdot n-1}{1 \cdot 2} d^i + \frac{n \cdot n-1 \cdot n-2}{1 \cdot 2 \cdot 3} d^{ii} \pm \frac{n \cdot n-1 \cdot n-2 \cdot n-3}{1 \cdot 2 \cdot 3 \cdot 4} d^{iii} \\
 &\quad + \frac{n \cdot n-1 \cdot n-2 \cdot n-3 \cdot n-4}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} d^{iv} \text{ etc. to } m+1 \text{ terms}
 \end{aligned}$$

QUES-

QUESTION LVII.

What is the n^{th} power of $a+b$?

The following powers may be easily obtained by multiplication; viz.

$$a+b = a^1 + b^1$$

$$a+b^2 = a^2 + 2ab + b^2$$

$$a+b^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$a+b^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

And by
 coefficients of the
 in which the number
 it appears
 that the
 term of a
 series, whose
 differences
 are equal, is

$$\begin{matrix} a^1 + b^1 & D; \\ a^2 + 2ab + b^2 & + \Delta; \\ a^3 + 3a^2b + 3ab^2 + b^3 & + 3\Delta + \delta; \\ a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4 & + 6\Delta + 4\delta + d; \\ \text{etc.} & \end{matrix}$$

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QUESTION LVII. continued.

COROL. 1.

If A, B, C , etc. represent the first, second, third, etc. term of the above expression;
Then;

$$\frac{a+b^n}{a+b} = a^n + \frac{n}{1} \times \frac{b}{a} A + \frac{n+1}{2} \times \frac{b^2}{a^2} B + \frac{n+2}{3} \times \frac{b^3}{a^3} C + \frac{n+3}{4} \times \frac{b^4}{a^4} D \text{ etc.}$$

COROL. 1.

COROL. 2.

If the Power of b be required: Then,

$$\frac{a-b^n}{a-b} = a^n - \frac{n}{1} \times \frac{b}{a} A + \frac{n+1}{2} \times \frac{b^2}{a^2} B - \frac{n+2}{3} \times \frac{b^3}{a^3} C + \frac{n+3}{4} \times \frac{b^4}{a^4} D \text{ etc.}$$

QUESTION LVIII.

In a series of numbers whose first differences are equal, two terms are given; to find the third?

Let a , b , and c , represent the three terms, of an ascending series:

Then $b - a = c - b$;

Or $c - 2b + a = 0$.

- Th. Case {
1. If a , and b ; be given, $2b - a = c$;
 2. If b , and c ; be given, $2b - c = a$;
 3. If a , and c ; be given, $\frac{a+c}{2} = b$.

QUESTION LIX.

In a series of numbers, whose second differences are equal, three terms are given; to find the fourth?

Let a , b , c , and d , represent four terms of an ascending series;

	Terms	First difference.	Second difference.
Then {	d	$d - c$	$d - 2c + b$
	c	$c - b$	
	b	$b - a$	$c - 2b + a$
	a		

Th. $d - 2c + b = c - 2b + a$

- Th. Case {
1. If a , b , and c ; be given, $\frac{c - b}{3} \times 3 + a = d$;
 2. If a , b , and d ; be given, $\frac{d - a + 3b}{3} = c$;
 3. If a , c , and d ; be given, $\frac{3c - d - a}{3} = b$;
 4. If b , c , and d ; be given, $d - c - b \times 3 = a$.

QUES.

QUESTION LX.

In a series of numbers, whose third differences are equal; four terms are given to find the fifth?

If a, b, c, d , and e , represent five numbers of an ascending series;

Terms	First difference.	Second difference.	third difference.
e	$e-d$	$e-2d+c$	$e-3d+3c-b$
d	$d-c$	$d-2c+b$	$d-3c+3b-a$
c	$c-b$	$c-2b+a$	
b	$c-a$		
a			

Th. $e-3d+3c-b=d-3c+3b-a$

Or $e-4d+6c-4b+a=0$.

- Th. Case
1. If a, b, c , and d ; be given, $e = \frac{4d-6c+4b-a}{1}$
 2. If a, b, c , and e ; be given, $d = \frac{e+6c-4b+a}{4}$
 3. If a, b, d , and e ; be given, $c = \frac{d+b \times 4 - e - a}{6}$
 4. If a, c, d , and e ; be given, $b = \frac{e+6c-4d+a}{4}$
 5. If b, c, d , and e ; be given, $a = \frac{4d-6c-4b-e}{1}$

Corol. If a, b, c, d, e, f , etc. represent a series whose m^{th} differences are equal; then if $m+1$ terms be given, another term may be found, by the following equation; viz.

$$a - \frac{m+1}{1} \cdot b + \frac{m+1 \cdot m}{1 \cdot 2} c - \frac{m+1 \cdot m \cdot m-1}{1 \cdot 2 \cdot 3} d \text{ etc. to } m+2 \text{ terms} = 0.$$

See process in quest. 56.

QUES.

QUESTION LXI.

All the divisors of the number six, are required?

The number six, is composed of the prime numbers two, and three;

Or generally, ab is composed of a and b ;

But ab may be divided by 1, a , b , and ab ;

Therefore (ab or) 6 has 4 divisors, 1, 2, 3, and 6.

QUESTION LXII.

All the divisors of the number 30 (or abc) which is composed of the 3 prime numbers 2, 3, and 5; are required?

abc may be divided by $\begin{cases} 1, \\ a, b, c, \\ ab, ac, bc, \\ abc. \end{cases}$

The 30, has 8 divisors, viz. $\begin{cases} 1, \\ 2, 3, 5, \\ 6, 10, 15, \\ 30. \end{cases}$

QUES-

QUESTION LXIII.

Tis required to find all the divisors of 210 (or $abcd$) which is composed of the four prime numbers 2, 3, 5, and 7.

$abcd$, may be divided by

$$\left\{ \begin{array}{l} 1; \\ a, b, c, d; \\ ab, ac, ad, bc, bd, cd; \\ abc, abd, acd, bcd; \\ abcd. \end{array} \right.$$

Th. 210 has 16 divisors, viz.

$$\left\{ \begin{array}{l} 1; \\ 2, 3, 5, 7; \\ 6, 10, 14, 15, 21, 35; \\ 30, 42, 70, 105; \\ 210. \end{array} \right.$$

Corol. Hence a number, or quantity that is composed of n prime numbers, or letters, has 2^n divisors.

Schol. The three last questions relate to elections;

For out of two things proposed, a , and b ; neither, either; or both, may be chosen;

Out of three things proposed, a , b , and c ; neither any one; any two; or all three, may be chosen: etc.

Therefore the number of elections in n things, is 2^n .

QUES.

QUESTION LXIV.

All the divisors of $729 = 3^6$ are required?

aa has 3 divisors 1. a , aa ;

aaa has 4 divisors 1. a , aa , a^3 ;

$aaaa$ has 5 divisors 1. a , aa , a^3 , a^4 ;

etc.

etc.

Th. a^n has $n+1$ divisors 1. a , a^2 , a^3 , etc. to a^n ,

And $729 = 3^6$, has 7 divisors, viz. 1, 3, 9, 27, 81, 243, 729.

QUESTION LXV.

It is required to find the number of the divisors of the quantity $a^n b$?

By last a^n has $n+1$ divisors 1, a , a^2 , a^3 etc;

Beside which $a^n b$ has b , ab , a^2b , a^3b etc.

Th. $a^n b$, has $n+1 \times 2$ divisors.

QUESTION LXVI

How many divisors has the quantity $a^n bc$?

The divisors of $a^n bc$ are

$$\left\{ \begin{array}{l} 1, a, a^2, a^3, \text{ etc.} \\ b, ba, ba^2, ba^3, \text{ etc.} \\ c, ca, ca^2, ca^3, \text{ etc.} \\ bc, bca, bca^2, bca^3, \text{ etc.} \end{array} \right.$$

Th. $a^n bc$, has $\overline{n+1} \times 4$ divisors.

COROL.

Since $a^n b$, has $\overline{n+1} \times 2$,

And $a^n bc$, has $\overline{n+1} \times 4$,

Th. $a^n bcd$, has $\overline{n+1} \times 8$,

$a^n bcde$, has $\overline{n+1} \times 16$,

divisors;

And putting the number of single letters b, c, d, e , and $c = m$.

Then $a^n bcde$ etc. has $\overline{n+1} \times 2^m$, divisors;

And $a^n b^r c^s d^t f^u$ etc. has $\overline{n+1} \times \overline{r+1} \times \overline{s+1} \times \overline{t+1} \times \overline{u+1}$, divisors.

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QUESTION LXVII

How many ways may the order of n things be varied?

If the given things, 2 in number, be represented by a , and b ;

Then they may be placed in two different orders; viz. $\left. \begin{array}{l} ab, ba; \end{array} \right\}$

If the number of things be three, viz. a, b , and c ;

Then they may be placed in six different orders; viz. $\left. \begin{array}{l} abc, acb; \\ bac, bca; \\ cab, cba; \end{array} \right\}$

If the number of things be four, viz. a, b, c , and d ;

Then they may be placed in twenty-four different orders; viz. $\left\{ \begin{array}{l} abcd, abdc, acbd, acdb, adcb, adbc; \\ bacd, badc, bcad, bcda, bdac, bdca; \\ cabd, cadb, cbad, cbda, cdab, cdba; \\ dabc, dacb, dbac, dbca, dcab, dcba; \end{array} \right.$

But $2 = 1 \times 2$; $6 = 1 \times 2 \times 3$; and $24 = 1 \times 2 \times 3 \times 4$;

Th. The permutation of n things $= 1 \times 2 \times 3 \times 4$ etc. to n

In

In computing the interest of money.

If $\left\{ \begin{matrix} p \\ r \\ n \\ m \end{matrix} \right\}$ represent $\left\{ \begin{matrix} \text{the principal,} \\ \text{the interest of 1 £. for 1 time,} \\ \text{the number of times,} \\ \text{the amount of principal and int.} \end{matrix} \right\}$
 Then

QUESTION LXVIII.

Given p, r and n ; to find m ?
 Now $pr = \text{the interest of } p \text{ for 1 time,}$
 $2pr = D^o = \frac{2}{3};$
 $3pr = D^o = \frac{3}{3};$
 etc. etc.
 Th. $npr = \text{the interest of } p \text{ for } n \text{ times;}$
 And $p + npr = m.$

QUESTION LXIX.

Given p, r , and m ; to find n ?
 (Since $p + npr = m$; $npr = m - p$;
 Th. $\frac{1}{r} \left(\frac{m - p}{p} \right) = n = \frac{m - p}{pr}$

QUESTION LXX.

Given p, n , and m , to find r ?
 (Since $npr = m - p$; Th. $r = \frac{m - p}{np}$

QUES.

QUESTION LXXI.

Given m , r , and x ; to find y .

(Since $p + nr = m$) Th. $p = \frac{m}{1+nr}$.

QUESTIONS LXXII.

An usurer lent 186 $\text{\textit{l}}$. for x months and gained thereby 31 pounds; and at the same rate of interest lending 360 $\text{\textit{l}}$. for y months, he gained 90 pounds; the values of x and y are required, when $x+y=20$?

If $r =$ the interest of 1% for 1 year;

Then $186r =$ the interest of 186 for 1 year;

And $360r = D^o = 360;$

Also $\left\{ \begin{array}{l} \frac{186x}{12} = D^o \quad 186 \text{ for } x \\ \frac{360y}{12} = D^o \quad 360 \text{ for } y \end{array} \right\} \text{Months.}$

Th. $\frac{186x}{12} = 31$ Or $\frac{x}{12} = \left(\frac{31}{186} \right) = \frac{1}{6x};$

Ad $\frac{360y}{12} = 90;$ Or $\frac{y}{12} = \left(\frac{90}{360} \right) = \frac{1}{4y};$

Th. $\frac{1}{6x} \pm \frac{1}{4y} = \frac{1}{20}$ Or $\frac{1}{4y} = \frac{1}{20} - \frac{1}{6x};$

But $x+y=20;$ Th. $x=20-y;$

Whence $4y = (6 \times 20 - y) \frac{1}{20} - 6y;$

Or $10y = 120;$ Th. $y=12.$

QUESTION LXXIV.

QUESTION LXXV.

The duties of certain goods amounted to 2460, out of which a discount of $2\frac{5}{40}$ per cent. was allowed in consideration of prompt payment on the sum actually paid: What did the discount amount to?

Let $2460 = m$; $\left(\frac{2,5}{100} = \frac{1}{40}\right)$ and n be the discount required.

(Where m , r , and n are of the same kind as in the 4 last questions; that is $m = d = p$.)

Then by quest. 71. $m - d = \frac{m}{1 + nr}$

Th $\left(m - \frac{m}{1 + nr} = \frac{mnr}{1 + nr} = d\right)$

In this example- $d = \left(\frac{2460 \times \frac{1}{40}}{1 + \frac{1}{40}} = \frac{2460}{41} = 60\right)$

$d = \frac{mnr}{1 + nr}$

$\frac{d}{m} = \frac{nr}{1 + nr}$

$\frac{d}{m} + \frac{d}{m} = \frac{nr}{1 + nr} + \frac{nr}{1 + nr}$

$\frac{d}{m} + \frac{d}{m} = \frac{2nr}{1 + nr}$ QUES.

QUESTION LXXIV

A person has now due to him 320 £. and at the end of 5 years 96 £. more, will be due from the same debtor; they agree that the whole shall be discharged at one payment, at that time when the simple interest of the 320 £. shall be equal to the discount of the 96 £. both being calculated at 5 £. per. cent. per annum: the time of payment is required?

Let $320 = p$; $5 = t$; $96 = m$; $\left(\frac{5}{100} = \right) \frac{1}{20} = r$;

And $x =$ time required;

Then by qu. 68. $x \times pr =$ interest of p for x ;

And by quest. 73. $\frac{mx}{1+xr} =$ discount of m for x ;

Th. $(x \times pr =) xpr - \frac{mx}{1+xr} = 0$

Or $xpr - \frac{mx}{1+xr} = 0$

Or $xpr + \frac{mx}{1+xr} = 0$

Or $prx^2 + mx = 0$

Th. $x^2 + \frac{m+pr}{pr}x = \left(\frac{tp}{pr} = \right) \frac{t}{r}$

Substitute $\frac{m+pr}{pr} = 2b$

Then $xx + 2bx = \frac{t}{r}$

But $xx + 2bx + bb = \frac{t}{r} + bb$

Th. $x + b = \sqrt{\frac{t}{r} + bb}$; and $x = \sqrt{\frac{t}{r} + bb} - b$.

QUESTION LXXIV. continued.

In this example $\left(\frac{96 + 320 - 5.320 \times \frac{1}{2}}{320 \times \frac{1}{2}} \right) 21 = 2b.$

And $x = \sqrt{5 \times 20 + \frac{21 \times 21}{4} - \frac{21}{2}} = 4;$

Th. $(5 - 4) = 1$ year = time required.

QUESTION LXXV.

The discounting of a bill came to 5*l.* 12*s.* 0*d.* Had the rate per cent. been 1*l.* more, it would have cost 6*l.* 6*s.* 0*d.* and if the rate per cent. had been 1*l.* less, only 4*l.* 16*s.* 0*d.*

The value of the bill, time when due, and rate of interest are required?

Let p = principal, n = time, and r = interest of 100*l.* in 1 year, a = 5*l.* 12*s.* 0*d.* b = 6*l.* 6*s.* 0*d.* and c = 4*l.* 16*s.* 0*d.*

Then by quest. 73. $\left\{ \begin{array}{l} \frac{rtp}{100 + rt} = a \\ \frac{r + 1 \times tp}{100 + r + 1 \times t} = b \\ \frac{r - 1 \times tp}{100 + r - 1 \times t} = c \end{array} \right\}$ therefore $\left\{ \begin{array}{l} p = \frac{100a + art}{rt} \\ p = \frac{100b + bt \times r + 1}{r + 1 \times t} \\ p = \frac{100c + ct \times r - 1}{r - 1 \times t} \end{array} \right.$

Th.

QUESTION LXXV. continued.

$$\text{Th. } \frac{100a + art}{r} = \frac{100b + btr + 1}{r + 1},$$

$$\text{Or } r + 1 \times 100a + r + 1 \times art = 100br + btr + 1;$$

$$\text{Th. } \frac{r + 1 \times 100a - 100br}{b - ar + r + 1} = t;$$

$$\text{Also } \frac{100a + art}{r - 1} = \frac{100c + ctr + 1}{r - 1},$$

$$\text{Or } r - 1 \times 100a + r - 1 \times art = 100cr + ctr + 1;$$

$$\text{Th. } t = \frac{100cr - r - 1 \times 100a}{a - cr + r - 1};$$

$$\text{Th. } \frac{r + 1 \times 100a - 100br}{b - ar + r + 1} = \frac{100cr - r - 1 \times 100a}{a - cr + r - 1};$$

$$\text{By reduction } b + car - 2br = \frac{b - cr + r - 1}{b - ar + r + 1};$$

$$\text{Th. } r = \frac{b - cr + r - 1}{b + car - 2br};$$

$$\text{In this example } = \left(\frac{6,3 - 4,8 \times 5,6}{6,3 + 4,8 \times 5,6 - 2 \times 6,3 \times 4,8} \right) 5.$$

QUESTION LXXVI.

The discounting of two notes whose value taken together was 308*l.* 6*s.* 8*d.* cost 8*l.* 6*s.* 8*d.*; one of them was due at the end of 6, the other at the end of 8 months; and the interest of the two sums of money (each for its respective time) is 4*s.* 8*d.* $\frac{2}{3}$ more than the discount: the value of each note is required?

Let x = value of the note due at 6 months end;

And r = the interest of 1*l.* for 1 year;

$$\text{Now } 308\text{i}l. 6\text{i}s. 8\text{i}d. = \frac{925}{3} \text{ and } 8\text{i}l. 6\text{i}s. 8\text{i}d. = \frac{25}{3} \text{ and}$$

$$4\text{i}s. 8\text{i}d. \frac{2}{3} = \frac{17}{72}$$

Then

QUESTION LXXVI. continued.

Then $\frac{925}{3} - x =$ value of the note due at 8 months end.

Now by quest. 68. $\left\{ \begin{array}{l} \frac{rx}{2} = \text{int. of the 1st. for 6 months.} \\ \frac{2}{3} \times \frac{925r}{3} - \frac{2rx}{3} = \text{int. of the 2d. for 8 months.} \end{array} \right.$

Their sum $\frac{1850r}{9} - \frac{2rx}{6} =$ sum of interest's:

By quest. 73. $\left\{ \begin{array}{l} \frac{\frac{1}{2}rx}{1+\frac{1}{2}r} \text{ or } \frac{rx}{2+r} = \text{dis. of 1st.} \\ \left(\frac{2}{3} \times \frac{925r}{3} - \frac{2rx}{3} \right) \text{ or } \frac{1850r}{3} - \frac{2rx}{3} = \text{dis. of 2d.} \end{array} \right.$

Their sum $\frac{3700r - 3rx + 1850r^2}{3 \times 6 + r + 2rr} =$ sum of dis.

But by quest. $\frac{3700r - 3rx + 1850r^2}{3 \times 6 + r + 2rr} = \frac{25}{3}$

Th. $\frac{3525r + 1800r^2 - 150}{3r} = x$

Also by quest. $\frac{1850r}{9} - \frac{rx}{6} = \frac{25}{3} + \frac{17}{72}$

Th. $\frac{14800r - 617}{12r} = x$

Now $\frac{3525r + 1800r^2 - 150}{3r} = \frac{14800r - 617}{12r}$

Th. $\frac{7}{72}r = \frac{17}{7200}$

And $\left(\frac{7}{144} + \frac{1}{720} = \frac{36}{720} \right) \frac{1}{20} = r$

Now $x = \left(\frac{14800 \times \frac{1}{20} - 617}{12 \times \frac{1}{20}} = \frac{14800 - 12340}{12} = \frac{2460}{12} = 205 \right)$

QUES.

QUESTION LXXVII.

To compute the amount of an annuity in arrear by simple interest?

Let $\left\{ \begin{array}{l} a \\ r \\ n \end{array} \right\}$ represent $\left\{ \begin{array}{l} \text{the annuity;} \\ \text{the Interest of } 1\% \text{ for } 1 \text{ time;} \\ \text{the number of times;} \\ \text{the sum due at the end of } n \text{ times;} \end{array} \right.$

Then $\left\{ \begin{array}{l} a \\ a + ar \\ a + 2ar \\ a + 3ar \\ \text{etc.} \\ a + n - 1 \times ar \end{array} \right\}$ becomes due $\left\{ \begin{array}{l} \text{first} \\ \text{second} \\ \text{third} \\ \text{fourth} \\ \text{etc.} \\ n^{\text{th}} \end{array} \right\}$ time;

Th. their Sum $na + \frac{n \cdot n - 1}{1 \cdot 2} ar = m$. See quest. 3.

QUESTION LXXVIII.

Given a , n , and m : to find r ?

From above, $\frac{n \cdot n - 1}{1 \cdot 2} ar = m - na$

Th. $r = \frac{m - na \times 2}{n \cdot n - 1 \cdot a}$

QUESTION LXXIX.

Given m , n , and r : to find a ?

From above $2na + \frac{n \cdot n - 1}{1 \cdot 2} ar = 2m$

Th. $a = \frac{2m}{2n + \frac{n \cdot n - 1}{2} r}$

Or $a = \frac{2m}{2 + n - 1 \cdot r \times n}$

QUEST

QUESTION LXXX.

Given a , r , and m ; to find n ?

From above $2na + n \cdot n - 1 \cdot ar = 2m$,

That is $2na + n^2 ar - nar = 2m$,

Or $arnn + \frac{2-r}{2} \times an = 2m$,

Or $nn + \frac{2-r}{r} n = \frac{2m}{ar}$;

But $nn + \frac{2-r}{r} n + \left(\frac{2-r}{2r}\right)^2 = \frac{2m}{ar} + \frac{2-r}{2r}$;

Th. $n + \frac{2-r}{2r} = \sqrt{\frac{2m}{ar} + \frac{2-r}{2r}}$;

And $\sqrt{\frac{2m}{ar} + \frac{2-r}{2r}} - \frac{2-r}{2r} = n$.

QUESTION LXXXI.

The amount of an annuity in arrear computed at simple interest was 215 £ ; Now if the annuity had continued unpaid one year longer, it would have amounted, to 275 £ ; but if one year less, to no more than 157 £ . 10 s . the annuity, time, and rate are required?

If a = annuity; n = time; and r = interest of 1 $\%$.

Also 215 = b ; 275 = c ; and 157, 5 = d ;

Then $\left\{ \begin{array}{l} d = n-1 \times a + \frac{n-1 \cdot n-2}{1 \cdot 2} ar. \\ b = na + \frac{n \cdot n-1}{1 \cdot 2} ar, \\ c = n+1 \times a + \frac{n+1 \cdot n}{1 \cdot 2} ar \end{array} \right\} \text{by qu. 77.}$

QUESTION LXXXI continued.

$$\text{Or } \left\{ \begin{array}{l} \frac{b-na}{n \cdot n-1} \times 2 = ar, \\ \frac{c-n+1 \times a \times 2}{n+1 \cdot n} = ar, \\ \frac{d-n-1 \times a \times 2}{n-1 \cdot n-2} = ar; \end{array} \right.$$

$$\text{Th. } \frac{b-na \times 2}{n \cdot n-1} = \frac{c-n+1 \times a \times 2}{n+1 \cdot n},$$

$$\text{Th. } \frac{n+1 \cdot b-n-1 \cdot c}{n+1} = a;$$

$$\text{Again } \frac{b-na \times 2}{n \cdot n-1} = \frac{d-n-1 \times a \times 2}{n-1 \cdot n-2},$$

$$\text{Th: } a = \frac{nd-n-2 \cdot b}{n};$$

$$\text{Now } \frac{n+1 \cdot b-n-1 \cdot c}{n+1} = \frac{nd-n-2 \cdot b}{n},$$

$$\text{Th. } n - \frac{c-d}{d+c-2b} \cdot n = -\frac{2b}{d+c-2b};$$

$$\text{Th. } \frac{c-d}{2d+2c-4b} - n = \frac{\sqrt{c-d^2-4b \times 2d+2c-4b}}{2d+2c-4b};$$

$$\text{And } \frac{c-d - \sqrt{c-d^2-4b \times 2d+2c-4b}}{2d+2c-4b} = n = 4.$$

In a geometrical progression (that is a series of numbers whose ratio's are equal)

Let $\left\{ \begin{array}{l} a \\ z \\ n \\ r \\ s \end{array} \right\}$ the least term,
the greatest term,
the number of terms,
the ratio, or the quotient of any two adjoining terms,
the sum of all the terms.

QUESTIONS LXXXII and LXXXIII.

When a , r , and n , are given; to find z and s .

Then a ,
 ar ,
 ar^2 ,
 ar^3 ,
etc.
 ar^{n-1}

$\left\{ \begin{array}{l} \text{will repre-} \\ \text{sent the} \end{array} \right\}$

$\left\{ \begin{array}{l} \text{first,} \\ \text{second,} \\ \text{third,} \\ \text{fourth,} \\ \text{etc.} \\ n^{\text{th}} \end{array} \right\}$ term;

Th. $ar^{n-1} = z$.

Or if A , R , and Z be the logarithms of a , r , and z ;

Then $A + \frac{n-1}{1} \times R = Z$.

Since, $(1 : r :: a + ar + ar^2 \text{ etc.} : ar + ar^2 + ar^3 \text{ etc.})$
That is) $1 : r :: s - z : s - a$;

Th. $s - a = sr - zr$;

But $z = ar^{n-1}$ by last

Th. $s - a = sr - ar^n$,

And $\frac{ar^n - a}{r - 1} = s$.

In Logarithms (Let $nR = B$, then) $A + \frac{B-1}{1} = L \frac{r-1}{r-1}$
 $= S$. where $L \frac{b-1}{b-1}$ signifies the Logarithm of $\frac{b-1}{b-1}$.

QUESTIONS LXXXIV. and LXXXV.

In a geometrical progression are given a , r , and z ;
to find n and s ?

By quest 82; $A + \frac{z-A}{r-1} \times R = Z$;

Th. $n = \frac{Z-A}{R} + 1$.

From Equat. 3
in quest. 83. 3

Or $zr - z + z - a = sr - s$;

Or $\frac{zr - z + z - a}{r-1} = \frac{sr - s}{r-1}$;

Th. $z + \frac{z-a}{r-1} = s$.

QUESTIONS LXXXVI, and LXXXVII.

In a geometrical progression are given a , r , and s ;
to find z , and n ?

By Equ. 2; in quest. 85; $zr - a = sr - s$;

Th. $z = \frac{sr - s + a}{r}$;

Or $z = s - \frac{s-a}{r}$.

By quest 82,

By quest. 87

Th.

Or

Th.

$$z = ar^{n-1};$$

$$z = \frac{sr - s + a}{r};$$

$$ar^{n-1} = \frac{sr - s + a}{r};$$

$$ar^n = sr - s + a;$$

$$r^n = \frac{r-1 \times s}{a} + 1;$$

In

QUESTION LXXXVII continued.

In Logarithms; Let $\frac{r-1 \times s}{a} + 1 = b;$

Then $n = \frac{B}{R}.$

QUESTION LXXXVIII and LXXXIX.

In a geometrical progression are given, a , z , and s ; to find r and n ?

From equation $\left\{ \begin{array}{l} s-a = sr-zr; \end{array} \right\}$ first in quest. 82.

Th. $\frac{s-a}{s-z} = r.$

By last; $L. s-a - L. s-z = R;$

By quest. 84; $n = \frac{Z-A}{R} + 1;$

Th. $n = \frac{Z-A}{L. s-a - L. s-z} + 1.$

QUESTIONS XC and XCI.

In a geometrical progression are given, a , n , and z ; to find r and s ?

By quest. 82; $A + n-1 \times R = Z;$

Th. $R = \frac{Z-A}{n-1};$

By last; $R = \frac{Z-A}{n-1};$

By quest. 85; $s = z + \frac{z-a}{r-1}.$

Or

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QUESTION XCII and XCIII.

In a geometrical progression are given a , s , and r ; to find r , and x ?

By equation }
4th quest. 87. } $ars = sr - s + a$;

Th. $ars - sr + s - a = 0$.

But $\frac{ars - sr + s - a}{r - 1} = \frac{ars - 1 + ars - 2}{r - 1}$ etc. to $ar - s - a$;

Th. $ars - 1 + ars - 2 + ars - 3$ etc. to $ar - s - a = 0$.

If $r = -1$; and s be an $\begin{cases} \text{even} \\ \text{odd} \end{cases}$ No. $\begin{cases} s \\ s-a \end{cases} = 0$,

If $r = 0$; Then $s - a = 0$,

If $r = 1$; Then $s - na = 0$;

Th. (r being either a whole number or rational fraction) when s is an $\begin{cases} \text{even} \\ \text{odd} \end{cases}$ number, if among the re-

spective divisors of $\begin{cases} s \\ s-a \end{cases}$, $s-a$, and $s-na$; there

be found three, that differ by d (some divisor of a):

(That is n , being $\begin{cases} \text{even} \\ \text{odd} \end{cases}$ if $m+d$, m and $m-d$

where $m =$ any whole number, are severally divisors of

$\begin{cases} s \\ s-a \end{cases}$, $s-a$, and $s-na$) Then $\frac{m}{d} = r$.

From above r (if rational) $= \frac{m}{d}$;

By quest. 86. $x = s - \frac{s-a}{r}$

^a This is an application of Sir Isaac Newton's method of finding divisors (before cited in part 1st.) to the general solution of this question.

Q U E S -

QUESTION XCII. continued.

Example; Required the ratio of that geometrical progression, whose least term is 5; number of terms 10; and sum 5115?

$$\left. \begin{array}{l} s = 5115, \\ s - a = 5110, \\ s - na = 5065, \end{array} \right\} \text{Its divisors } \left\{ \begin{array}{l} 1, 3, 5, 11, 15, 31, \\ 1, 2, 5, 7, 10, 35; \\ 1, 5, 1013; \end{array} \right.$$

Where $\left\{ \begin{array}{l} 3, 2, 1 \\ 15, 10, 5 \end{array} \right\}$ differ by $\left\{ \begin{array}{l} 1 \\ 5 \end{array} \right\}$ a divisor of 5;

And $\frac{2}{1} = \frac{10}{5} = 2 = r.$

QUESTIONS XCIV and XCV.

In a geometrical progression are given, z , r , and n ; to find a , and s ?

By quest. 82; $A + n - 1 \times R = Z$;

Th. $A = Z - n - 1 \times R.$

By equation 1st. } $s - a = sr - zr$;

And (by quest. 82) $ar^{n-1} = z$,

Or $a = \frac{z}{r^{n-1}}$;

Th. $s - \frac{z}{r^{n-1}} = sr - zr$,

Th. $\left(\frac{zr^n - z}{r^n - r^{n-1}} = \right) \frac{r^{n-1} \times z}{r - 1 \times r^{n-1}} = s$;

In Logarithms (if $nR = B$)

Then $L. b - 1 + Z - L. r - 1 - n - 1 \times R = S$;

QUESTIONS XCVI and XCVII.

In a geometrical progression are given, x , r , and s , to find a , and n ?

By equation } $s - a = sr - xr$;
in quest. 83. }

Th. $s - \frac{r-1}{s-1} \times r = a$.

By equ. 5. qu. 87; $r^n = \frac{r-1 \times s}{a} + 1$;

Th. (by above) $r^n = \left(\frac{r-1 \times s}{s - \frac{r-1}{s-1} \times r} + 1 \right) = \frac{xr}{s - \frac{r-1}{s-1} \times r}$;

In Logarithms; let $b = \frac{xr}{s - \frac{r-1}{s-1} \times r}$;

Then $n = \frac{B}{R}$.

QUESTIONS XCVIII and XCIX.

In a geometrical progression are given n , r , and s , to find a , and z ?

By equation 4 } $arn = sr - s + a$,
Question 87; }

Th. $a = \frac{s \times r - 1}{r^n - 1}$;

In logarithms (if $nR = B$) $A = S + L \cdot \overline{r-1} - L \cdot \overline{b-1}$.

By quest. 95 $s = \frac{r^n - 1 \times z}{r - 1 \times r^{n-1}}$;

Th. $\frac{r^n - 1 \times r - 1 \times s}{r^n - 1} = z$;

In logarithms; if $nR = B$;

Then $Z = \overline{n-1 \times R} + \overline{L \cdot r-1} + \overline{S-L \cdot b-1}$.

QUES.

QUESTION C and CI.

In a geometrical progression are given x , z , and s ; to find r and a ?

By quest. 95. $zr^n - z = sr^n - sr^{n-1}$;

Th. $0 = s - z \times r^n - sr^{n-1} + z$;

But $\frac{s - z \times r^n - sr^{n-1} + z}{r - 1} = \frac{s - z \times r^{n-1} - zr^{n-2} - zr^{n-3} \text{ etc. to } z}{1}$;

Th. $s - z \times r^{n-1} - zr^{n-2} - zr^{n-3} - zr^{n-4} \text{ etc. to } - zr - z = 0$.

If $r = -1$; and n an $\begin{cases} \text{even} \\ \text{odd} \end{cases}$ No. $\begin{cases} s - z \\ z \end{cases} = 0$;

If $r = 0$; then $z = 0$;

If $r = 1$; then $nz - s = 0$;

Th. (r being either an integer or rational fraction)

when n is an $\begin{cases} \text{even} \\ \text{odd} \end{cases}$ number, if among the respective

divisors of $\begin{cases} s - z \\ s - z \end{cases}$, z , and $nz - s$; there be found

three that differ by d (some divisor of $s - z$): (that is n being $\begin{cases} \text{even} \\ \text{odd} \end{cases}$ if $m \pm d, m, m - d$, where $m =$ any

whole number, are severally divisors of $\begin{cases} s - z \\ s - z \end{cases}$

z , and $nz - s$); then $\frac{m}{d} = r$.

By last ; r , (if rational) $= \frac{m}{d}$

By quest. 96; $a = s - s - z \times r$.

Example; Required the ratio of that geometrical progression, whose greatest term is 2560; number of terms 10; and sum 5115?

$s = 5115$,
 $z = 2560$,
 $nz - s = 20485$,
 its divisors $\begin{cases} 1, 3, 5, 11, 15, 31, \text{ etc.} \\ 1, 2, 4, 5, 8, 10, 16, \text{ etc.} \\ 1, 5, 17, 85, 241, \text{ etc.} \end{cases}$

And $\frac{2}{5} = \frac{1}{5} = 2 = r$.

QUESTION CII.

In a geometrical progression are given, the ratio, number of terms, and product of the first and last term, to find the terms?

Here r , n , and ax , are given; to find the rest;

By quest. 82. $x = \frac{ar^n - 1}{r - 1}$;

Th. $ax = aar^n - 1$; Hence $\sqrt[n]{\frac{ax}{a-1}} = a$.

QUESTION CIII.

Three numbers in geometrical progression are required, so that the difference of the first and second may be 6; and of the second and third 15?

If x , y , and z , be the numbers required;

Then $y - x = 6$; Or $y - 6 = x$;

And $z - y = 15$; $z = 15 + y$;

But $x : y :: y : z$

That is $y - 6 : y :: y : 15 + y$

Th. $yy + 9y - 90 = yy$, Th. $y = 10$.

QUESTION CIV.

'Tis required to find three numbers in geometrical progression, the sum of the first and second of which may be 14; and of the second and third 35?

If x , y , and z , be the numbers required;

Then $x + y = 14$; Or $x = 14 - y$;

And $y + z = 35$; Or $z = 35 - y$;

But $x : y :: y : z$

That is $14 - y : y :: y : 35 - y$;

Th. $490 - 49y + yy = yy$, Th. $10 = y$.

QUES.

QUESTION CV.

There are three numbers in geometrical progression, whose product is 512; and the sum of the first and last 34: What are the numbers?

If x , y , and z , be the numbers required;

Then $x : y :: y : z$; Th. $xz = yy$;

And $(xyz = xz \times y =) y^3 = 512$;

Th. $y = 8$;

And $(yy =) xz = 64$

But by quest. $z + x = 34$;

Th. by $\begin{cases} z = \frac{34 + \sqrt{34 \times 34 - 4 \times 64 \times \frac{1}{2}}}{2} = 32 \\ \text{qu. 160. } x = \frac{34 - \sqrt{34 \times 34 - 4 \times 64 \times \frac{1}{2}}}{2} = 2. \end{cases}$

QUESTION CVI.

What three numbers in geometrical progression are those, whose sum is 95; and the sum of their squares 3225?

If x , y , and z be the numbers required;

Then, $x : y :: y : z$; Th. $xz = yy$;

And, $x + y + z = 95$; Th. $x + z = 95 - y$;

Also, $xx + yy + zz = 3225$; Th. $xx + xz + z^2 = 3225$.

Now (by 2^d.) $xx + 2xz + zz = 9025 - 190y + yy$

And (by first) $xz = yy$;

Th. $xx + xz + zz = 9025 - 190y$ by subtraction;

Whence $3225 = 9025 - 190y$;

Th. $y = \left(\frac{9025 - 3225}{190} \right) = 30$;

Th. $x + z = (95 - 30) = 65$; and $xz = (33 \times 30) = 900$;

And by $\begin{cases} x = \frac{65 + \sqrt{65 \times 65 - 4 \times 900 \times \frac{1}{2}}}{2} = 45 \\ \text{qu. 160. } z = \frac{65 - \sqrt{65 \times 65 - 4 \times 900 \times \frac{1}{2}}}{2} = 20. \end{cases}$

QUES

QUESTIONS CVII.

There are three numbers in geometrical progression; if the second of them be taken from the sum of the first and third, and that difference be severally multiplied by the sum of the first and third, and by the sum of the three numbers; the products will be 1120, and 1456: What are those numbers?

If x , y , and z , represent the numbers required;
And $x+z-y$ (the said difference) $=u$;

Then $x+z \times u = 1120$; Or $x+z = \frac{1120}{u}$;

And $x+z \times u+y = 1456$; Or $uy = 336$;

But $(x+z-x+z-y) y = \frac{1120}{u} - u$;

Th. $\frac{1120}{u} - u = (y =) \frac{336}{u}$, (by second)

Th. $28 = (u =) x+z-y$, and $xz = yy = 144$

And $y = 12$,

Also $x+z = 40$:

Th. by quest. $\begin{cases} z = (40 + \sqrt{40^2 - 4 \times 144} \times \frac{1}{2}) = 36, \\ x = (40 - \sqrt{40^2 - 4 \times 144} \times \frac{1}{2}) = 4. \end{cases}$

QUESTION CVIII.

There are three numbers in geometrical progression, the third exceeds the first by 15; and the sum of their squares is 525. What are those numbers?

If x , y , and z , represent the numbers required:

Then $x+15=z$; And $x+15^2 = zz$:

But $x:y::y, x+15$; Th. $xx+15x = yy$;

By quest. $xx + xx+15x + xx+30x+225 = 525$,

But $xx+15x + \frac{15^2}{2} = \left(\frac{225}{4} + 100 =\right) \frac{625}{4}$;

Th. $x + \frac{15}{2} = \frac{25}{2}$; And $x = \left(\frac{25-15}{2} =\right) 5$.

QUES-

QUESTION CIX.

Of three numbers in geometrical progression, there are given the sum of the first and second 14; and the difference of the third and second 15: To find the numbers?

If x , y , and z , be the numbers required;
 Then $x + y = 14$; Or $x = 14 - y$;
 And $z - y = 15$; Or $z = 15 + y$;
 But $x : y :: y : z$; per qu. That is $14 - y : y :: y : 15 + y$;
 Th. $210 - yy = yy$,
 Or $2y + y = 210$, Or $yy + \frac{1}{2}y = 105$;
 But $yy + \frac{1}{2}y + \frac{1}{8} = (105 + \frac{1}{8}) = \frac{1681}{16}$;
 Th. $y + \frac{1}{4} = \frac{41}{4}$; And $y = \left(\frac{41 - 1}{4}\right) = 10$.

QUESTION CX.

There are three numbers in geometrical progression the greatest of which exceeds the least by 15; also the difference of the squares of the greatest and least numbers, is to the sum of the squares of all the three numbers; as 5, to 7?

If x , y , z , represent the three numbers;
 Then $x + 15 = z$; by quest.
 And $x : y :: y : x + 15$; Th. $xx + 15x = yy$;
 Now $(x + 15)^2 - xx = 30x + 225 = zz - xx$,
 And $(x^2 + x^2 + 15x + x + 15^2) = 3x^2 + 45x + 225 = xx + yy + zz$;
 Th. $30x + 225 : 3x^2 + 45x + 225 :: 5 : 7$ per quest.
 Th. $210x + 1575 = 15xx + 225x + 1125$,
 Th. $30 = xx + x$; And $5 = x$.

QUES

MATHEMATICAL QUESTION CXI.

The sum (35) of three numbers in geometrical progression, and the proportion of the mean term to the difference of the extremes (as 2 to 3) being given; to find the numbers?

If x , y , and z , be the numbers required?

Then $x + y + z = 35$; Or $y = 35 - x - z$;

And $y : z - x :: 2 : 3$; Or $3y = 2z - 2x$;

Th. $2x - 2x = (3y =) 105 - 3x - 3z$,

Th. $z = \frac{105 - x}{5}$;

Now $3y = \left(\frac{210 - 2x}{5} - 2x = \right) \frac{210 - 12x}{5}$;

Th. $y = \frac{70 - 4x}{5}$.

But by qu. $x : \frac{70 - 4x}{5} :: \frac{70 - 4x}{5} : \frac{105 - x}{5}$;

Th. $\frac{105x - xx}{5} = \frac{4900 - 560x + 16xx}{25}$,

Th. $xx - \frac{155}{3}x = -\frac{700}{3}$; And $5 = x$.

QUESTION CXII.

There are three numbers in geometrical progression, whose sum is 28 and the sum of their cubes 4672; What are those numbers?

If x , y , and z , denote the numbers required.

Then by quest. $\begin{cases} x + y + z = 28, \\ x^3 + y^3 + z^3 = 4672; \end{cases}$

Or by Transposition $\begin{cases} x + z = 28 - y, \\ x^3 + z^3 = 4672 - y^3; \end{cases}$

Th. $x + z = 28 - y^3$;

Now (the diff. of two last is) $3x^2z + 3xz^2 = 17280 - 2352y + 84y^2$,

But $x + z = 28 - y$; And $xz = yy$;

Th. $= 28 - y \times 3yy = 17280 - 2352y + 84y^2$, Th.

QUESTION CXII. continued.

Th. $y^3 - 784y + 5760 = 0$: Where $y = 8$
 Now $(yy =) xx = 64$; and $28 - y =) x + z = 20$;

Th. by $\begin{cases} z = \frac{20 + \sqrt{20 \times 20 - 4 \times 64}}{2} \times \frac{1}{2} = 16, \\ \text{quest. 160. } x = \frac{20 - \sqrt{20 \times 20 - 4 \times 64}}{2} \times \frac{1}{2} = 4. \end{cases}$

QUESTION CXIII.

There are three numbers in geometrical progression whose sum is 14; and the difference of the squares of the greatest and least is 60: What are those numbers?

If $x =$ the least number; and y the ratio;

Then x , xy , and xy^2 will be the numbers required;

Now by quest. $\begin{cases} x + xy + xy^2 = 14, \\ x^2 y^4 - xx = 60; \end{cases}$

Th. $\begin{cases} x = \frac{14}{1 + y + y^2}, \\ xx = \frac{60}{y^4 - 1}; \end{cases}$

But (by 3^d) $\frac{196}{1 + 2y + 3y^2 + 2y^3 + y^4} = xx$;

Th. $\frac{196}{1 + 2y + 3y^2 + 2y^3 + y^4} = \frac{60}{y^4 - 1}$;

Whence $34y^4 - 30y^3 - 45y^2 - 30y - 64 = 0$, by reduction;

But $\frac{34y^4 - 30y^3 - 45y^2 - 30y - 64}{y - 2} = 34y^3 + 38y^2 + 31y + 32$

QUESTION CXIV.

Tis required to find four mean proportionals; between 5 and 160?

Let $r =$ the ratio of the progression,

Then 5 , $5r$, $5r^2$, $5r^3$, $5r^4$, $160 = 5r^5$, are six terms in geometrical progression;

Th. $\left(\frac{160}{5} =\right) 32 = r^5$,

Th. $\left(\sqrt[5]{32} =\right) 2 = r$;

Whence $5r = 10$; $5r^2 = 20$; $5r^3 = 40$; and $5r^4 = 80$ are the numbers required.

QUESTION CXV.

There are four numbers in geometrical progression, the sum of the first and third of which is 25; and the sum of the second and fourth is 50. What are those numbers?

If $x, y, u,$ and $z,$ represent the numbers required;
Then $x + u = 25$; and $y + z = 50$ per question;

Also $x : y :: u : z,$

Or $x : u :: y : z,$

Or $x : x + u :: y : y + z;$

Th. $x : y :: x + u : y + z;$

That is $x : y :: 25 : 50;$

Th. $50x = 25y$; and $2x = y$

Also $(x : 2x :: 2x : 4x = u)$

But $(x + 4x =) 5x = 25$; Th. $x = 5$

QUESTION CXVI.

What four numbers in geometrical progression, The sum of the two least of which is 15; and the sum of the two greater 60?

If $x, y, u,$ and $z,$ represent those numbers;

Then $x + y = 15$; and $u + z = 60$ per quest.

But $x : y :: y : u,$

Th. $x + y : y :: y + u : u;$

Also $y : u :: u : z,$

Th. $y + u : u :: u + z : z;$

Whence $x + y : y + u :: y + u : u + z,$

That is $15 : y + u :: y + u : 60;$

Th. $(15 \times 60 =) 900 = y + u^2,$

And $30 = y + u.$

Now by second $15 : y :: 30 : u,$

That is $15u = 30y,$ And $u = 2y$

Th. $(y + 2y =) 3y = 30,$ And $y = 10.$

QUES-

QUESTION CXVII.

There are four numbers in geometrical progression, the product of the two least terms is 50; and of the two greatest 800: What are those numbers?

If $x, y, u,$ and $z,$ represent those numbers;

Then $xy = 50$; Or $x = \frac{50}{y}$,

$uz = 800$; Or $z = \frac{800}{u}$,

per quest.

And $xz = uy$; Or $\frac{50}{y} \times \frac{800}{u} = uy$;

Th. $50 \times 800 = u^2 y^2$,

Th. $50 \times 4 = uy$, And $\frac{200}{y} = u$;

But $x : y :: y : u$,

That is $\frac{50}{y} : y :: y : \frac{200}{y}$;

Th. $\frac{10000}{yy} = yy$, Th. $10 = y$.

QUESTION CXVIII.

Tis required to find four numbers in geometrical progression, so that the sum of the means may be 30; and the sum of the extremes 45:

If $x, y, u,$ and $z,$ represent the numbers required;

Then $\begin{cases} y + u = 30; \text{ Or } u = 30 - y, \\ x : y :: y : u; \text{ Or } x = \frac{yy}{30 - y}; \\ y : u :: u : z; \text{ Or } z = \frac{30 - y^2}{y}. \end{cases}$

But $(x + z) = \frac{yy}{30 - y} + \frac{(30 - y)^2}{y} = 45$ by quest.

And $y = 20$; Or $10 = y$.

QUES.

QUESTION CXIX.

What four numbers are in geometrical progression, the sum of whose extremes is 45 and the sum of the cubes of the means 9000?

If $x, y, u,$ and $z,$ be the numbers required :

$$\text{Then } \left\{ \begin{array}{l} x+z=45; \text{ Or } x=45-z, \\ x:y::u:z; \text{ Or } xz=uy; \\ x:y::y:u; \text{ Or } xu=yy; \\ y:u::u:z; \text{ Or } zy=uu; \end{array} \right\} \text{ by quest.}$$

$$\text{Therefore } \left\{ \begin{array}{l} 45-z \times z = xz; \text{ Or } xz = 45z - zz, \\ xuy = y^3; \text{ Or } xxx = y^3; \\ xyu = u^3; \text{ Or } xxx = u^3; \end{array} \right.$$

$$\begin{array}{l} \text{But } (xz \times x+z) = 45z - zz \times 45 = xxx + xxx, \\ \text{And } (y^3 + u^3) = 9000 = xxx + xxx; \\ \text{Th. } 2025z - 45zz = 9000; \text{ and } z=40. \end{array}$$

QUESTION CXX.

What four numbers are in geometrical progression, the sum of the cubes of whose means is 9000; and the sum of the cubes of the extremes 64125?

If x, y, u and z represent the numbers required,
And if by substitution $a=x+z$;

$$\text{Then } a^3 = (x+z)^3 = x^3 + 3x^2z + 3xz^2 + z^3,$$

$$\text{And by last } y^3 = xxz, \text{ and } u^3 = xzx.$$

$$\text{Th. } a^3 = x^3 + 3y^3 + 3u^3 + z^3.$$

$$\text{But } y^3 + u^3 = 9000; \text{ Th. } 3y^3 + 3u^3 = 2700$$

$$\text{And } x^3 + z^3 = 64125; \text{ Th. } a^3 = 91125.$$

$$\text{Th. } a = (\sqrt[3]{91125}) = 45 = x+z,$$

$$\text{Now } x = 45-z, \text{ And } xz = 45z - zz.$$

$$\text{Also } x+z \times xz = (x^2z + xz^2 = y^3 + u^3) = 9000,$$

$$\text{That is } 9000 = 45 \times 45z - z^2, \text{ Th. } z = 40.$$

QUES-

QUESTION CXXI.

'Tis required to find four numbers in geometrical progression; so that their sum may be 75; and the sum of the squares of the mean numbers 500?

If $x, y, u,$ and $z,$ represent the numbers required;

Then $x : y :: y : u$; Or $x = \frac{yy}{u}$:

And $y : u :: u : z$; Or $z = \frac{uu}{y}$:

Now by quest. $yy + uu = 500$,

Th. $\frac{yy}{u} + \frac{uu}{y} = 500 \times \frac{y+u}{yu}$,

That is $y^3 + y^2u + yu^2 + u^3 = 500y + 500u$;

Also by quest. $75 = \frac{yy}{u} + y + u + \frac{uu}{y}$;

Th. $y^3 + y^2u + yu^2 + u^3 = 75uy$;

And (putt. $u + y = a$.) $500a = 75uy$;

Also (bec. $yy + uu = 500$) $a^2 = 500 + 2uy$:

Now $\frac{500a}{75} = (uy =) \frac{a^2 - 500}{2}$,

Th. $a = \left(\frac{70 + 20}{3} = \right) 30 = u + y$,

And $uy = \frac{500 \times 30}{75} = 200$;

Th. by quest. 160. $\begin{cases} u = 30 + \sqrt{30 \times 30 - 4 \times 200} \times \frac{1}{2} = 20, \\ y = 30 - \sqrt{30 \times 30 - 4 \times 200} \times \frac{1}{2} = 10. \end{cases}$

QUES.

QUESTION CXXII.

There are four numbers in geometrical progression whose sum is 75; and the sum of the squares of the extremes is 1625?

Let x , y , u , and z , represent the numbers required;
And let $y+z=a$; Then $x+z=75-a$:

Now $(x+z)^2 = 1625 + 2xz = 5625 - 150a + a^2$,

Or $2xy (= 2xz) = 4000 - 150a + a^2$:

Now (per last) $y^3 + y^2u + yu^2 + u^3 = 75xy$,

And $y^3 + y^2u + yu^2 + u^3 = yy + uu \times a$,

Also $yy + 2xy + uu = a^2$;

The diff. of third and last $yy + uu = 150a - 4000$;

Th. $y^3 + y^2u + yu^2 + u^3 = 150a^2 - 4000a$,

Whence $75xy = 150a^2 - 4000a$,

Or $3xy = 6a^2 - 160a$;

Th. $\frac{6a^2 - 160a}{3} = (xy) = \frac{4000 - 150a + a^2}{2}$,

Th. $a = \left(\frac{335 - 65}{9} = \right) 30 = y + u$;

And $xy = \frac{6 \times 30 \times 30 - 160 \times 30}{3} = 200$.

Th. $x = 20$ $y = 10$, as in the last.

QUESTION CXXIII.

There are four numbers in geometrical progression, the sum of the squares of the two means of which is 500; and the sum of the squares of the extremes 1625: What are those numbers?

If x , y , u , and z , represent the numbers required;

Now because x , y , u , and z , } are in geometrical
Therefore x^2 , y^2 , u^2 , and z^2 , } progression.

Q U E S -

QUESTION CXXIII continued.

Then by q. 116; $x^2 + y^2 : y^2 + u^2 :: y^2 + u^2 : u^2 + z^2$:

But $\left. \begin{array}{l} x^2 + z^2 = 1625, \\ y^2 + u^2 = 500, \end{array} \right\} \text{ by quest.}$

Th. $x^2 + y^2 + u^2 + z^2 = 2125$:

Now (if $x^2 + y^2 = a$; Then) $u^2 + z^2 = 2125 - a$;

And $a : 500 :: 500 : 2125 - a$;

Th. $2125a - a^2 = 2500$:

Whence ($a =$) $x^2 + y^2 = 125$:

And $u^2 + z^2 = (2125 - 125) = 2000$:

But by q. 116; $x^2 + y^2 : y^2 :: y^2 + u^2 : u^2$,

That is $125 : y^2 :: 500 : u^2$;

Th. $125u^2 = 500y^2$; Or $u^2 = 4y^2$:

But ($u^2 + y^2 = 4y^2 + y^2 =$) $5y^2 = 500$;

Th. $y^2 = 100$:

And $y = 10$.

QUESTION CXXIV.

The sum 75, of four numbers in geometrical progression; and the sum of their squares, 2185, being given; to find the numbers?

Let $x, y, u,$ and $z,$ represent those numbers;

Then $x + y + u + z = 75,$
 And $x^2 + y^2 + u^2 + z^2 = 2125, \left. \vphantom{\begin{array}{l} x + y + u + z = 75, \\ x^2 + y^2 + u^2 + z^2 = 2125, \end{array}} \right\} \text{ by quest.}$

Now $x + y + u + z^2 - x^2 + y^2 + u^2 + z^2 = 3500,$

Or $2xy + 2xu + 2xz + 2yu + 2yz + 2uz = 3500,$

Ques.

QUESTION CXXIV continued.

$$\text{Or } xy + \left\{ \frac{xu}{y} \right\} + \left\{ \frac{xz}{yu} \right\} + yu + \left\{ \frac{yz}{uu} \right\} \cdot uz = 1750;$$

$$\text{Or } xy + yu + yu + yu + uz = 1750;$$

$$\text{Or (if } y+u=a) \quad xy + ua + uz = 1750;$$

$$\text{Th. } \quad \quad \quad xy + uz = 1750 - aa;$$

$$\text{But } \left\{ \frac{x^2y}{x^2u} + \frac{y^2}{y^2u} + \frac{u^2y}{u^2} + \frac{z^2y}{z^2u} \right\} \\ = \frac{x^2 + y^2 + u^2 + z^2}{x^2 + y^2 + u^2 + z^2} \times y + u,$$

$$\text{And } \left\{ \frac{x^2y}{xuz} + \frac{xy^2}{yuz} + \frac{xyu}{u^2z} + \frac{xyz}{uz^2} \right\} \\ = \frac{x + y + u + z}{x + y + u + z} \times xy + uz;$$

Now by the
property of ::

$$\left\{ \begin{array}{l} yu = xu; \text{ Th. } xy^2 = x^2u; \\ xu = yy; \text{ Th. } xyu = y^3; \\ xz = yu; \text{ Th. } xyz = y^2u; \\ xz = yu; \text{ Th. } xuz = yu^2; \\ yz = uu; \text{ Th. } yuz = u^3; \\ uu = yz; \text{ Th. } u^2z = yz^2; \end{array} \right.$$

$$\text{Th. } \frac{x^2 + y^2 + u^2 + z^2}{x^2 + y^2 + u^2 + z^2} \times y + u = \frac{x + y + u + z}{x + y + u + z} \times xy + uz,$$

That is

$$2125a = 75 \times 1750 - aa;$$

$$\text{Whenc } a^2 + \frac{85}{3}a = 1750, \text{ And } (a =) y + u = 30.$$

$$\text{Now } x + z = (75 - 30 =) 45; \text{ and } x + z \times y + u = 1350;$$

$$\text{That is } xy + yz + ux + uz = 1350;$$

$$\text{The difference of } \left\{ \begin{array}{l} 2yu = (1750 - 1350 =) 400; \end{array} \right.$$

$$\text{Th. } yu = 200; \text{ And } u = 20; y = 10. \text{ See qu. 121.}$$

QUES-

QUESTION CXXV continued.

$$\text{By D}^{\circ}. \begin{cases} u = \frac{14 + \sqrt{14 \times 14 - 4 \times 45}}{2} \times \frac{1}{2} = 9; \\ y = \frac{14 - \sqrt{14 \times 14 - 4 \times 45}}{2} \times \frac{1}{2} = 5; \end{cases}$$

$$\text{By D}^{\circ}. \begin{cases} z = \frac{18 + \sqrt{18 \times 18 - 4 \times 45}}{2} \times \frac{1}{2} = 15; \\ x = \frac{18 - \sqrt{18 \times 18 - 4 \times 45}}{2} \times \frac{1}{2} = 3; \end{cases}$$

QUESTION CXXVI.

There are four numbers in geometrical progression; the sum of the second and fourth is 50; and the difference between the third and first is 15: What are those numbers?

If $x, y, u,$ and $z,$ represent those numbers;

Then $y + z = 50$; Or $z = 50 - y$;

And $u - x = 15$; Or $x = u - 15$;

Also (because $yy = ux$) Th. $yy = u \times u - 15$;

Th. $\sqrt{u \times u - 15} = y$; And $z = 50 - \sqrt{u \times u - 15}$;

Now $u - 15$: $\sqrt{u \times u - 15} :: u$: $50 - \sqrt{u \times u - 15}$,

Th. $u - 15 \times 50 - u - 15 \times \sqrt{u \times u - 15} = u \times \sqrt{u \times u - 15}$;

Or $2u - 15 \times \sqrt{u \times u - 15} = u - 15 \times 50$;

Th. $\frac{u - 15 \times 50}{2u - 15} = \sqrt{u \times u - 15}$,

And $\frac{(u - 15 \times 50)^2}{(2u - 15)^2} = u \times u - 15$,

Or $4u^3 - 60u^2 - 2275u + 37500 = 0$: Th. $u = 20$.

QUES.

QUESTION CXXVII.

There are four numbers in geometrical progression, the sum of the two least is 15; and the difference of the two greatest 20: what are those numbers?

If $x, y, z,$ and $u,$ be the numbers required;

Then $x+y=15$; Th. $x=15-y$;

And $z-u=20$; Th. $z=20+u$;

Also (because $u=\frac{yy}{x}$;) Th. $z=20+\frac{yy}{15-y}$;

But $15-y:y::\frac{yy}{15-y}:20+\frac{yy}{15-y}$;

That is $15-y \times 20 + yy = \frac{yy^2}{15-y}$, And $y=10$.

QUESTION CXXVIII.

The sum of the means, 30; and the sum of the squares of the extremes, 1625; of 4 numbers in geometrical progression being given, to find the numbers?

If $x, xy, xy^2,$ and $xy^3,$ represent the numbers required;

Then $xy+xy^2=30$; Or. $x=\frac{30}{1+yy}$;

And $x^2+x^2y^6=1625$; Or $x^2=\frac{1625}{1+y^6}$;

Th. $\left(\frac{30}{1+yy}\right)^2 = \frac{900}{yy+2y^3+y^6} = \frac{1625}{1+y^6}$;

Th. $36y^6-65y^4-130y^2-65y^2+36=0$: And $y=2$

QUES.

QUESTION CXXIX.

The sum 75; and the sum of the cubes, 73125; of four numbers in geometrical progression, being given; to find the numbers?

Let $x, y, z,$ and $u,$ represent the numbers required; And let $y + u = a$; Then $x + z = 75 - a$:

Now $y + u^3 = a^3$,

And $x + z^3 = 421875 - 16875a + 225a^2 - a^3$;

Th. $y + u^3 + x + z^3 = 421875 - 16875a + 225a^2$:

But, $y + u^3 + x + z^3 = \begin{cases} x^3 + y^3 + u^3 + z^3 + 3yu \times y + u \\ + 3xz \times x + z, \\ = 73125 + 3yu \times x + y + u + z, \\ = 73125 + 3yu \times 75; \end{cases}$

Th. $73125 + 225yu = 421875 - 16875a + 225a^2$;

Th. $yu = 1550 - 75a + a^2$:

But $\begin{cases} y^3 = x^2 z \\ u^3 = xz^2 \end{cases}$ see quest. 119;

$\begin{cases} = x^3 + x^2 z + xz^2 + z^3, \\ = x + z^3 - 2xz \times x + z, \\ \text{Th. } 73125 = x + z^3 - 2xz \times x + z, \\ = 421875 - 16875a + 225a^2 - a^3 \\ = -yu \times 150 - 2a; \end{cases}$

Or $150 - 2a \times yu = 348750 - 16875a + 225a^2 - a^3$;

Th. $yu = \frac{348750 - 16875a + 225a^2 - a^3}{150 - 2a}$:

Th. $1550 - 75a + a^2 = \frac{348750 - 16875a + 225a^2 - a^3}{150 - 2a}$:

By reduction $a = a^3 - 75a^2 + 2525a + 136250$:

Th. $a = 30$;

And $yu = (1550 - 75 \times 30 + 30 \times 30) = 200$:

Th. $\begin{cases} u = 20 \\ y = 10 \end{cases}$ as in quest. 121.

QUESTION CXXX.

There are four numbers in geometrical progression whose sum is 9620; and if the product of the two extremes be added to the product of their cube roots, the sum will be 3581730: What are those numbers?

If, x^3, y^3, u^3 , and z^3 , represent the numbers required;

Then $x^3 + y^3 + u^3 + z^3 = 3581730$ by quest.

But $x^3 + y^3 + u^3 + z^3 = 3581730$
 $2x + 153 = x^2 + 153xz + 23410$

Th. $2x = 153$

Now $x^2 = y^2$; And $x^2 = u^2$ (by quest. 119)

Th. $x^3 + x^2z + xz^2 + z^3 = 9620$ by quest.

Substitute $x = z$ $x + z = a$;

Th. $x^3 + 3x^2z + 3xz^2 + z^3 = a^3$;

And $(2x^2z + 2xz^2) = 306a - a^3 - 9620$, by subtr.

Th. $a^3 - 306a - 9620 = 0$: And $(a =) x + z = 26$.

And by $\{ \begin{aligned} z &= (26 + \sqrt{26 \times 26 - 4 \times 153 \times \frac{1}{2}}) \frac{1}{2} = 17; \\ \text{qu. 160. } x &= (26 - \sqrt{26 \times 26 - 4 \times 153 \times \frac{1}{2}}) \frac{1}{2} = 9. \end{aligned}$

QUESTION CXXXI.

Of five numbers in geometrical progression, there are given, the sum of the first and third 25; and the sum of the third and fifth 100; to find the numbers?

If x, y, u, e , and z , represent those numbers;

Then $x + u = 25$; And $u + z = 100$ per quest.

Now $x : y :: y : u :: u : e :: e : z$

Th. $x : x + u :: y : y + e$

Again $y : u :: u : z$

Th. $y : y + e :: u : u + z$

Whence $x + u : y + e :: y + e : u + z$

That is $25 : y + e :: y + e : 100$;

Th. $(100 \times 25 =) 2500 = y + e^2$, and $50 = y + e$

But $x : y :: x + u : y + e$

That is $x : y :: 25 : 50$; Th. $2x = y$;

Now $x : 2x :: 2x : u$; Th. $4x = u$;

And $(x + 4x =) 5x = 25$; Th. $x = 5$.

QUEST.

QUESTION CXXXII.

Of 5 numbers in geometrical progression, there are given, the sum of the first and second, 15; and the sum of the fourth and fifth, 120; to find those numbers?

If x, y, u, e , and z , represent those numbers;
Then $x+y=15$; and $e+z=120$ per quest.

Now by quest. 116; $\begin{cases} x+y : y+u :: y+u : u+e, \\ y+u : u+e :: u+e : e+z; \end{cases}$

That is $\begin{cases} 15 : y+u :: y+u : u+e, \\ y+u : u+e :: u+e : 120; \end{cases}$

Th. $\begin{cases} 15 \times u+e = y+u^2, \\ \frac{u+e}{120} = y+u; \end{cases}$

Whence $\frac{u+e}{120 \times 120} = 15 \times u+e,$

Th. $(\sqrt[3]{120 \times 15}) 60 = u+e,$

And $(\frac{60 \times 60}{120} =) 30 = y+u:$

But because $x : y :: y : u$ $x : x+y :: y : y+u,$

That is $x : 15 :: y : 30;$

Th. $30x=15y;$ And $2x=y;$

And $(x+y=x+2x=) 3x=15;$ Th. $x=5.$

QUESTION CXXXIII.

Of five numbers in geometrical progression, there is given the sum of the three least 35; and the sum of the three greatest, 140; to find those numbers?

If x, y, u, e , and z , represent those numbers;

Then $x+y+u=35$; And $e+z+u=140:$

But $x : y :: u : e;$

Th. $x+y : y :: u+e : e;$

But $y : e :: u : z;$

Th. $x+y : u :: u+e : z;$

Whence $x+y+u : u :: u+e+z : z;$

That is $35 : u :: 140 : z;$

Th. $z=4u:$

QUESTION CXXXIII continued.

But $zu = ee$: Th. $4u \times u = ee$, and $2u = e$
 Now $(u + 2u + 4u =) 7u = 140$; Th. $u = 20$.

QUESTION CXXXIV.

In five numbers in geometrical progression, there are given the sum of the two extremes 85; and the sum of the three means, 70... To find those numbers?

If x, y, u, e , and z , be the numbers required;
 Then $x + z = 85$; and $y + u + e = 70$ per quest.

Now $ye = uu$; Th. $2ye = 2uu$;

Also $yy = xu, ee = zu$;

Th. $y + e = \frac{y^2 + 2ye + e^2}{2} = \frac{yy + 2ye + ee}{2} = \frac{xu + 2uu + zu}{2}$.

But $y + e = 70 - u$; Th. $y + e = \frac{4900 - 140u + uu}{2}$;

And $\frac{xu + 2uu + zu}{2} = \frac{85 + 2uu}{2}$;

Th. $85u + 2uu = 4900 - 140u + uu$;

Or $uu + 225u = 4900$; and $u = 20$;

Now $ye = (uu =) 400$; And $y + e = (70 - 20 =) 50$;

Th. by qu. 160. $\begin{cases} e = 50 + \sqrt{50 \times 50 - 4 \times 400 \times \frac{1}{2}} = 40 \\ u = 50 - \sqrt{50 \times 50 - 4 \times 400 \times \frac{1}{2}} = 10 \end{cases}$

QUESTION CXXXV.

In five numbers in geometrical progression, there are given the sum of the first, third, and fifth, 105; and the sum of the second, and fourth 50: to find those numbers?

If x, y, u, e , and z , represent the numbers required;
 Then $x + u + z = 105$; And $y + e = 50$.

Now (by last) $x + z + 2uu = y + e$;

That is $105 + uu = (50 \times 50 =) 2500$;

Th. $u = 20$;

Whence the rest may be found as in the last.

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QUESTION CXXXVI.

In five numbers in geometrical progression, there is given the mean term 20; and the sum of the other four, 135; to find the numbers?

If $x, y, 20, e,$ and z , represent the numbers required; Then $x+y+e+z=135$ per quest.

Substitute, $a=y+e$; Then $x+z=135-a$.

Now (by qu. 134) $x+z+40 \times 20 = y+e$;

That is $\left\{ \begin{array}{l} 135-a+40 \times 20 \\ 175-a \times 20 \end{array} \right\} = a$;

Th $aa+20a=3500$,

And $(a=y+e)=50$;

Ans $(20 \times 20=) 400=y+e$;

Whence $\left\{ \begin{array}{l} e=40 \\ y=10 \end{array} \right\}$ as in q. 134.

QUESTION CXXXVII.

In five numbers in geometrical progression; the sum of the extremes, 85; and the sum of the squares of the second and fourth, 1700, are given, to find the numbers?

If $x, y, u, e,$ and z , represent the numbers required;

Then $x+z=85$; And $y+e=1700$ per quest.

But $x+z+2ux= y+e$ (by quest 134)

That is $85+2ux=y+e$;

Th. $85u=(y+e)=1700$,

And $u=20$; Also $(uu=)xz=400$;

Th. by qu. 160 $\left\{ \begin{array}{l} z = 85 + \sqrt{85 \times 85 - 4 \times 400} \times \frac{1}{2} = 80; \\ x = 85 - \sqrt{85 \times 85 - 4 \times 400} \times \frac{1}{2} = 5; \end{array} \right.$

QUES

QUESTION CXXXVIII.

The sum of five numbers in geometrical progression is, 155; and the sum of their squares, 8525: what are those numbers?

If x, y, u, e , and z , represent those numbers;

And $y + e = z$; Then $x + z = 155 - a - u$;

Now $a^2 = (yy + 2ye + ee) = yy + 2uu + ee$;

Th. $a^2 - uu = yy + uu + ee$;

And $x + z = 24025 - 310a - 310u + a^2 + 2au + uu$;

But $2az = 2uu$;

Th. $x^2 + z^2 = 24025 - 310x - 310u + x^2 + 2az - uu$;

Now $8525 = x^2 + y^2 + u^2 + e^2 + z^2$ by quest;

Th. $8525 = 24025 - 310a - 310u + 2a^2 + 2au - 2uu$

But $y + e^2 = x + z + 2u \times u$ (by quest. 134)

That is $d^2 = (155 - x + u \times u) = 155u - au + uu$,

Or $155u = a^2 + au - uu$;

Th. $310u = 2a^2 + 2au - 2uu$;

Now $8525 = 24025 - 310a - 310u + 310u$,

Or $310a = (24025 - 8525) = 15500$;

Th. $a = 50 = y + e$;

But (above) $a^2 = 155u - au + uu$,

That is $2500 = (155u - 50u + uu) = uu + 105u$;

Th. $20 = u$;

And the rest will be found as in quest. 134.

QUESTION CXXXIX.

What is the sum M , of n terms of the series 1, 3, 7, 15, 31, etc. the terms of which are the successive sums of the geometrical progression 1, 2, 4, 8, 16, etc.

If $a=1$; and $r=2$; then the sum of

1	$\left. \begin{array}{l} \text{Terms of the geometrical pro.} \\ a, ar, ar^2, ar^3, \text{ etc. will be} \end{array} \right\}$	$\frac{ar-a}{r-1} = \frac{a}{r-1} \times \frac{a}{r-1}$	$\left. \begin{array}{l} \text{By quest. 83.} \end{array} \right\}$
2		$\frac{ar^2-a}{r-1} = \frac{a}{r-1} \times \frac{a}{r-1}$	
3		$\frac{ar^3-a}{r-1} = \frac{a}{r-1} \times \frac{a}{r-1}$	
4		$\frac{ar^4-a}{r-1} = \frac{a}{r-1} \times \frac{a}{r-1}$	
etc.		etc. etc.	
n		$\frac{ar^n-a}{r-1} = \frac{a}{r-1} \times \frac{a}{r-1}$	

$$\text{Th. } M = \frac{a}{r-1} \times \left\{ \begin{array}{l} n \text{ terms of } r, r^2, r^3, \text{ etc.} \\ - n \text{ terms of } 1+1+1 \text{ etc.} \end{array} \right.$$

$$\text{But } n \text{ terms of } r, r^2, r^3, \text{ etc.} = \frac{r^{n+1}-r}{r-1} \times \frac{r}{r-1} \text{ by q. 83.}$$

$$\text{Th. } M = \frac{r \times r^{n+1} - 1}{r-1} - n \times \frac{a}{r-1} :$$

In this example if $n=5$; Then,

$$M = \left(\frac{2 \times 2^5 - 1}{2-1} - 5 \times \frac{1}{2-1} = 2 \times 31 - 5 = \right) 57.$$

In computations relating to compound interest and annuities

Let $\left\{ \begin{array}{l} p \\ r \\ n \end{array} \right\}$ represent $\left\{ \begin{array}{l} \text{the principal or present worth,} \\ \text{the annuity,} \\ \text{the interest of 1 l. in 1 time,} \\ \text{the number of times,} \\ \text{the amount;} \end{array} \right.$

Then, QUESTION CXLI.

p , r , and n , are given; to find m ?

Now bec. 1 l. will in 1 time amount to $1+r$
 $\left\{ \begin{array}{l} p \times 1 + r \\ p \times 1 + r^2 \\ \text{etc.} \\ p \times 1 + r^n \end{array} \right\}$ will amount to $\left\{ \begin{array}{l} p \times 1 + r \\ p \times 1 + r^2 \\ p \times 1 + r^3 \\ \text{etc.} \\ p \times 1 + r^n \end{array} \right\}$
 $\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \\ \text{etc} \\ n \end{array} \right\}$ times
 $\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \\ \text{etc} \\ n \end{array} \right\}$ the amount of p at the end of

Th. $p \times 1 + r^n = m$;

In Logarithms $P + n \times L, 1 + r = M$.

QUESTION CXLII.

When p , r , and m are given, to find n ?

Then $n = \frac{M - P}{L, 1 + r}$

QUESTION CXLIII.

When p , n , m , are given; to find r ?

Then $L, 1 + r = \frac{M - P}{n}$.

QUESTION CXLIV.

When r , n , and m , are given to find p ?

Then $P = M - n \times L, 1 + r$.

QUES.

QUESTION CXLV.

In annuities computed by compound interest a , r , and n , are given; to find m ?

Then $\left\{ \begin{array}{l} a \\ a \times 1 + r \\ a \times 1 + r^2 \\ a \times 1 + r^3 \\ \text{etc.} \\ a \times 1 + r^n \end{array} \right\}$ becomes the $\left\{ \begin{array}{l} \text{1st.} \\ \text{2d.} \\ \text{3th.} \\ \text{4th.} \\ \text{etc.} \\ \text{nth.} \end{array} \right\}$ Time.

Th. their sum $\left\{ \frac{a \times 1 + r^n - a}{r} = m \text{ by queff. 83.} \right.$

In logar. let $n \times L. 1 + r = B$; then $A + L. b - 1 - R = M.$

QUESTION CXLVI.

When m , r , and n , are given; to find a ?

Then $a = \frac{rm}{1 + r^n - 1} :$

Or (if $n \times L. 1 + r = B$) $A = M + R - L. b - 1.$

QUESTION CXLVII.

When a , m , and r , are to find n ?

Then $a \times 1 + r^n = mr + a ;$

Th. $1 + r^n = \frac{mr + a}{a}$

And $n = \frac{L. \frac{mr + a}{a} - A}{L. 1 + r}$

QUESTION CXLVIII.

When a , n , and m , are given; to find r .

$$\text{Then } \frac{a \times 1 + r^n - a}{1 + r - 1} = m,$$

$$\text{Or } \frac{a \times 1 + r^n - a}{1 + r - 1} = m \times 1 + r - m;$$

$$\text{Th. } \frac{a \times 1 + r^n - a}{1 + r - 1} - m \times 1 + r - m - a = 0;$$

If $1+r$ could reasonably be expected to be a whole number, or rational fraction, it might be found in the same manner as r in quest. 92; but as that will seldom or never happen; use the following approximation*.

$$\text{Let } D = \frac{M - A - N}{\frac{1}{2} \times n - 1}; \text{ and } e = \frac{6}{n + 1};$$

$$\text{Also } F = \frac{L \times 2 \times d - 1 + e + E}{2}; \text{ Then } r = f - e.$$

For example; if an annuity of 50 *l.* forborn 18 years amounts to 1342 *l.* 15 *s.* what rate of interest was allowed?

$$\begin{array}{ll} \text{Here } 50 = a, & M = 3,12800, \\ 18 = n, & A = 1,69897, \\ 1342,75 = m, & N = 1,25527; \\ 8,5 = \frac{1}{2} \times n - 1, & 8,5) 0,17375 (0,02044 = D \\ & 0,31579 = \frac{6}{n + 1} = e; \text{ And } 1,04819 = d. \end{array}$$

$$\text{Then } 2 \times 0,04819 = 0,09638, \quad E = 1,49940, \\ = 0,31579, L \times 2 \times d - 1 + e = 1,61508,$$

$$\text{Th. } 2 \times d - 1 + e = 0,41217; \quad 2 \sqrt{1,11448},$$

$$\text{The } (f =) 0,36078 - 0,31579 = (0,04499) 0,045 = r.$$

Answer $4\frac{1}{2}$ *l.* per cent.

* The most excellent mathematician William Jones Esq. hath given leave for the publication of this approximation in his books relating to logarithms.

QUES-

QUESTION CXLIX.

In annuities computed by compound interest a , r , and n , are given; to find p

Then $\left\{ \begin{array}{l} \frac{a}{1+r} \\ \frac{a}{1+r^2} \\ \frac{a}{1+r^3} \\ \text{etc} \\ \frac{a}{1+r^n} \end{array} \right\}$ is the present value of the sum payable at the end of the $\left\{ \begin{array}{l} 1^{\text{st}} \\ 2^{\text{d}} \\ 3^{\text{d}} \\ \text{etc.} \\ n^{\text{th}} \end{array} \right\}$ times;

Th. their sum $\left\{ \frac{1+r^n - 1 \times a}{r \times 1+r^n} = p \text{ by quest. 95} \right.$

In Log. Let $n \times L.1+r = B$;

Then $A + L.b - 1 - R - B = P$.

QUESTION CL.

When p , r , and n , are given; to find a

Then $a = \frac{pr \times 1+r^n}{1+r^n - 1}$;

Or (if $n \times L.1+r = B$; $A = \frac{P+R+B}{L.b-1}$).

QUESTION CLI.

When a , p , and r are given; to find n

Then $1+r^n \times a = pr \times 1+r^n + a$;

Th. $\frac{1+r^n}{1+r^n} = \frac{a}{a-pr}$;

And $n = \frac{A - L.a - pr}{L.1+r}$.

QUESTION CLII

When a , n , and p , are given to find r ?

Then
$$\frac{a \times 1 + r^n - a}{1 + r - 1 \times 1 + r^n} = p.$$

Th. $p \times 1 + r^{n+1} - a + p \times 1 + r^n + a = 0$

And here, as in quest. 148 (since the value of $1 + r$ is not to be expected in a whole number or rational fraction) the following approximation (quoted from the same gentleman) may be used:

Let $G = \frac{A + N - P}{\frac{1}{2} \times n + 1}$; $b = \frac{6}{n-1}$;

$K = \frac{L \cdot b - 2 \times g - 1 + H}{2}$; Then $r = b - k$:

For example; suppose a bookfeller purchases a work for 40 l . and pays for printing a thousand copies thereof 15 l . for paper 20 l . and for advertising and other incident charges 10 l . Now if he sells the edition at 3 s . each copy in 10 years (*i. e.* 100 copies every year) what does he gain per cent.?

Here the bookfeller lays out 85 l . to purchase an annuity of 15 l . per year, to continue 10 years;

Th. $15 = a$, $A = 1,17609$.

$10 = n$, $N = 1,00000$,

$85 = p$, $2,17609$,

$\frac{1}{2} = \frac{6}{n-1} = b$, $P = 1,92942$,

$5,5 = \frac{1}{2} \times n + 1$; $5,5)0,24667 (=0,04485 = G$;

Th. $1,10879 = g$.

$k = 0,66667$, $H = 1,82391$,

$2 \times 0,10879 = 0,21758$, $L \cdot b - 2 \times g - 1 = 5,65233$,

$k - 2 \times g - 1 = 0,44909$; $2)1,47624$,

$K = 1,73812$,

Then $0,66667 - (k)0,54717 = 0,11950 = r$.

Answer 11. 19. per cent.

QUES.

QUESTION CLIII.

In annuities computed by compound interest a , p , and r , are given; to find m ?

By quest. 151 $\frac{a}{1+r^n} = \frac{a}{a-pr}$;

By quest. 141 $\frac{m}{1+r^n} = \frac{m}{p}$;

Th $\frac{m}{p} = \frac{a}{a-pr}$. And $m = \frac{pa}{a-pr}$.

QUESTION CLIV.

When a , r , and m , are given; to find p ?

Then $\frac{ma}{a+mr} = p$.

QUESTION CLV.

When a , p , and m , are given; to find r ?

Then $\frac{m-p+a}{mp} = r$.

QUESTION CLVI.

When p , r , and m , are given; to find a ?

Then $a = \frac{m+p}{m-r}$.

QUESTION CLVII.

In computing annuities by compound interest, p , m , and n , are given; to find a ?

By quest. 141. $1+r^n = \frac{a}{p}$

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QUESTION CLVII continued.

Th. $r = \frac{m}{p} \left| \frac{1}{n} - 1 \right|$

By quest. 146. $a = \frac{rm}{1+r^n-1}$

If $M-P=B$, and $\frac{M-P}{n} = C$;

Then $A = \frac{L.r-1+M}{L.b-1}$

QUESTION CLVIII.

When a , p , and m , are given to find n ?

By qu. 155; $r = \frac{m-p \times a}{mp}$;

Then by qu. 142; $n = \frac{M-P}{L.1+r}$

QUESTION CLIX.

When p , n , and a , are given to find m ?

First find r by quest 152; and then m by quest. 153.

QUESTION CLX.

When m , n , and a , are given to find p ?

First find r by quest. 148; and then p by quest. 154.

In computing the values of annuities in reversion by compound interest;

Let $\left\{ \begin{matrix} a \\ p \\ t \\ x \\ r \end{matrix} \right\}$ repr. the $\left\{ \begin{matrix} \text{annuity,} \\ \text{present worth thereof,} \\ \text{time of its duration,} \\ \text{time before it commences,} \\ \text{interest of 1\% in 1 time;} \end{matrix} \right.$

Then QUESTION CLXI.

a , t , x , and r , are given to find p ?

By quest. 149.
the present worth of
 a to continue
 x years to

$$\frac{1+r^x}{r \times 1 + r^x} \times a - a = \frac{1+r^x}{r \times 1 + r^x} \times a - a = p.$$

Th. $\frac{1+r^x}{r \times 1 + r^x} \times a - a = p.$

In Logarithms (if $t \times L. 1+r = B$)

Then $A + L.b - 1 - R - x + t \times L. 1+r = P.$

QUESTION CLXII.

When p , t , x , and r , are given to find a ?

Then $A - P + R + x + t \times L. 1+r = L.b - 1.$

QUESTION CLXIII.

When p , a , t , and r , are given to find x ?

Then $\frac{A + L.b - 1 - R - P - B}{L. 1+r} = x.$

QUESTION CLXIV.

When p , a , x and r , are given to find t ?

If $P + R + x \times L. 1+r = C$;

Then $t = \frac{A - L.a - C}{L. 1+r}.$

QUES-

IX. QUESTION CLXV.

When p , a , x , and r , are given to find r .

The following approximation (by the gentleman mentioned in quest. 148) may be used.

$$\text{Let } \frac{2x+n+1}{2} = b; \quad u = \frac{12b}{n-1};$$

$$\text{And } \frac{A+T-P}{b} = v; \quad z = u - \frac{v-1}{v} \times 2;$$

$$\text{Then } \frac{Z+U}{2} = E; \quad \text{And } u-E = r.$$

QUESTION CLXVI.

A gentleman who had 10 different annuities of 100*l.* each, the longest being to continue 60 years, the second 59, the third 58 years, etc. fold them all at 5*l.* per cent. compound interest; how much money did he receive?

If $a=100$, and $r=0,05$; then the present worth of 100*l.* a year to continue,

$$\left. \begin{array}{l} 60 \\ 59 \\ 58 \\ \text{etc.} \end{array} \right\} \text{years will be } \left\{ \begin{array}{l} \frac{1+r^{60}-1}{r \times 1+r^{60}} \times a = 1 - \frac{1}{1+r^{60}} \times \frac{a}{r} \\ \frac{1+r^{59}-1}{r \times 1+r^{59}} \times a = 1 - \frac{1}{1+r^{59}} \times \frac{a}{r} \\ \frac{1+r^{58}-1}{r \times 1+r^{58}} \times a = 1 - \frac{1}{1+r^{58}} \times \frac{a}{r} \\ \text{etc.} \end{array} \right\} \text{by quest. 149.}$$

Therefore the value of 10 such annuities will be

$$10 - \frac{1+r^{10}-1}{r \times 1+r^{60}} \times \frac{a}{r} \text{ by quest. 83;}$$

$$\text{Now } (1+0,05^{10}) = \frac{1,05^{10}}{1,05^{10}-1} = 1,628895;$$

$$\text{Th. } \frac{1,05^{10}}{1,05^{10}-1} = 0,628895;$$

$$\begin{array}{rcl}
 \text{Also} & \frac{1,05^{60}}{0,628895} & = 18,679186; \\
 \text{Th.} & \frac{0,05 \times 18,679186}{10 - 0,673368 \times \frac{200}{0,05}} & = 0,673368: \\
 \text{And} & & = 18653,26 \text{ l.}
 \end{array}$$

QUESTION CLXVII.

If to enjoy the benefit of an estate for 23 years, after the expiration of 8 years, be worth 400 l. present money, what will the same estate be worth for 21 years, after the expiration of 10 years; allowing compound interest at 5 per cent?

Suppose the estate was a l. per year, and x = the number required;

$$\begin{array}{rcl}
 \text{Then} & \frac{1,05^{23} - 1}{0,05 \times 1,05^{23}} \times a & = 400 \\
 \text{And} & \frac{1,05^{21} - 1}{0,05 \times 1,05^{21}} \times a & = x.
 \end{array}
 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{per quest.}$$

$$\text{That is} \quad a = \frac{400 \times 0,05 \times 1,05^{23}}{1,05^{23} - 1}$$

$$\text{And} \quad a = \frac{x \times 0,05 \times 1,05^{21}}{1,05^{21} - 1}$$

$$\text{Th.} \quad \frac{x \times 0,05 \times 1,05^{21}}{1,05^{21} - 1} = \frac{400 \times 0,05 \times 1,05^{23}}{1,05^{23} - 1}$$

$$\text{Th.} \quad x = \frac{400 \times 1,05^{23} - 1}{1,05^{23} - 1}$$

$$\text{That is} \quad x = \left(\frac{400 \times 1,785963}{2,071524} \right) = 344,8597.$$

QUES-

QUESTION CLXVIII.

If 150 ℓ . be lent, on condition that 12 $\%$. per annum be paid, until the principal and its interest at 5 per cent. be satisfied; and that at every payment, the interest then due, shall be discharged, and the remainder, by which such payment exceeds that interest, applied to reduce the principal: How many years must the said payment continue?

If $p=150$; $a=12$; $r=.05$; and n —the time required;

$$\text{Then } \left\{ \begin{array}{l} p + pr \\ -a \end{array} \right\} + pr \quad \begin{array}{l} = \text{sum due bef.} \\ = \text{sum due after} \end{array} \left. \vphantom{\begin{array}{l} p + pr \\ -a \end{array}} \right\} \begin{array}{l} 1^{\text{st}} \text{ pay.} \end{array}$$

$$\text{And } \left\{ \begin{array}{l} p - ar + pr^2 \\ -2a \end{array} \right\} + 2p - ar + pr^2 \quad \begin{array}{l} = \text{inter. due at} \\ = \text{sum due after} \end{array} \left. \vphantom{\begin{array}{l} p - ar + pr^2 \\ -2a \end{array}} \right\} \begin{array}{l} 2^{\text{d}} \text{ pay.} \end{array}$$

$$\text{Also } \left\{ \begin{array}{l} p - 2ar + 2p - ar^2 + pr^3 \\ -3a \end{array} \right\} + 3p - 3ar + 3p - ar^2 + pr^3 \quad \begin{array}{l} = \text{int. due at} \\ = \text{f. due aft.} \end{array} \left. \vphantom{\begin{array}{l} p - 2ar + 2p - ar^2 + pr^3 \\ -3a \end{array}} \right\} \begin{array}{l} 3^{\text{d}} \text{ paym.} \end{array}$$

QUES.

QUESTION CLXVIII continued.

Again

$$\left\{ \begin{aligned} & \left\{ -4a \right\} + \frac{p}{4a} + \frac{p-3axn+3p-3ax^2+3p-ax^3+p^4}{4a-6ax+6p-4ax^2+4p-ax^3+p^4} = \text{interest due at } 4^{\text{th}} \text{ pay.} \\ & \text{sum due after} \end{aligned} \right.$$

$$\text{Th. } \left\{ \begin{aligned} & p + npr + \frac{n \cdot n-1}{1 \cdot 2} p^2 + \frac{n \cdot n-1 \cdot n-2}{1 \cdot 2 \cdot 3} p^3 \text{ etc.} \\ & na - \frac{n \cdot n-1}{2} ar - \frac{n \cdot n-1 \cdot n-2}{1 \cdot 2 \cdot 3} a^2 - \frac{n \cdot n-1 \cdot n-2 \cdot n-3}{1 \cdot 2 \cdot 3 \cdot 4} a^3 \text{ etc.} \end{aligned} \right.$$

will be the sum due after the n^{th} payment; which by the question is nothing:

$$\text{Th. } p \cdot 1 + n + \frac{n \cdot n-1}{1 \cdot 2} r^2 \text{ etc.} = a \cdot n + \frac{n \cdot n-1}{1 \cdot 2} r + \frac{n \cdot n-1 \cdot n-2}{1 \cdot 2 \cdot 3} r^2 \text{ etc.}$$

$$\text{Or } p \cdot 1 + r^n = a \cdot x \frac{1+r^{n-1}}{r},$$

$$\text{Or } \frac{p \cdot 1 + r^n}{a} = \frac{1+r^n-1}{r},$$

QUESTION CLXVIII continued.

$$\text{Or } 1 = 1 - \frac{pr}{a} \times 1 + r^n;$$

$$\text{Or } 1 = \frac{a - pr}{a} \times 1 + r^n;$$

$$\text{Th. } \frac{a}{a - pr} = 1 + r^n.$$

$$\text{And } \frac{L(a - pr)}{L(1 + r)} = n$$

Scholium. By comparing this result with quest. 151. it appears; that, where a debt is discharged by many equal payments, and the interest due at the time of each payment is cleared before any part of the principal, compound interest is allowed to the lender.

But it is both legal and customary, when money is paid in part of a debt, to deduct the interest then due, out of such payment; and to apply, only, the remaining part to the discharge of the principal.

Therefore, in computing the present values of annuities, etc. the rules found on the principles of compound interest, will give their legal value.

QUES-

QUESTION CLXIX.

In a geometrical progression infinitely decreasing are given, the greatest term x , and the ratio, r ; to find the sum s ?

By quest. 85, $x + \frac{x-r}{r-1} = s$

Where a , in a finite progression, signifies the least term; but in an infinite progression the least term is inconsiderable;

Th. $x + \frac{x-r}{r-1} = s$, Or $\frac{rx}{r-1} = s$

QUESTION CLXX.

What is the present value p of an estate of a per year, in fee simple, allowing compound interest at r per pound per annum?

Now $p = \frac{a}{1+r} + \frac{a}{1+r^2} + \frac{a}{1+r^3}$ etc. ad infinitum.

Th. $p = \left(\frac{a}{1+r} \times \frac{1}{1+r-1} \right)$ by quest. last.

QUESTION CLXXI.

QUESTION CLXXI.

In a geometrical progression infinitely decreasing are given, the greatest term a , and the ratio r ; to find the sum S .

What is present value p ; of the reversion of an estate of a per annum, in fee simple; to commence at the end of x years; allowing compound interest at r per pound per annum?

By quest. 149, an annuity of a to continue x years } $= \frac{a \times 1 + r^x - a}{r \times 1 + r^x}$

By quest. 170 a perpetuity } $= \frac{a}{r}$

$$\text{Th. } \frac{a}{r} - \frac{a \times 1 + r^x - a}{r \times 1 + r^x} = p$$

$$\text{Or } \frac{a \times 1 + r^x - a}{r \times 1 + r^x} = \frac{a}{r} - p$$

$$\text{Th. } \frac{a}{r \times 1 + r^x} = \frac{a}{r} - p$$

$$\text{Or } A - R - x \times L.1 + r = P$$

$$\text{Corol. 1; } A - P + R + x \times L.1 + r =$$

$$\text{Corol. 2; } x = \frac{A - P - R}{L.1 + r}$$

QUESTION CLXXII.

'Tis required to find the sum of the infinite series of the reciprocals of the triangular numbers; $\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15}$, &c. ?

Let $1 = \frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15}$, &c. ad infinitum ;

Then $1 - 1 = \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21} + \frac{1}{28} + \frac{1}{36} + \frac{1}{45}$, &c. by transposition ;

And the diff. of these two } $1 = \frac{2-1}{1 \times 2} + \frac{3-2}{2 \times 3} + \frac{4-3}{3 \times 4} + \frac{5-4}{4 \times 5}$, &c. equat. is

That is $1 = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5}$, &c.

Twice which is $2 = \frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15}$, &c.

Th. $2 = \frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15}$, &c.

QUESTION CLXXIII.

The sum of n terms of the series, $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$,
 &c. is required?

$$\text{If } n = 4, \text{ then } 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}, \text{ \&c. to } \frac{1}{n}.$$

$$\text{Then } 1 + \frac{1}{n+1} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}, \text{ \&c. to } \frac{1}{n+1};$$

$$\text{Th. } 1 - \frac{1}{n+1} = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20}, \text{ \&c. to } \frac{1}{n \cdot n+1};$$

$$\text{Or } - \frac{n}{n+1} = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20}, \text{ \&c. to } \frac{1}{n \cdot n+1};$$

$$\text{Th. } - \frac{2n}{n+1} = \frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10}, \text{ \&c. to } \frac{2}{n \cdot n+1};$$

QUESTION CLXXIV.

*Tis required to find the sum of the infinite series of the reciprocals of the pyramidal numbers, $\frac{1}{1} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36}$, &c. ?

By g. 172. $z = \frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10}$
 $[+ \frac{1}{15}, \&c.]$

Then $2 - 1 = \frac{1}{2} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15}$
 $+ \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{84} + \frac{1}{100} + \frac{1}{126} + \frac{1}{140} + \frac{1}{168} + \frac{1}{180} + \frac{1}{210} + \frac{1}{252} + \frac{1}{280} + \frac{1}{315} + \frac{1}{360} + \frac{1}{420} + \frac{1}{490} + \frac{1}{560} + \frac{1}{630} + \frac{1}{720} + \frac{1}{840} + \frac{1}{945} + \frac{1}{1080} + \frac{1}{1260} + \frac{1}{1470} + \frac{1}{1680} + \frac{1}{1960} + \frac{1}{2240} + \frac{1}{2520} + \frac{1}{2835} + \frac{1}{3150} + \frac{1}{3528} + \frac{1}{3920} + \frac{1}{4410} + \frac{1}{4914} + \frac{1}{5488} + \frac{1}{6120} + \frac{1}{6720} + \frac{1}{7448} + \frac{1}{8190} + \frac{1}{9000} + \frac{1}{9882} + \frac{1}{10800} + \frac{1}{11840} + \frac{1}{12960} + \frac{1}{14168} + \frac{1}{15456} + \frac{1}{16830} + \frac{1}{18288} + \frac{1}{19840} + \frac{1}{21480} + \frac{1}{23208} + \frac{1}{25020} + \frac{1}{26928} + \frac{1}{28940} + \frac{1}{31072} + \frac{1}{33330} + \frac{1}{35712} + \frac{1}{38220} + \frac{1}{40864} + \frac{1}{43640} + \frac{1}{46560} + \frac{1}{49632} + \frac{1}{52860} + \frac{1}{56256} + \frac{1}{59820} + \frac{1}{63568} + \frac{1}{67500} + \frac{1}{71632} + \frac{1}{75960} + \frac{1}{80504} + \frac{1}{85260} + \frac{1}{90240} + \frac{1}{95472} + \frac{1}{100980} + \frac{1}{106784} + \frac{1}{112890} + \frac{1}{119312} + \frac{1}{126060} + \frac{1}{133152} + \frac{1}{140600} + \frac{1}{148416} + \frac{1}{156610} + \frac{1}{165208} + \frac{1}{174220} + \frac{1}{183664} + \frac{1}{193560} + \frac{1}{203920} + \frac{1}{214768} + \frac{1}{226020} + \frac{1}{237704} + \frac{1}{249840} + \frac{1}{262460} + \frac{1}{275584} + \frac{1}{289230} + \frac{1}{303432} + \frac{1}{318110} + \frac{1}{333288} + \frac{1}{348984} + \frac{1}{365220} + \frac{1}{382024} + \frac{1}{399420} + \frac{1}{417440} + \frac{1}{436104} + \frac{1}{455440} + \frac{1}{475480} + \frac{1}{496248} + \frac{1}{517770} + \frac{1}{539984} + \frac{1}{562920} + \frac{1}{586616} + \frac{1}{611100} + \frac{1}{636408} + \frac{1}{662570} + \frac{1}{689632} + \frac{1}{717630} + \frac{1}{746592} + \frac{1}{776550} + \frac{1}{807544} + \frac{1}{839608} + \frac{1}{872770} + \frac{1}{907072} + \frac{1}{942550} + \frac{1}{979232} + \frac{1}{1017160} + \frac{1}{1056384} + \frac{1}{1096840} + \frac{1}{1138584} + \frac{1}{1181650} + \frac{1}{1226088} + \frac{1}{1271940} + \frac{1}{1319264} + \frac{1}{1368010} + \frac{1}{1418248} + \frac{1}{1469940} + \frac{1}{1523144} + \frac{1}{1577920} + \frac{1}{1634328} + \frac{1}{1692420} + \frac{1}{1752264} + \frac{1}{1813920} + \frac{1}{1877450} + \frac{1}{1942928} + \frac{1}{2010410} + \frac{1}{2079984} + \frac{1}{2151720} + \frac{1}{2225704} + \frac{1}{2302010} + \frac{1}{2380728} + \frac{1}{2461940} + \frac{1}{2545744} + \frac{1}{2632220} + \frac{1}{2721472} + \frac{1}{2813600} + \frac{1}{2908710} + \frac{1}{2996928} + \frac{1}{3088360} + \frac{1}{3183064} + \frac{1}{3281100} + \frac{1}{3382528} + \frac{1}{3487400} + \frac{1}{3595792} + \frac{1}{3707780} + \frac{1}{3823448} + \frac{1}{3942870} + \frac{1}{4066144} + \frac{1}{4193360} + \frac{1}{4324620} + \frac{1}{4459928} + \frac{1}{4599380} + \frac{1}{4743088} + \frac{1}{4891140} + \frac{1}{5043640} + \frac{1}{5199704} + \frac{1}{5359440} + \frac{1}{5522970} + \frac{1}{5690408} + \frac{1}{5861860} + \frac{1}{6037456} + \frac{1}{6217300} + \frac{1}{6401608} + \frac{1}{6590500} + \frac{1}{6784112} + \frac{1}{6982584} + \frac{1}{7185960} + \frac{1}{7394392} + \frac{1}{7607920} + \frac{1}{7826688} + \frac{1}{8050840} + \frac{1}{8280528} + \frac{1}{8515800} + \frac{1}{8756808} + \frac{1}{8993680} + \frac{1}{9236568} + \frac{1}{9485620} + \frac{1}{9740992} + \frac{1}{10002840} + \frac{1}{10271328} + \frac{1}{10546620} + \frac{1}{10828888} + \frac{1}{11118280} + \frac{1}{11414968} + \frac{1}{11719120} + \frac{1}{12030928} + \frac{1}{12350560} + \frac{1}{12678208} + \frac{1}{13014060} + \frac{1}{13358328} + \frac{1}{13711200} + \frac{1}{14072928} + \frac{1}{14443720} + \frac{1}{14823808} + \frac{1}{15213400} + \frac{1}{15612728} + \frac{1}{16022080} + \frac{1}{16441688} + \frac{1}{16871760} + \frac{1}{17312624} + \frac{1}{17764600} + \frac{1}{18227928} + \frac{1}{18702848} + \frac{1}{19189600} + \frac{1}{19688528} + \frac{1}{20199880} + \frac{1}{20723928} + \frac{1}{21260920} + \frac{1}{21811128} + \frac{1}{22374800} + \frac{1}{22952208} + \frac{1}{23543680} + \frac{1}{24149488} + \frac{1}{24769920} + \frac{1}{25405328} + \frac{1}{26056000} + \frac{1}{26722288} + \frac{1}{27404560} + \frac{1}{28103128} + \frac{1}{28818400} + \frac{1}{29550728} + \frac{1}{30300480} + \frac{1}{31069024} + \frac{1}{31856720} + \frac{1}{32663928} + \frac{1}{33491000} + \frac{1}{34338328} + \frac{1}{35206400} + \frac{1}{36095688} + \frac{1}{37006640} + \frac{1}{37939728} + \frac{1}{38895320} + \frac{1}{39873928} + \frac{1}{40876000} + \frac{1}{41892024} + \frac{1}{42931520} + \frac{1}{43995028} + \frac{1}{45083000} + \frac{1}{46195928} + \frac{1}{47334320} + \frac{1}{48498728} + \frac{1}{49689680} + \frac{1}{50907728} + \frac{1}{52153440} + \frac{1}{53427328} + \frac{1}{54730000} + \frac{1}{56062024} + \frac{1}{57423920} + \frac{1}{58816288} + \frac{1}{60239680} + \frac{1}{61694624} + \frac{1}{63181680} + \frac{1}{64691428} + \frac{1}{66224400} + \frac{1}{67790328} + \frac{1}{69389840} + \frac{1}{71023528} + \frac{1}{72691120} + \frac{1}{74393328} + \frac{1}{76130720} + \frac{1}{77903928} + \frac{1}{79713680} + \frac{1}{81560624} + \frac{1}{83445520} + \frac{1}{85369028} + \frac{1}{87331920} + \frac{1}{89335024} + \frac{1}{91379080} + \frac{1}{93464828} + \frac{1}{95593040} + \frac{1}{97764424} + \frac{1}{99979720} + \frac{1}{102239728} + \frac{1}{104545200} + \frac{1}{106897024} + \frac{1}{109296080} + \frac{1}{111743288} + \frac{1}{114239680} + \frac{1}{116786224} + \frac{1}{119384000} + \frac{1}{122034024} + \frac{1$

And by } $1 = \frac{3-1}{1 \times 3} + \frac{6-3}{3 \times 6} + \frac{10-6}{6 \times 10} + \frac{15-10}{10 \times 15}, \text{ \&c.}$
substr. }

That is $1 = \frac{2}{1 \cdot 3} + \frac{3}{3 \cdot 6} + \frac{4}{6 \cdot 10} + \frac{5}{10 \cdot 15}, \text{ \&c.}$

Or $\therefore 1 = \frac{2}{3} + \frac{1}{6} + \frac{1}{15} + \frac{1}{30}, \&c,$

$$\text{Th.} \quad \frac{3}{e} = \frac{1}{1} + \frac{1}{4} + \frac{1}{10} + \frac{1}{20}, \text{ \&c.}$$

QUESTION CLXXV.

The sum of n terms of the series, $\frac{1}{1} + \frac{1}{2} + \frac{1}{15} + \frac{1}{20}$, &c. is required?

By quest. 173.

$$\frac{2n}{n+1} = \frac{1}{1} + \frac{1}{2} + \frac{1}{6} + \frac{1}{15}, \&c. \text{ to } \frac{2}{n \cdot n+1};$$

$$\text{Th. } \frac{2n}{n+1} - 1 + \frac{2}{n+1 \cdot n+2} = \frac{1}{2} + \frac{1}{6} + \frac{1}{15} + \frac{1}{20}, \&c. \text{ to } \frac{2}{n+1 \cdot n+2};$$

$$\text{Th. } 1 - \frac{1}{n+1 \cdot n+2} = \frac{1}{2} + \frac{1}{6} + \frac{1}{15} + \frac{1}{20}, \&c. \text{ to } \frac{4}{n \cdot n+1 \cdot n+2};$$

$$\text{Or } \frac{n+3 \times n}{n+1 \times n+2} = \frac{1}{2} + \frac{1}{6} + \frac{1}{15} + \frac{1}{20}, \&c. \text{ to } \frac{4}{n \cdot n+1 \cdot n+2};$$

$$\text{Th. } \frac{n+3 \times 3n}{2 \cdot n+1 \cdot n+2} = \frac{1}{2} + \frac{1}{4} + \frac{1}{15} + \frac{1}{20}, \&c. \text{ to } \frac{2 \cdot 3}{n \cdot n+1 \cdot n+2}.$$

"It is required to find the sum of the infinite series, $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$, &c.?"

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QUESTION CLXXVII.

The sum of n terms of the series, $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$, &c. is required?

By quest. 175. $\frac{3^n n + 3}{2 \cdot n + 1 \cdot n + 2} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$, &c. to n terms;

Th. $\frac{3^n n + 3}{2 \cdot n + 1 \cdot n + 2} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$, &c. to ditto;

Th. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$, &c. to ditto;

And $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$, &c. to ditto;

QUESTION CLXXVII. [Continued.]

COROL. I.

Since $\left\{ \begin{array}{l} \frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10}, \&c. = \frac{2}{1} \\ \frac{1}{1} + \frac{1}{4} + \frac{1}{10} + \frac{1}{20}, \&c. = \frac{3}{2} \\ \frac{1}{1} + \frac{1}{5} + \frac{1}{15} + \frac{1}{35}, \&c. = \frac{4}{3} \end{array} \right\}$ as before

Th. $\left\{ \begin{array}{l} \frac{1}{1} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20}, \&c. = \frac{5}{4} \\ \frac{1}{1} + \frac{1}{7} + \frac{1}{28} + \frac{1}{49}, \&c. = \frac{6}{5} \\ \frac{1}{1} + \frac{1}{8} + \frac{1}{36} + \frac{1}{80}, \&c. = \frac{7}{6} \end{array} \right\}$

&c. &c.

COROL. II.

Since n terms of $\left\{ \begin{array}{l} \frac{1}{1} + \frac{1}{3} + \frac{1}{6}, \&c. = \frac{2}{1} - \frac{2}{1} \times \frac{1}{n+1} \\ \frac{1}{1} + \frac{1}{4} + \frac{1}{10}, \&c. = \frac{3}{2} - \frac{3}{2} \times \frac{1 \cdot 2}{n+1 \cdot n+2} \\ \frac{1}{1} + \frac{1}{5} + \frac{1}{15}, \&c. = \frac{4}{3} - \frac{4}{3} \times \frac{1 \cdot 2 \cdot 3}{n+1 \cdot n+2 \cdot n+3} \end{array} \right.$

Therefore the manner of continuing these rules is also evident.

QUESTION CLXXVIII.

The sum of the infinite series, $\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \frac{1}{4 \cdot 5 \cdot 6} + \dots$ is required?

By equation 4, $\left. \begin{array}{l} \text{quest. 172.} \end{array} \right\} 1 = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \dots$

Then $1 - \frac{1}{2} = \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{5}{5 \cdot 6} + \dots$

Th. $\frac{1}{2} = \frac{6-2}{1 \cdot 4 \cdot 3} + \frac{12-6}{2 \cdot 9 \cdot 4} + \frac{20-12}{3 \cdot 16 \cdot 5} + \dots$

Or $\frac{1}{2} = \frac{4}{1 \cdot 4 \cdot 3} + \frac{6}{2 \cdot 9 \cdot 4} + \frac{8}{3 \cdot 16 \cdot 5} + \dots$

Or $\frac{1}{2} = \frac{2}{1 \cdot 2 \cdot 3} + \frac{2}{2 \cdot 3 \cdot 4} + \frac{2}{3 \cdot 4 \cdot 5} + \dots$

Th. $\left(\frac{1}{2} = \right) \frac{2}{2} = \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \dots$

QUES.

QUESTION CLXXIX.

The sum of n terms of the series $\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5}$, &c. is required?

If $x = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{1}{n \cdot n+1} + \frac{1}{n+1 \cdot n+2}$;

Then $x - \frac{1}{2} + \frac{1}{n+1 \cdot n+2} = \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{1}{n+1 \cdot n+2}$;

Th. $\frac{1}{2} - \frac{1}{n+1 \cdot n+2} = \frac{2}{1 \cdot 2 \cdot 3} + \frac{2}{2 \cdot 3 \cdot 4} + \frac{2}{3 \cdot 4 \cdot 5}$, &c. to n terms;

Th. $\frac{1}{2} - \frac{1}{2 \cdot n+1 \cdot n+2} = \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5}$, &c. to n terms.

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QUESTION CLXXX.

The sum of the infinite series $\frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \frac{1}{2 \cdot 3 \cdot 4 \cdot 5} + \frac{1}{3 \cdot 4 \cdot 5 \cdot 6}$, &c. is required?

$$\text{By q. 178. } \frac{1}{4} = \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5}, \text{ \&c.}$$

$$\text{Then } \frac{1}{3} - \frac{1}{6} = \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \frac{1}{4 \cdot 5 \cdot 6}, \text{ \&c.}$$

$$\text{Th. } \frac{1}{6} = \frac{24-6}{1 \cdot 4 \cdot 9 \cdot 4} + \frac{60-24}{2 \cdot 9 \cdot 16 \cdot 5} + \frac{120-60}{3 \cdot 16 \cdot 25 \cdot 6}, \text{ \&c.}$$

$$\text{That is, } \frac{1}{6} = \frac{18}{1 \cdot 4 \cdot 9 \cdot 4} + \frac{36}{2 \cdot 9 \cdot 16 \cdot 5} + \frac{60}{3 \cdot 16 \cdot 25 \cdot 6}, \text{ \&c.}$$

$$\text{Th. } \left(\frac{1}{1 \cdot 2 \cdot 3 \cdot 3} \right) \frac{1}{18} = \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \frac{1}{2 \cdot 3 \cdot 4 \cdot 5} + \frac{1}{3 \cdot 4 \cdot 5 \cdot 6} \text{ [\&c.]}$$

QUESTION CLXXXI.

The sum of n terms of the series, $\frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \frac{1}{2 \cdot 3 \cdot 4 \cdot 5} + \frac{1}{3 \cdot 4 \cdot 5 \cdot 6}$, &c. is required?

By reasoning as in the preceeding questions;

$$\left. \begin{aligned} \frac{1}{6} - \frac{1}{n+1 \cdot n+2 \cdot n+3} &= \frac{3}{1 \cdot 2 \cdot 3 \cdot 4} \\ \text{Th. } \frac{1}{18} - \frac{1}{3 \cdot n+1 \cdot n+2 \cdot n+3} &= \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} \end{aligned} \right\} \begin{array}{l} \text{\&c. to } n \\ \text{terms.} \end{array}$$

QUES.

QUESTION CLXXXI. [Continued.]

COROL. I.

Therefore	Since	
$\frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \frac{1}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7} + \text{&c.} = \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 5}$	$\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \frac{1}{4 \cdot 5 \cdot 6} + \text{&c.} = \frac{1}{1 \cdot 2 \cdot 3 \cdot 3}$	$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \text{&c.} = \frac{1}{1 \cdot 1}$
	$\frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \frac{1}{2 \cdot 3 \cdot 4 \cdot 5} + \frac{1}{3 \cdot 4 \cdot 5 \cdot 6} + \text{&c.} = \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 4}$	
		$\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \frac{1}{4 \cdot 5 \cdot 6} + \text{&c.} = \frac{1}{1 \cdot 2 \cdot 2}$
		$\frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \frac{1}{2 \cdot 3 \cdot 4 \cdot 5} + \frac{1}{3 \cdot 4 \cdot 5 \cdot 6} + \text{&c.} = \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 4}$
		$\frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} + \frac{1}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \text{&c.} = \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 5}$

as before ;

QUES.

QUESTION CLXXXI. [Continued.]

COROL. II.

Since n terms of

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3}, \text{ \&c.} = \frac{1}{1 \cdot 1} - \frac{1}{n+1}$$

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4}, \text{ \&c.} = \frac{1}{1 \cdot 2 \cdot 2} - \frac{1}{2 \cdot n+1 \cdot n+2}$$

$$\frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \frac{1}{2 \cdot 3 \cdot 4 \cdot 5}, \text{ \&c.} = \frac{1}{1 \cdot 2 \cdot 3 \cdot 3} - \frac{1}{3 \cdot n+1 \cdot n+2 \cdot n+3}$$

Therefore

$$\frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} + \frac{1}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6}, \text{ \&c.} = \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 4} - \frac{1}{4 \cdot n+1 \cdot n+2 \cdot n+3 \cdot n+4}$$

QUES.

QUESTION CLXXXII.

In the series of the reciprocals of the natural numbers, $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$, &c. infinitely continued; 'tis required to find what proportion the sum of the odd terms, has to the sum of the even terms?

$$\text{Now } \left\{ \begin{array}{l} \frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \\ \frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \frac{1}{24} \\ \frac{1}{5} + \frac{1}{10} + \frac{1}{20} + \frac{1}{40} \\ \frac{1}{7} + \frac{1}{14} + \frac{1}{28} + \frac{1}{56} \end{array} \right\} \text{ \&c. ad infinit. } \left\{ \begin{array}{l} = (\frac{1}{1} \times 2 =) \frac{2}{1}, \\ = (\frac{1}{3} \times 2 =) \frac{2}{3}, \\ = (\frac{1}{5} \times 2 =) \frac{2}{5}, \\ = (\frac{1}{7} \times 2 =) \frac{2}{7}, \end{array} \right\} \text{ by quest. 169.} \\ \text{\&c.}$$

$$\text{And (by transposition) } \left\{ \begin{array}{l} \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \\ \frac{1}{6} + \frac{1}{12} + \frac{1}{24} \\ \frac{1}{10} + \frac{1}{20} + \frac{1}{40} \\ \frac{1}{14} + \frac{1}{28} + \frac{1}{56} \end{array} \right\} \text{ \&c. ad infinit. } \left\{ \begin{array}{l} = (\frac{2}{2} - \frac{1}{2} =) \frac{1}{2}, \\ = (\frac{2}{3} - \frac{1}{3} =) \frac{1}{3}, \\ = (\frac{2}{5} - \frac{1}{5} =) \frac{1}{5}, \\ = (\frac{2}{7} - \frac{1}{7} =) \frac{1}{7}; \end{array} \right.$$

$$\text{Th. by addition } \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}, \text{ \&c.} \\ = \frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7}, \text{ \&c.}$$

QUES.

QUESTION CLXXXIII.

In the infinite series of the reciprocals of the square numbers, $\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16}$, &c. the proportion which the sum of the odd terms, has to the sum of the even terms, is required?

$$\text{Now } \left\{ \begin{array}{l} \frac{1}{1} + \frac{1}{4} + \frac{1}{16} + \frac{1}{64}, \text{ \&c.} = \frac{4}{3}, \\ \frac{1}{9} + \frac{1}{36} + \frac{1}{144} + \frac{1}{576}, \text{ \&c.} = \frac{4}{3 \times 9}, \\ \frac{1}{25} + \frac{1}{100} + \frac{1}{400} + \frac{1}{1600}, \text{ \&c.} = \frac{4}{3 \times 25}, \end{array} \right\} \text{ per quest. 169.}$$

$$\text{And by Addition } \left\{ \frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16}, \text{ \&c.} = \frac{4}{3} + \frac{4}{3 \times 9} + \frac{4}{3 \times 25}, \text{ \&c.} \right.$$

$$\text{Or } \frac{3}{1} + \frac{3}{4} + \frac{3}{9} + \frac{3}{16}, \text{ \&c.} = \frac{4}{1} + \frac{4}{9} + \frac{4}{25} + \frac{4}{49}, \text{ \&c.}$$

$$\text{Or } \frac{3}{4} + \frac{3}{16} + \frac{3}{36} + \frac{3}{64}, \text{ \&c.} = \frac{1}{1} + \frac{1}{9} + \frac{1}{25} + \frac{1}{49}, \text{ \&c.}$$

$$\text{Th. } \frac{1}{1} + \frac{1}{9} + \frac{1}{25} + \frac{1}{49}, \text{ \&c.} : \frac{1}{4} + \frac{1}{16} + \frac{1}{36} + \frac{1}{64}, \text{ \&c.} \\ [\therefore 3 : 1.]$$

QUES.

QUESTION CLXXXIV.

In the infinite series of the reciprocals of the cube numbers $\frac{1}{1} + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \frac{1}{125}$, &c. the proportion which the sum of the odd terms, has to the sum of the even terms, is required?

$$\text{Ans. } \left\{ \begin{array}{l} \frac{1}{1} + \frac{1}{8} + \frac{1}{64}, \&c. = \frac{8}{7} \\ \frac{1}{27} + \frac{1}{216} + \frac{1}{1728}, \&c. = \frac{8}{7 \times 27} \\ \frac{1}{125} + \frac{1}{1000} + \frac{1}{8000}, \&c. = \frac{8}{7 \times 125} \end{array} \right\} \text{by quest. 169.}$$

And by addition. $\frac{1}{1} + \frac{1}{8} + \frac{1}{27}, \&c. = \frac{8}{7} + \frac{8}{7 \times 27} + \frac{8}{7 \times 125}, \&c.$

Or $\frac{7}{1} + \frac{7}{8} + \frac{7}{27}, \&c. = \frac{8}{1} + \frac{8}{27} + \frac{8}{125}, \&c.$

Or $\frac{7}{8} + \frac{7}{64} + \frac{7}{216}, \&c. = \frac{1}{1} + \frac{1}{27} + \frac{1}{125}, \&c.$

Th. $\frac{1}{1} + \frac{1}{27} + \frac{1}{125}, \&c. : \frac{1}{8} + \frac{1}{64} + \frac{1}{216}, \&c.$

$[:: 7 : 1.]$

QUESTION CLXXXIV. [Continued.]

C O R O L

Since by the three preceding questions

$$\frac{1}{1} + \frac{1}{3} + \frac{1}{5}, \text{ \&c.} :: \frac{1}{2} + \frac{1}{4} + \frac{1}{6}, \text{ \&c.} :: 1 : 1$$

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2}, \text{ \&c.} :: \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2}, \text{ \&c.} :: 3 : 1$$

$$\frac{1}{1^3} + \frac{1}{3^3} + \frac{1}{5^3}, \text{ \&c.} :: \frac{1}{2^3} + \frac{1}{4^3} + \frac{1}{6^3}, \text{ \&c.} :: 7 : 1$$

Therefore

$$\frac{1}{1^n} + \frac{1}{3^n} + \frac{1}{5^n}, \text{ \&c.} :: \frac{1}{2^n} + \frac{1}{4^n} + \frac{1}{6^n}, \text{ \&c.} :: 2^n - 1 : 1$$

Q U E S.

QUESTION CLXXXV.

The sum (a) of the infinite series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$, &c. is required?

This is solv'd by quest. 169: but as it is an introductory question to the following class, the sum may be investigated as follows:

Let $\frac{x}{2} = x;$

Then $a = x + x^2 + x^3 + x^4 + x^5, \&c.$

Put $\frac{x}{1-x} = (a =) x + x^2 + x^3 + x^4, \&c.$

Then $x = 1 - x \times x + x^2 + x^3 + x^4, \&c.$

But $x = 1 - x \times x + x^2 + x^3 + x^4, \&c.$

See the work.

$$x + x^2 + x^3 + x^4 + x^5, \&c.$$

$$1 - x$$

$$x + x^2 + x^3 + x^4 + x^5, \&c.$$

$$-x^2 - x^3 - x^4 - x^5, \&c.$$

$$x + 0 + 0 + 0 + 0, \&c.$$

Th. $x = x,$

And $\frac{x}{1-x} = (a =) x + x^2 + x^3 + x^4, \&c.$

QUES.

QUESTION CLXXXVI.

The sum (a) of the infinite series, $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16}$, &c. is required?

Put $\frac{x}{1+x} = x$; And let $\frac{x}{1+x} = a$;

Then $\frac{x}{1+x} = x - x^2 + x^3 - x^4 + x^5$, &c.

And $x = \frac{x - x^2 + x^3 - x^4 + x^5}{1+x}$, &c.

But $a = \frac{x - x^2 + x^3 - x^4 + x^5}{1+x}$, &c.

See the work.

$$\begin{array}{r} x - x^2 + x^3 - x^4 + x^5 - x^6, \text{ \&c.} \\ \hline 1 + x \\ \hline x - x^2 + x^3 - x^4 + x^5 - x^6, \text{ \&c.} \\ + x^2 - x^3 + x^4 - x^5 + x^6, \text{ \&c.} \\ \hline x + 0 + 0 + 0 + 0 + 0, \text{ \&c.} \end{array}$$

Th. $\therefore \frac{x}{1+x} = x$

And $\therefore \frac{x}{1+x} = (a) x - x^2 + x^3 - x^4 + x^5$, &c.

In this Example $\left\{ \frac{\frac{1}{2}}{1+\frac{1}{2}} = \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32}, \text{ \&c.} \right.$

That is $\left\{ \frac{1}{2} \right\} \frac{1}{2} \left(\frac{1}{2} = \frac{1}{2} = \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32}, \text{ \&c.} \right.$

QUESTION CLXXXVII.

The sum (*b*) of the infinite series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$, &c. is required?

Put $\frac{1}{2} = x$; And let $\frac{x}{1-x} = b$;

Then $\frac{x}{1-x} = x + 2x^2 + 3x^3 + 4x^4 + 5x^5$, &c.

And $x = \frac{1}{1-x} \times x + 2x^2 + 3x^3 + 4x^4$, &c.

But $x = \frac{1}{1-x} \times x + 2x^2 + 3x^3 + 4x^4$, &c.

[as will appear by actually multiplying.

Th. $\frac{x}{1-x} = x + 2x^2 + 3x^3 + 4x^4 + 5x^5$, &c.

And $\frac{x}{1-x} = (b =) x + 2x^2 + 3x^3 + 4x^4$, &c.

In this exam. $\frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$, &c.

That is $\frac{1}{2} \cdot \frac{1}{\frac{1}{2}} (\frac{1}{2} = 2 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$, &c.

Corol. $\frac{x}{1+x} = (b =) x - 2x^2 + 3x^3 - 4x^4$, &c.

That is $\frac{1}{2} \cdot \frac{1}{\frac{3}{2}} (\frac{1}{3} = \frac{2}{3} = \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32}$, &c.

This will appear by a similar process.

QUES.

Q U E

QUESTION CLXXXVIII.

'Tis required to find the sum (c) of the infinite series,
 $\frac{a}{m} + \frac{a+d}{mr} + \frac{a+2d}{mr^2} + \frac{a+3d}{mr^3} + \frac{a+4d}{mr^4}$, &c. arising by
 dividing the terms of an arithmetical, by those of a geo-
 metrical progression?

Let $\frac{1}{r} = x$; And $\frac{a}{m \times 1 - x} = c$

Then $\frac{x}{m \times 1 - x} = \frac{a}{m} + \frac{a+d}{mr} + \frac{a+2d}{mr^2}$, &c.

Or $\frac{x}{1 - x} = a + \frac{a+d}{r} + \frac{a+2d}{r^2}$, &c.

That is $\frac{x}{1 - x} = a + a \times x + d \times x + a + 2d \times x^2$, &c.

Th. $\therefore \frac{x}{1 - x} = \frac{1}{1 - x} \times \{a + a \times x + d \times x + a + 2d \times x^2\}$, &c.

But $\frac{1}{1 - x} \times a + d \times x$
 $= \frac{1}{1 - x} \times a + a \times x + d \times x + a + 2d \times x^2$, &c.

Which will appear by performing the Operation.

QUESTION CLXXXVIII. [Continued.]

Th. $z = a \times 1 - x + dx,$

And

$$\frac{a \times 1 - x + dx}{m \times 1 - x^2} = (c =) \frac{a}{m} + \frac{a+d}{mr} + \frac{a+2d}{mr^2}, \text{ \&c.}$$

COROL.

$$\frac{a \times 1 + x - dx}{m \times 1 + x^2} = (c =) \frac{a}{m} - \frac{a+d}{mr} + \frac{a+2d}{mr^2}, \text{ \&c.}$$

This will be obtained by a similar process.

Now if $\frac{1}{r}$ be wrote for x ; Then

$$c = \frac{a \times 1 - \frac{1}{r} + \frac{d}{r}}{m \times 1 - \frac{2}{r} + \frac{1}{rr}} = \frac{a \times r - 1 + d}{m \times r - 1^2}$$

$$t = \frac{a \times 1 + \frac{1}{r} - \frac{d}{r}}{m \times 1 + \frac{2}{r} - \frac{1}{rr}} = \frac{a \times r + 1 - d}{m \times r + 1^2}$$

QUES.

QUESTION CLXXXVIII.

The sum (p) of n terms of the series $\frac{a}{m} + \frac{a+d}{mr} + \frac{a+2d}{mr^2} + \frac{a+3d}{mr^3}$, &c. is required?

The $n+1^{\text{th}}$ term of this series is $\frac{a+nd}{mr^n}$;

And the series after the n^{th} term will be $\frac{a+nd}{mr^n} + \frac{a+n+1 \times d}{mr^{n+1}}$, &c.

Th. putting $a+nd$ for a ; and mr^n for m in the result of the last question,

The sum of all the terms after the n^{th} will be $\frac{a+nd \times 1 - x + dx}{mr^n \times 1 - x}$;

But the sum of n terms

$$p = \frac{a \times 1 - x + dx}{m \times 1 - x} - \frac{a+nd \times 1 - x + dx}{mr^n \times 1 - x}$$

$$\text{Or } p = \frac{r^n - 1 \times a - nd \times r - 1 + r^n - 1 \times d}{mr^n \times r - 1} r$$

QUES.

QUESTION CXCI.

'Tis required to find the sum (s) of the infinite series,
 $\frac{a}{m} + \frac{a+D}{mr} + \frac{a+2D+d}{mr^2} + \frac{a+3D+3d}{mr^3}, \&c.$ arising by dividing the terms of a series whose second differences are equal, by those of a geometrical progression?

Let $\frac{1}{r} = x$; And $\frac{z}{m \times 1 - x^3} = s$;

Then $\frac{z}{m \times 1 - x^3} = \frac{a}{m} + \frac{a+D}{mr} + \frac{a+2D+d}{mr^2}, \&c.$

Or $\frac{z}{1-x^3} = a + a+D \times x + a+2D+d \times xx, \&c.$

Th. $z = \frac{1}{1-x^3} \times \left\{ \begin{array}{l} a + a+D \times x + \\ a+2D+d \times x^2, \&c. \end{array} \right.$

See the operation:

$$\begin{array}{r} a + a+D \times x + a+2D+d \times xx, \&c. \\ 1 - \quad \quad \quad 3x + \quad \quad \quad 3x^2 \quad \quad - x^3 \\ \hline a + a+D \times x + a+2D+d \times x^2, \&c. \\ - 3a \quad \quad \quad \times x - 3a+3D \quad \quad \times x^2, \&c. \\ \quad \quad \quad + 3a \quad \quad \quad \times x^2, \&c. \\ \hline a - 2ax + Dx + ax^2 - Dx^2 + dx^2; \quad + 0 \\ \text{Th.} \quad z = a \times \frac{1-2x+x^2}{1-x^3} + D \times \frac{x-x^2}{1-x^3} + dx^2, \\ \text{Or} \quad z = a \times \frac{1-x^2}{1-x^3} + D \times \frac{1-xx}{1-x^3} + dx^2; \end{array}$$

QUES.

QUESTION CXc. [Continued.]

$$\text{Th. } \frac{a \times \overline{1-x}^2 + Dx \times \overline{1-x} + dx^2}{m \times \overline{1-x}^3}$$

$$= (e =) \frac{a}{m} + \frac{a+D}{mr} + \frac{a+2D+d}{mrr}, \text{ \&c.}$$

$$\text{Cor. } \frac{a \times \overline{1+x}^2 - Dx \times \overline{1+x} + dx^2}{m \times \overline{1+x}^3}$$

$$= (e =) \frac{a}{m} - \frac{a+D}{mr} + \frac{a+2D+d}{mrr}, \text{ \&c.}$$

If $\frac{1}{r}$ be wrote for x ; Then

$$e = \frac{a \times \overline{1-\frac{2}{r}} + \frac{1}{rr} + \frac{D}{r} \times \overline{1-\frac{1}{r}} + \frac{d}{rr}}{m \times \overline{1-\frac{3}{r}} + \frac{3}{rr} - \frac{1}{r^3}}$$

$$= \frac{a \times \overline{r-1}^2 + D \times \overline{r-1} + d}{m \times \overline{r-1}^3} r; \text{ and}$$

$$e = \frac{a \times \overline{r+1}^2 - D \times \overline{r+1} + d}{m \times \overline{r+1}^3} r$$

Q

QUES-

QUESTION CXCL.

The sum (q) of n terms of the series $\frac{a}{m} + \frac{a+D}{mr} + \frac{a+2D+d}{mrr} + \frac{a+3D+3d}{mr^2} + \dots$ is required?

The $n+1^{\text{th}}$ term of this series is $\frac{a+nD+\frac{1}{2}n \cdot n-1 \cdot d}{mr^n}$,

The $n+2^{\text{th}}$ term is $\frac{a+n+1 \cdot D + \frac{1}{2}n+1 \cdot n \cdot d}{mr^{n+1}}$;

Th. (putting $\left\{ \begin{array}{l} a+vD+\frac{1}{2}n \cdot n-1 \cdot d, \text{ for } a, \\ D, \text{ for } D, \\ mr^n, \text{ for } m, \end{array} \right\}$ in the result of the last quest.)

QUESTION CXCI. [Continued.]

The sum of all the series after the n^{th} term $\sum \frac{a+nD+\frac{1}{2}n \cdot n-1 \cdot d \times 1-x + D+nd \times 1-x \times x+dx^2}{m^n \times 1-x}$

And $q = \frac{a \times 1-x^2 + D \times x \times 1-x + dx^2}{m \times 1-x^3}$

$\frac{a+nD+\frac{1}{2}n \cdot n-1 \cdot d \times 1-x^2 + D+nd \times x \times 1-x+dx^2}{m^n \times 1-x}$

Or $q = \frac{r^n-1 \times a-nD-\frac{1}{2}n \cdot n-d \times 1-x + r^n-1 \times D+nd \times x \times 1-x+r^n-1 \times dx^2}{m^n \times 1-x}$

If $\frac{1}{r}$ be wrote for x Then,

$q = \frac{r^n-1 \times a-nD-\frac{1}{2}n \cdot n-1 \cdot d \times r-1 + r^n-1 \times D+nd \times r-1+r^n-1 \times d}{m^n \times r-1}$

QUES.

QUESTION CXXII.

The sum of the infinite series $\frac{a}{m} + \frac{a+D}{mr} + \frac{a+2D+\Delta}{mr^2} + \frac{a+3D+3\Delta+d}{mr^3}$

$\frac{a+4D+6\Delta+4d}{mr^4}$, &c. arising from the division of the terms of a series whose third

differences are equal, by those of a geometrical progression, is required?

Now if $\frac{1}{r} = x$; And $\frac{z}{m \times 1-x} = f$;

Then $\frac{z}{m \times 1-x} = \frac{a}{m} + \frac{a+D}{mr} + \frac{a+2D+\Delta}{mr^2}$, &c.

Or $\frac{z}{1-x} = a + a+D \times x + a+2D+\Delta \times x^2$, &c.

Th. $z = 1-x \times a + a+D \times x + a+2D+\Delta \times x^2 + a+3D+3\Delta+d \times x^3$, &c.

But $a-3ax+Dx+3ax^2-2Dx^2+\Delta x^3-ax^3+Dx^3-\Delta x^3+dx^3$, is the product of that multipⁿ.

Th.

QUESTION CXCH. [Continued.]

$$\text{Th. } z = a \times 1 - x^3 + Dx \times 1 - x^3 + \Delta x^2 \times 1 - x + dx^3 ;$$

And writing $\frac{1}{r}$ for x ,

$$f = \frac{a}{m} + \frac{a+D}{mr} + \frac{a+2D+\Delta}{mr^2} + \frac{a+3D+3\Delta+d}{mr^3}, \text{ \&c.}$$

$$= \frac{a \times r - 1^3 + D \times r - 1^2 + \Delta \times r - 1 + d}{m \times r - 1^4}$$

$$\text{Also } f = \frac{a}{m} - \frac{a+D}{mr} + \frac{a+2D+\Delta}{mr^2} - \frac{a+3D+3\Delta+d}{mr^3}, \text{ \&c.}$$

$$= \frac{a \times r + 1^3 - D \times r + 1^2 + \Delta \times r + 1 - d}{m \times r + 1^4}$$

Q 3

COROL

COROL.

From the solutions of the preceding questions it will follow, that the sum of an infinite series of fractions, the numerators of which are a rank of quantities whose p^{th} differences are equal, and their denominators the terms of a geometrical progression (putting

a = the numerator of the first term ; m = the denominator of the first term ;

d' = the first of the first differences ; d'' = the first of the second differences, &c.

d^p = the p^{th} difference ; and r = the ratio of the geom. progr.) will be

$$\frac{a \times r - 1}{d' \times r - 1} + \frac{d' \times r - 1}{d'' \times r - 1} + \frac{d'' \times r - 1}{d^p \times r - 1} + \dots$$

QUES.

QUESTION CXIII.

The sum (*s*) of *n* terms of the series $\frac{a}{m} + \frac{a+D}{mr} + \frac{a+2D+\Delta}{mr^2}$

+ $\frac{a+3D+3\Delta+d}{mr^3}$, &c. is required?

By using a process similar to that in quest. 191; it will appear that

$$s = \frac{1}{mr^n \times 1-x} \times \left\{ \begin{aligned} & \frac{r^3-1 \times a-nD-\frac{1}{2}n \cdot n-1 \cdot \Delta-\frac{1}{6}n \cdot n-1 \cdot n-2 \cdot d \times 1-x}{1+r^3-1 \times D-n\Delta-\frac{1}{2}n \cdot n-1 \cdot d \times x \times 1-x} \\ & + \frac{1+r^3-1 \times \Delta-n d \times x^2 \times 1-x + r^3-1 \times dx^3}{1+r^3-1 \times a-nD-\frac{1}{2}n \cdot n-1 \cdot \Delta-\frac{1}{6}n \cdot n-1 \cdot n-2 \cdot d \times r-1} \\ & + \frac{1+r^3-1 \times D-n\Delta-\frac{1}{2}n \cdot n-1 \cdot d \times r-1}{1+r^3-1 \times \Delta-n d \times r-1 + r^3-1 \times d} \end{aligned} \right\}$$

Q 4

COROL.

COROL.

Hence the sum of n terms of a series of fractions, the numerators of which, are a rank of quantities whose p^{th} differences are equal; and their denominators the terms of a geometrical progression; (the symbols being as in Corol. Quest. 192.) will be

$$\begin{aligned}
 & \frac{1}{r^{n-1} \times r - 1} + 1 \\
 & \times \left\{ \begin{aligned} & \frac{r-1}{r-1} \times \frac{r^2-1}{r^2-1} \times a - n d' - \frac{n \cdot n - 1}{1 \cdot 2} d'' - \frac{n \cdot n - 1 \cdot n - 2}{1 \cdot 2 \cdot 3} d''' , \text{ \&c.} + \\ & \frac{r-1}{r-1} \times \frac{r^3-1}{r^3-1} \times d - n d'' - \frac{n \cdot n - 1}{1 \cdot 2} d''' - \frac{n \cdot n - 1 \cdot n - 2}{1 \cdot 2 \cdot 3} d^{iv} , \text{ \&c.} + \\ & \frac{r-1}{r-1} \times \frac{r^4-1}{r^4-1} \times d' - n d'' - \frac{n \cdot n - 1}{1 \cdot 2} d''' - \frac{n \cdot n - 1 \cdot n - 2}{1 \cdot 2 \cdot 3} d^{iv} , \text{ \&c.} + \\ & \text{\&c. to} \\ & 1 \times \frac{r^n-1}{r^n-1} \times d^p . \end{aligned} \right.
 \end{aligned}$$

QUES.

QUESTION CXCV.

'Tis required to find a series (1) which shall have the same relation to $\frac{1}{1-n}$; that 2^d hath to $\left(\frac{1}{1-n}\right)^2$; that is to find a series expressing the logarithm of $\frac{1}{1-n}$?

Since by qu. 185, $\frac{1}{1-n} = 1+n+n^2+n^3, \&c. \text{ ad infin.}$

Let $L. \frac{1}{1-n} = xn+yn^2+zn^3, \&c.$

Then $L. \left(\frac{1}{1-n}\right)^2 = 2xn+2yn^2+2zn^3, \&c. \text{ by q.}$

But $L. \left(\frac{1}{1-n}\right)^2 = \left(\frac{1}{1-2n+nn}\right) = \frac{1}{1-2n+nn};$

And

$$L. \frac{1}{1-2n+nn} = x \times \frac{1}{2n+nn} + y \times \frac{1}{2n+nn}^2 + z \times \frac{1}{2n+nn}^3, \&c.$$

By the above assumption; writing therein $\frac{1}{2n+nn}$ for n :

Q 5

Now

QUESTION CXCV. [Continued.]

Now $2n - nn \times x = 2xn - xnn;$

$2n - nn^2 \times y = +4ynn - 4yn^3 + yn^5;$

$2n - nn^3 \times z = +8xn^3 - 12zn^4 + 6zn^5;$

$2n - nn^4 \times u = +16un^4 + 32un^5$

Th. L. $\frac{1}{1-n} = 2xn - xn^2 - 4yn^3 + yn^4 + 6zn^5, \&c.$

$+4y + 8z - 12z - 32u \&c.$

$+16u + 32e \&c.$

Now by COROL. QUEST. 251, PART I.

$2x = 2x;$

$2y = 4y - y;$

$2z = 8z - 4y;$

$2u = 16u - 12z + y;$

$2e = 32e - 32u + 6z$

$\&c. \quad \&c.$

x may be assumed;

$x = 2y;$

$4y = 6z;$

$12z = 14u + y;$

$32u = 30e + 6z;$

$\&c. \quad \&c.$

$\therefore \begin{cases} y = \frac{1}{2}x; \\ 2x = 6z; \\ 4x = 14u + \frac{1}{2}x; \\ 8x = 30e + 2x; \\ \&c. \quad \&c. \end{cases}$

Whence $\begin{cases} \frac{1}{3}x = z; \\ \frac{1}{4}x = u; \\ \frac{1}{5}x = e; \\ \&c. \end{cases}$

Th. L. $\frac{1}{1-n} = xn + \frac{x}{2}n^2 + \frac{x}{3}n^3 + \frac{x}{4}n^4 + \frac{x}{5}n^5, \&c.$

COROL. By a similar process, the logarithm of

$1+n = xn - \frac{x}{2}n^2 + \frac{x}{3}n^3 - \frac{x}{4}n^4 + \frac{x}{5}n^5, \&c.$

QUES.

QUESTION CXCV.

A series expressing the logarithm of any number (m) is required?

Let $m = \frac{1+n}{1-n}$; Then $m - nm = 1 + n$;

Or $m - 1 = mn + n$; Th. $\frac{m-1}{m+1} = n$.

Now $L. \frac{1}{1-n} = xn + \frac{x}{2}n^2 + \frac{x}{3}n^3 + \frac{x}{4}n^4 + \frac{x}{5}n^5$, &c.

And $L. \frac{1+n}{1-n} = xn + \frac{x}{2}n^2 + \frac{x}{3}n^3 - \frac{x}{4}n^4 + \frac{x}{5}n^5$, &c.

But the sum of two logarithms, is the logarithm of the product of their corresponding numbers:

Th. $L. \frac{1+n}{1-n} = 2xn + \frac{2x}{3}n^3 + \frac{2x}{5}n^5$, &c.

And $L. m = 2x \times \frac{\frac{m-1}{m+1} + \frac{1}{3} \times \frac{\frac{m-1}{m+1}}{3}}{\frac{m-1}{m+1}} + \frac{1}{3} \times \frac{\frac{m-1}{m+1}}{5}$ &c.

QUESTION CXCVI.

The logarithms of two numbers, a and b , whose difference is 2, being given; to find the logarithm of $(\frac{1}{2} \times \overline{a+b})$, the intermediate number?

$$\text{Now } \overline{\frac{1}{2} \times a + b} = \frac{1}{4}aa + \frac{1}{2}ab + \frac{1}{4}bb$$

$$\text{And } \overline{\frac{1}{2} \times a + b} - ab = (\frac{1}{4}aa - \frac{1}{2}ab + \frac{1}{4}bb) = \overline{\frac{1}{2} \times a - b}$$

$$\text{But } \frac{1}{2} \times a - b = 1, \text{ by quest.}$$

$$\text{Th. } \overline{\frac{1}{2} \times a + b} - ab = 1.$$

By writing $\frac{k}{p}$ for m in quest. 196.

$$\text{L. } \frac{k}{p} = 2x \times \frac{k-p}{k+p} + \frac{1}{3} \times \frac{k-p}{k+p}^3 \&c.$$

$$\text{Th. L. } \frac{\overline{\frac{1}{2} \times a + b}}{ab} \text{ (putting } \overline{\frac{1}{2} \times a + b} + ab = d \text{ will be)}$$

$$= 2x \times \frac{1}{d} + \frac{1}{3d^3} + \frac{1}{5d^5}, \&c.$$

Now $\frac{1}{2}$ the logarithm of any square number, is the logarithm of its root;

$$\text{Th. L. } \frac{\overline{\frac{1}{2} \times a + b}}{\sqrt{ab}} = x \times \frac{1}{d} + \frac{1}{3d^3} + \frac{1}{5d^5}, \&c.$$

But

QUESTION CXCVI. [Continued.]

$$\text{But } L. \frac{a+b}{2} = L. \sqrt{ab} + x \times \frac{1}{d} + \frac{1}{3d^3}, \text{ \&c.}$$

$$\text{And } L. \sqrt{ab} = \frac{A+B}{2}; \text{ (where } A=L.a; \text{ and } B=L.b):$$

$$\text{Th. } L. \frac{a+b}{2} = \frac{A+B}{2} + x \times \frac{1}{d} + \frac{1}{3d^3}, \text{ \&c.}$$

EXAMPLE I.

What is the logarithm of the number 3?

$$\text{Here } a=4; \quad A=1,386294356;$$

$$\text{And } b=2; \quad B=0,693147178;$$

$$\text{Th. } a+b=6; \quad A+B=2,079441534;$$

$$\text{Th. } \frac{1}{2} \times a+b=3; \quad \frac{1}{2} \times A+B=1,039720767;$$

$$\text{And } \frac{1}{2} \times a+b=9;$$

$$\text{Also } (\frac{1}{2} \times a+b)^2 + ab = (9+8) \quad 17=d;$$

$$\text{Let } \frac{1}{17} = x; \text{ Then,}$$

$$x=$$

QUESTION CXCVI. [Continued.]

$$n = 0,058823529, \frac{1}{2} \text{ of which } 0,058823529;$$

$$n^2 = 0,000203542, \frac{1}{2} \text{ of which } 0,000067847;$$

$$n^3 = 0,000000704, \frac{1}{2} \text{ of which } 0,000000141;$$

$$\text{Th. the logarithm of 3 is } 1,098612284.$$

EXAMPLE II.

What is the logarithm of the number 5?

$$\text{Here } a = 6; \text{ And } A = 1,791759462;$$

$$\text{And } b = 4; \text{ And } B = 1,386294356;$$

$$\text{Th. } a + b = 10; \text{ And } A + B = 3,178053818;$$

$$\text{And } \frac{1}{2} \times a + b = 5; \frac{1}{2} \times A + B = 1,589026909;$$

$$\text{Now } \frac{1}{2} \times a + b = 25;$$

$$\text{And } (\frac{1}{2} \times a + b)^2 + ab = 25 + 24 = 49 = d;$$

Let $n = \frac{1}{49}$. Then,

$$n = 0,020408163, \frac{1}{2} \text{ of which } 0,020408163;$$

$$n^2 = 0,000008500, \frac{1}{2} \text{ of which } 0,000002833;$$

$$n^3 = 0,000000004, \frac{1}{2} \text{ of which } 0,000000001;$$

$$\text{Th. the logarithm of 5 is } 1,609437906.$$

COROL.

Now if the before found Logs. of 2, 3, and 5, be multiplied by 0.43429448, &c. *Briggs's Log.* of those numbers will be produced.

EXAMPLE III.

What is *Briggs's* logarithm of the number 7?

Here $a = 8$; $A = 0.90308997$;

And $b = 6$; $B = 0.77815125$;

Th. $a + b = 14$; $A + B = 1.68124122$;

And $\frac{1}{2} \times a + b = 7$; $\frac{1}{2} \times A + B = 0.84062061$;

Also $\frac{1}{2} \times a + b = 49$;

Th. $(\frac{1}{2} \times a + b + ab = 49 + 48 =) 97 = d$;

Let $\frac{1}{97} = n$;

Now $x = 0.434294482$,

And $n \times x = 0.004477263$, $\frac{1}{2}$ of which 0.00447726 ;

$n^2 \times x = 0.000000476$, $\frac{1}{2}$ of which 0.00000016 ;

Th. the logarithm of 7 is 0.84509803 .

COROL.

COROL. II.

From due observation of the 3 last examples; the following rule, for finding *Briggs's* logarithm to 7 places of figures, of any prime number (p) greater than 7, will be easily deduced:

$$L.p = \frac{L.p+1 + L.p-1}{2} + \frac{0.43429448}{pp+pp-1};$$

Which may be of use in examining the common tables of logarithms.

EXAMPLE

What is *Briggs's* logarithm of 11?

$$11 \times 11 = 121,$$

$$L. 12 = 1,07918124,$$

$$11 \times 11 - 1 = 120;$$

$$L. 10 = 1,00000000,$$

$$\underline{241}$$

$$2) 2,07918124:$$

$$\underline{1,03959062:}$$

$$\text{And } 241) 0,43429448 (0,00180205 = 0,00180205;$$

$$\underline{241}$$

$$L. 11 = 1,04139267.$$

$$\underline{1932}$$

$$\underline{1928}$$

$$\underline{494}$$

$$\underline{482}$$

$$\underline{1248}$$

Q U E

QUESTION CXCVII.

The sum (s) of n terms of the series, $\frac{1}{a} + \frac{1}{a-d} + \frac{1}{a-2d} + \frac{1}{a-3d}, \&c.$ is required.

Now $\frac{1}{a} = \frac{1}{a} + \frac{0}{aa} + \frac{0}{a^3} + \frac{0}{a^4}, \&c.$

$\frac{1}{a-d} = \frac{1}{a} + \frac{d}{aa} + \frac{d^2}{a^3} + \frac{d^3}{a^4}, \&c.$

$\frac{1}{a-2d} = \frac{1}{a} + \frac{2d}{aa} + \frac{4d^2}{a^3} + \frac{8d^3}{a^4}, \&c.$

$\frac{1}{a-3d} = \frac{1}{a} + \frac{3d}{aa} + \frac{9d^2}{a^3} + \frac{27d^3}{a^4}, \&c.$

Th. $s = n$ terms of $\left\{ \begin{array}{l} 1+1+1+1, \&c. \times \frac{1}{a} + \\ 0+1+2+3, \&c. \times \frac{1}{a} \times \frac{d}{a} + \\ 0+1+4+9, \&c. \times \frac{1}{a} \times \frac{d^2}{a^2} + \\ 0+1+8+27, \&c. \times \frac{1}{a} \times \frac{d^3}{a^3} + \\ 0+1+16+81, \&c. \times \frac{1}{a} \times \frac{d^4}{a^4}, \&c. \end{array} \right.$

But

QUESTION XCIV. [Continued.]

But by quest. 49. n terms of

$$1+1+1+1, \&c. = n,$$

$$0+1+2+3, \&c. = \frac{n^2}{2} - \frac{n}{2},$$

$$0+1+4+9, \&c. = \frac{n^3}{3} - \frac{nn}{2} + \frac{n}{6},$$

$$0+1+8+27, \&c. = \frac{n^4}{4} - \frac{n^3}{2} + \frac{nn}{4} - \frac{1}{4},$$

$$0+1+16+81, \&c. = \frac{n^5}{5} - \frac{n^4}{2} + \frac{n^3}{3} - \frac{nn}{2},$$

$\&c.$

$\&c.$

Th. (putting $\frac{d}{a} = q$)

$$\begin{aligned} & \left\{ \begin{aligned} & \frac{n^2 q}{2} - \frac{n q}{2} + \\ & \frac{n^3 q^2}{3} - \frac{n^2 q^2}{2} + \frac{n q^2}{6} + \\ & \frac{n^4 q^3}{4} - \frac{n^3 q^3}{2} + \frac{n^2 q^3}{3} - \\ & \frac{n^5 q^4}{5} - \frac{n^4 q^4}{2} + \frac{n^3 q^4}{3} - \frac{n q^4}{30}, \&c. \end{aligned} \right. \end{aligned}$$

Or

[QUESTION CXC VII. [Continued.]

$$\begin{array}{r}
 \left\{ \begin{array}{l}
 n + \frac{n^2 q}{2} + \frac{n^3 q^2}{3} + \frac{n^4 q^3}{4} + \frac{n^5 q^4}{5}, \&c. \\
 - \frac{nq}{2} - \frac{n^2 q^2}{2} - \frac{n^3 q^3}{2} - \frac{n^4 q^4}{2}, \&c. \\
 + \frac{nq^2}{6} + \frac{n^2 q^3}{4} + \frac{n^3 q^4}{3} + \frac{5n^4 q^5}{12}, \&c. \\
 - \frac{nq^4}{30} - \frac{n^2 q^5}{12} - \frac{n^3 q^6}{6} - \frac{7n^4 q^7}{24}, \&c. \\
 + \frac{nq^6}{42} + \frac{n^2 q^7}{12} + \frac{2n^3 q^8}{9} + \frac{n^4 q^9}{2}, \&c.
 \end{array} \right.
 \end{array}$$

$$\begin{array}{l}
 \left\{ \begin{array}{l}
 nq + \frac{n^2 q^2}{2} + \frac{n^3 q^3}{3}, \&c. = L. \frac{1}{1-nq}; \\
 nq + n^2 q^2 + n^3 q^3, \&c. = \frac{1}{1-nq} - 1; \\
 2nq + 3n^2 q^2 + 4n^3 q^3, \&c. = \frac{1}{1-nq^2} - 1; \\
 4nq + 10n^2 q^2 + 20n^3 q^3, \&c. = \frac{1}{1-nq^4} - 1;
 \end{array} \right.
 \end{array}$$

The

QUESTION CXCVII. [Continued.]

$$\text{Th. } \left\{ \begin{array}{l} 1 + \frac{n^2 q}{2} + \frac{n^3 q^2}{3}, \text{ \&c.} = \frac{1}{1-nq} \times L. \frac{1}{1-nq}; \\ \frac{nq}{2} + \frac{n^2 q^2}{2} + \frac{n^3 q^3}{2}, \text{ \&c.} = \frac{1}{1-nq} - 1 \times \frac{1}{2}; \\ \frac{nq^2}{6} + \frac{n^2 q^3}{4} + \frac{n^3 q^4}{3}, \text{ \&c.} = \frac{1}{1-nq} - 1 \times \frac{q}{12}; \\ \frac{nq^3}{30} + \frac{n^2 q^5}{12} + \frac{n^3 q^6}{6}, \text{ \&c.} = \frac{1}{1-nq} - 1 \times \frac{q^3}{120}; \end{array} \right.$$

$$\text{Th. } \left\{ \begin{array}{l} \frac{1}{q} \times L. \frac{1}{1-nq} - \frac{1}{1-nq} - 1 \times \frac{1}{2} + \frac{1}{1-nq^2} - 1 \times \frac{q}{12} \\ - \frac{1}{1-nq^4} - 1 \times \frac{q^3}{120} + \dots \end{array} \right.$$

Or by writing $\frac{d}{a}$ for q

$$\text{Th. } \left\{ \begin{array}{l} \frac{a}{d} \times L. \frac{a}{a-nd} - \frac{a}{a-nd} - 1 \times \frac{1}{2} + \frac{a}{a-nd^2} - 1 \times \\ \frac{d}{12a} - \frac{a}{a-nd^4} - 1 \times \frac{d^3}{120a^3}, \text{ \&c.} \end{array} \right.$$

Lastly,

QUESTION CXCVII. [Continued.]

Lastly, putting $\alpha = 6$; $\beta = 30$; $\gamma = 42$, &c. that is
 α, β, γ , &c. = the denominators of those fractions
 denoted by α, β, γ , &c. in quest. 49; and $\mathcal{Q} =$
 $\frac{\alpha}{\alpha - nd}$: Then,

$$s = \frac{L. \mathcal{Q}}{d} - \frac{\mathcal{Q}-1}{2\alpha} + \frac{\mathcal{Q}^2-1 \times d}{2\alpha\alpha\alpha} - \frac{\mathcal{Q}^3-1 \times d^3}{4\beta\alpha^4}, \text{ \&c.}$$

Where the *Neperian* (not the common) logarithm of $\mathcal{Q}$
 must be used in the first term.

COROL. I.

By a similar process, the sum ($\mathfrak{S}$) of n terms of the
 series $\frac{1}{e} + \frac{1}{e+d} + \frac{1}{e+2d}$, &c. may be found, viz.

(putting $\frac{e}{e+nd} = \mathcal{Q}$)

$$\mathfrak{S} = \left\{ \begin{array}{l} \frac{1}{d} \times L. \frac{1}{\mathcal{Q}} + \frac{1-\mathcal{Q}}{2e} + \frac{1-\mathcal{Q}^2 \times d}{2\alpha e e} - \\ \frac{1-\mathcal{Q}^4 \times d^3}{4\beta e^4} + \frac{1-\mathcal{Q}^6 \times d^5}{6\gamma e^6} - \frac{1-\mathcal{Q}^8 \times d^7}{8\delta e^8} \end{array} \right.$$

SCHOL

QUESTION CXCVII. [Continued]

SCHOLIUM.

The values of $\frac{1}{1}$, $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{5}{5}$, $\frac{6}{6}$, $\frac{7}{7}$, $\frac{8}{8}$, $\frac{9}{9}$, being severally found by division; the quotients added together, make 2,82896825 = x :

Then the sum (52) of n terms of the series $\frac{1}{1} + \frac{2}{2} + \frac{3}{3} + \frac{4}{4} + \frac{5}{5}$, &c. will be equal to

$$x + L. \frac{10+n}{10} + \frac{10+n-10}{2 \cdot 10+n \cdot 10} + \frac{10+n-10^2}{2 \cdot 10+n \cdot 10^2} - \frac{10+n-10^3}{4 \cdot 10+n \cdot 10^3} + \frac{10+n-10^4}{6 \cdot 10+n \cdot 10^4}, \&c.$$

Now when n is very great, in respect of 10;

$$\text{Then } \left(\frac{10+n-10}{10+n} \right) = \frac{n}{10+n} = 1, \text{ nearly,}$$

$$\text{And } 5 = x + L. \frac{10+n}{10} + \frac{1}{20} + \frac{1}{2 \cdot 10^2} - \frac{1}{4 \cdot 10^3}, \&c.$$

$$\text{But } x_0 + \frac{1}{2a_{10}} - \frac{1}{4b_{10}}, \quad \&c. = 0,05083190;$$

$$\text{Th. } x + \frac{1}{2a_{10}} + \frac{1}{4b_{10}}, \quad \&c. = 2,87980075;$$

$$\text{And } S = 2,87980075 + \text{Nep. L. } \frac{10+n}{10},$$

$$\text{Or } S = 2,87980075 + 2,3028509 \times \text{L. } \frac{10+n}{10};$$

$$\text{Now when } n = \begin{cases} 999999990 \\ 999999990 \\ 999999990 \end{cases} \quad \text{Then L. } \frac{10+n}{10} = \begin{cases} 8-1=7; \\ 9-1=8; \\ 10-1=9; \end{cases}$$

$$\text{And } S = 2,87980075 + \begin{cases} 7 \\ 8 \\ 9 \end{cases} \times 2,3025, \quad \&c. \begin{cases} = 18,99789638; \\ = 21,30048147; \\ = 23,60306656. \end{cases}$$

COROL

COROL. II.

Hence the sum of the whole series $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$, &c. is infinite; for as often as a place of figures is added to the number of terms n , so often is 2,3025, &c. added to S ; but when n is infinite, the number of its places of figures are infinite; and 2,3025, &c. must be added an infinite number of times.

EXAMPLE II.

The sum of 2000000004 terms of the series $\frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \frac{1}{5} - \frac{1}{6}$, &c. is required?

That is, putting $\frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} = x$,

1000000000 terms of $\frac{1}{3} + \frac{1}{13} + \frac{1}{17} + \frac{1}{21} = S$,

And 1000000000 terms of $\frac{1}{11} + \frac{1}{13} + \frac{1}{19} + \frac{1}{23} = S'$;

To find $x + S - S'$

Now $\frac{1}{2} = 1$

$\frac{1}{3} = 0,33333333$

$\frac{1}{3} = 0,2$

$\frac{1}{7} = 0,14285714$

Affirm. = 1,2

0,47619048

Neg. = 0,47619048

$x = 0,72380952$

To find $\mathfrak{D}$; $e=9$; $\pi=100000000$; $d=4$;

$$\mathfrak{L} = \frac{9}{9+4 \times 100000000} ; L \frac{1}{\mathfrak{L}} = L \frac{400000009}{9} ;$$

$$\text{And } \mathfrak{D} = \frac{L \cdot 400000000 - L \cdot 9}{4} + \frac{1}{2 \cdot 9} + \frac{4}{2 \cdot a \cdot 9^2}, \text{ \&c.}$$

To find $\mathfrak{D}'$; $e=11$; $\pi=100000000$; $d=4$;

$$\mathfrak{L} = \frac{11}{11+4 \times 100000000} ; L \frac{1}{\mathfrak{L}} = L \frac{4000000011}{11} ;$$

$$\text{And } \mathfrak{D}' = \frac{L \cdot 400000000 - L \cdot 11}{4} + \frac{1}{2 \cdot 11} + \frac{4}{2 \cdot a \cdot 11^2}, \text{ \&c.}$$

In both which $\mathcal{Q}$ is considered as inconsiderable, in any term after the first,

$$\text{And } L. 4000000000 = \begin{cases} L. 4000000000 \\ L. 4000000011 \end{cases}$$

Which is true in logarithms of no more than seven places,

$$Th. \mathcal{Q} - \mathcal{Q}' = \frac{L. 11 - L. 9}{4} + \frac{1}{2.9} + \frac{4}{2.9^2} - \frac{4^3}{4 \cdot 9^4} + \frac{4^5}{6 \cdot 9^6}, \text{ \&c.}$$

$$- \frac{1}{2.11} - \frac{4}{24.11} + \frac{4^3}{4 \cdot 11^4} + \frac{4^5}{6 \cdot 11^6}, \text{ \&c.}$$

$$L. 11 = 1,0413927$$

$$L. 9 = 0,9542425$$

$$4) 0,0871502$$

$$0,0217876 \times 23025, \text{ \&c.} = 0,0501678;$$

$$\begin{array}{rcl}
 \frac{L.11-L.9}{4} & = & 0,0501678; \\
 \frac{1}{2.9} & = & 0,0555556; \\
 \frac{4}{2.6.9^2} & = & 0,0041152; \\
 \frac{4^3}{4.30.11^4} & = & 0,0000364; \\
 \frac{4^5}{6.42.9^6} & = & 0,0000079; \\
 \text{Affirmative} & = & 0,1098829 \\
 \text{Negative} & = & 0,0482944 \\
 \text{---} & = & 0,0615885
 \end{array}$$

$$\begin{array}{rcl}
 \frac{1}{2.11} & = & 0,0454545; \\
 \frac{4}{2.6.11^2} & = & 0,0027548; \\
 \frac{4^3}{4.30.9^4} & = & 0,0000813; \\
 \frac{4^5}{6.42.11^6} & = & 0,0000023; \\
 \frac{4^7}{8.30.98} & = & 0,0000015; \\
 & & 0,0482944
 \end{array}$$

$$+ (x =) 0,72380952 = 0,7853980 \text{ the answer.}$$

COROL. III.

Hence the sum of an infinite series of the above form, may be found: *M. Leibnitz* has prov'd, that the area of a circle whose diameter is 1, is equal to $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9}$. &c. ad infinitum; which is a sufficient verification hereof; for the area of such a circle is 0,7853982, &c.

QUESTION CXCVIII.

The sum (s) of n terms of the series, $\frac{1}{aa} + \frac{1}{a-d^2} + \frac{1}{a-2d^2} + \frac{1}{a-3d^2}, \&c.$ is required.

Now $\frac{1}{aa} = \frac{1}{aa};$

$$\frac{1}{a-d^2} = \frac{1}{aa} + \frac{2 \cdot 1d}{a^3} + \frac{3 \cdot 1d^2}{a^4} + \frac{4 \cdot 1d^3}{a^5}, \&c.$$

$$\frac{1}{a-2d^2} = \frac{1}{aa} + \frac{2 \cdot 2d}{a^3} + \frac{3 \cdot 4d^2}{a^4} + \frac{4 \cdot 8d^3}{a^5}, \&c.$$

$$\frac{1}{a-3d^2} = \frac{1}{aa} + \frac{2 \cdot 3d}{a^3} + \frac{3 \cdot 9d^2}{a^4} + \frac{4 \cdot 27d^3}{a^5}, \&c.$$

Th. $s = n$ terms of

$$\left\{ \begin{array}{l} 1+1+1+1, \&c. \times \frac{1}{aa} + \\ 0+1+2+3, \&c. \times \frac{1}{aa} \times \frac{2d}{a} + \\ 0+1+4+9, \&c. \times \frac{1}{aa} \times \frac{3d^2}{a^2} + \\ 0+1+8+27, \&c. \times \frac{1}{aa} \times \frac{4d^3}{a^3}, \&c. \end{array} \right.$$

Th. (putting $\frac{d}{a} = q$)

QUESTION CXCVIII. [Continued.]

$$\frac{1}{1-nq} = \left\{ \begin{array}{l} n + \\ \frac{2n^2q}{2} - \frac{2nq}{2} + \\ \frac{3n^3q^2}{3} - \frac{3n^2q^2}{2} + \frac{3nq^2}{6} + \\ \frac{4n^4q^3}{4} - \frac{4n^3q^3}{2} + \frac{4n^2q^3}{4} + \\ \frac{5n^5q^4}{5} - \frac{5n^4q^4}{2} + \frac{5n^3q^4}{3} - \frac{5nq^4}{30}, \&c. \end{array} \right.$$

$$\text{But } \left\{ \begin{array}{l} 1 + nq + n^2q^2, \&c. = \frac{1}{1-nq}; \\ 2nq + 3n^2q^2 + 4n^3q^3, \&c. = \frac{1}{1-nq^2} - 1; \\ 3nq + 6n^2q^2 + 10n^3q^3, \&c. = \frac{1}{1-nq^3} - 1; \\ 5nq + 15n^2q^2 + 35n^3q^3, \&c. = \frac{1}{1-nq^5} - 1; \end{array} \right.$$

QUESTION CXCVIII. [Continued.]

$$\begin{aligned}
 & \left. \begin{aligned}
 & n + \frac{2n^2q}{2} + \frac{3n^3q^2}{3}, \&c. = \frac{n}{1-nq}; \\
 & \frac{2nq}{2} + \frac{3n^2q^2}{2} + \frac{4n^3q^3}{2}, \&c. = \frac{1}{1-nq} - 1 \times \frac{1}{2}; \\
 & \frac{3nq^2}{6} + \frac{4n^2q^3}{4} + \frac{5n^3q^4}{3}, \&c. = \frac{1}{1-nq} - 1 \times \frac{q}{6}; \\
 & \frac{5nq^4}{30} + \frac{6n^2q^5}{12} + \frac{7n^3q^6}{6}, \&c. = \frac{1}{1-nq} - 1 \times \frac{q^3}{30};
 \end{aligned} \right\} \text{Th.} \\
 & \text{and } t = \frac{1}{2} \times \left\{ \begin{aligned}
 & \frac{n}{1-nq} - \frac{1}{1-nq} - 1 \times \frac{1}{2} + \frac{1}{1-nq} - 1 \times \frac{q}{6} \\
 & - \frac{1}{1-nq} - 1 \times \frac{q^3}{30} + \frac{1}{1-nq} - 1 \times \frac{q^3}{42};
 \end{aligned} \right.
 \end{aligned}$$

Or by writing $\frac{d}{a}$ for q ; and $\alpha, \beta, \gamma, \&c.$ $\mathcal{Q}$ as in the last;

$$t = \frac{\mathcal{Q}-1}{ad} - \frac{\mathcal{Q}^2-1}{2a\alpha} + \frac{\mathcal{Q}^3-1 \times d}{a\alpha^3} - \frac{\mathcal{Q}^5-1 \times d^3}{\beta a^5} \&c.$$

QUES.

QUESTION CXCI.

The sum (v) of n terms of the series, $\frac{1}{a^3} + \frac{1}{a-d^3} + \frac{1}{a-2d^3} + \frac{1}{a-3d^3} + \frac{1}{a-4d^3} + \dots$ is required?

Now $\frac{1}{a^3} = \frac{1}{a^3}$;

$$\frac{1}{a-d^3} = \frac{1}{a^3} + \frac{3 \cdot 1d}{a^4} + \frac{6 \cdot 1d^2}{a^5} + \frac{10 \cdot 1d^3}{a^6} + \dots$$

$$\frac{1}{a-2d^3} = \frac{1}{a^3} + \frac{3 \cdot 2d}{a^4} + \frac{6 \cdot 4d^2}{a^5} + \frac{10 \cdot 8d^3}{a^6} + \dots$$

$$\frac{1}{a-3d^3} = \frac{1}{a^3} + \frac{3 \cdot 3d}{a^4} + \frac{6 \cdot 9d^2}{a^5} + \frac{10 \cdot 27d^3}{a^6} + \dots$$

Th. (putting $\frac{d}{a} = q$)

$$\begin{aligned} & \times \left\{ \begin{aligned} & n + \\ & \frac{3n^2q}{2} - \frac{3nq}{2} + \\ & \frac{6n^3q^2}{3} - \frac{6n^2q^2}{2} + \frac{6nq^2}{6} + \\ & \frac{10n^4q^3}{4} - \frac{10n^3q^3}{2} + \frac{10n^2q^3}{4} + \\ & \frac{15n^5q^4}{5} - \frac{15n^4q^4}{2} + \frac{15n^3q^4}{3} - \frac{15n^2q^4}{30} + \dots \end{aligned} \right. \end{aligned}$$

QUESTION CXCI. [Continued.]

But {

$$1 + \frac{1}{2}nq + 2n^2q^2 \&c. = \frac{2-nq}{2 \times 1-nq};$$

$$3nq + 6n^2q^2 + 10n^3q^3 \&c. = \frac{1}{1-nq} - 1;$$

$$4nq + 10n^2q^2 + 20n^3q^3 \&c. = \frac{1}{1-nq} - 1;$$

$$6nq + 21n^2q^2 + 56n^3q^3 \&c. = \frac{1}{1-nq} - 1;$$

Th. {

$$n + \frac{1}{2}n^2q + \frac{6}{3}n^3q^2 = \frac{n \times 2 - nq}{2 \times 1 - nq};$$

$$\frac{3nq}{2} + \frac{6n^2q^2}{2} + \frac{10n^3q^3}{2} = \frac{1}{1-nq} - 1 \times \frac{1}{2};$$

$$\frac{6nq^2}{6} + \frac{10n^2q^3}{4} + \frac{15n^3q^4}{3} = \frac{1}{1-nq} - 1 \times \frac{3q}{2 \cdot a};$$

$$\frac{15nq^4}{30} + \frac{21n^2q^5}{12} + \frac{28n^3q^6}{6} = \frac{1}{1-nq} - 1 \times \frac{5q^3}{2 \cdot \beta};$$

Th

QUESTION CXCI. [Continued.]

$$Th \cdot v = \frac{1}{a^3} \times \left\{ \begin{array}{l} \frac{n \times 2 - nq}{2 \times 1 - nq} - \frac{1}{1 - nq} - 1 \times \frac{1}{2} + \frac{1}{1 - nq} - 1 \times \frac{3q}{2a} \\ - \frac{1}{1 - nq} - 1 \times \frac{5q^3}{2\beta} + \frac{1}{1 - nq} - 1 \times \frac{7q^5}{2\gamma}, \text{ \&c.} \end{array} \right.$$

Or using the same symbols as in the preceding questions.

$$v = \frac{2^2 - 1}{2a^2d} - \frac{2^3 - 1}{2a^3} + \frac{2^4 - 1 \times 3d}{2a^4} - \frac{2^6 - 1 \times 5d^3}{2\beta a^6}, \text{ \&c.}$$

COROL. I.

If s = sum of the reciprocals, of the first power ; t , of the second powers ; v of the third powers, &c. and z of m^{th} powers, of an arithmetical progression : Then,

$$\begin{aligned}
 s &= \frac{L \cdot Q}{d} - \frac{Q-1}{2a} + \frac{Q^2-1 \times d}{2aa^2} - \frac{Q^4-1 \times d^3}{4ba^4} + \frac{Q^6-1 \times d^5}{6ya^6}, \text{ \&c.} ; \\
 t &= \frac{Q-1}{ad} - \frac{Q^2-1}{2aa} + \frac{Q^3-1 \times d}{aa^3} - \frac{Q^5-1 \times d^3}{6a^5} + \frac{Q^7-1 \times d^5}{7a^7}, \text{ \&c.} ; \\
 v &= \frac{Q^2-1}{2a^2d} - \frac{Q^3-1}{2a^3} + \frac{Q^4-1 \times 3d}{2aa^4} - \frac{Q^6-1 \times 5d^3}{2ba^6} + \frac{Q^8-1 \times 7d^5}{2ya^8}, \text{ \&c.} ; \\
 w &= \frac{Q^3-1}{3a^3d} - \frac{Q^4-1}{2a^4} + \frac{Q^5-1 \times 2d}{aa^5} - \frac{Q^7-1 \times 5d^3}{\beta a^7} + \frac{Q^9-1 \times 28d^5}{3ya^9}, \text{ \&c.} ; \\
 \text{\&c.} \quad \text{\&c.}
 \end{aligned}$$

$$\begin{aligned}
 x &= \left\{ \begin{aligned} &\frac{Q^{m-1}-1}{m-1 \times a^{m-1}d} - \frac{Q^m-1}{2a^m} + \frac{Q^{m+1}-1 \times md}{2aa^{m+1}} - \frac{Q^{m+3}-1 \times m \cdot m+1 \cdot m+2 \cdot d^3}{2 \cdot 3 \cdot 4 \cdot \beta a^{m+3}} \\ &+ \frac{Q^{m+5}-1 \times m \cdot m+1 \cdot m+2 \cdot m+3 \cdot m+4 \cdot d^5}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot \gamma a^{m+5}}, \text{ \&c.} \end{aligned} \right.
 \end{aligned}$$

COROL

COROL. II.

If Z represent the sum of a terms of the series $\frac{1}{e^m} + \frac{1}{e+d^m} + \frac{1}{e+2d^m} + \frac{1}{e+3d^m} + \dots$

(where $m \square 1$) put $\frac{e}{e+nd^m} = Q$; Then

$$Z = \left\{ \frac{1-Q^{m-1}}{m-1 \times e^{m-1}d} + \frac{1-Q^m}{2e^m} + \frac{1-Q^{m+1}}{2ae^{m+1} \times nd} - \frac{1-Q^{m+3}}{2.3.4.\beta.e^{m+3} \times m.m+1.m+2.d^3} + \dots \right.$$

$$\left. \frac{1-Q^{m+5}}{2.3.4.5.6.\gamma.e^{m+5} \times m.m+1.m+2.m+3.m+4.d^5} - \dots \right.$$

Sec. See COROL. I. QUEST. 197.

COROL. III.

But when n is a very great number, $\frac{1}{n} = \mathcal{Q}$ will be very small, and may be rejected; and then

$$Z = \frac{1}{m-1 \times m^{m-1} d} + \frac{1}{2 \times m^m} + \frac{m d}{2 \times m+1} - \frac{m \cdot m+1 \cdot m+2 \cdot d^3}{2 \cdot 3 \cdot 4 \cdot m+3}, \text{ \&c.}$$

EXAMPLE

EXAMPLE I.

What is the sum of the infinite series $\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16}$, &c.?

Let the sum of the first 9 terms be found } = 1,53976773
by so many divisions

And the sum of the remaining terms } = 0,10516634
beginning with the tenth, by Cor. 3.

Then $\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16}$, &c. ad infin. = 1,64463407

SCHOLIUM I.

M. *John Bernoulli* has proved that the above series is $\frac{1}{2}$ of the square of the circumference of a circle whose diameter is 1: See his WORKS, Vol. IV. Page 20, 21, 22, &c.

Now the circumference of a circle, whose diameter is 1 = 3,14159265; &c.

And $\frac{3,14159265, \&c.}{6} = 1,64493406, \&c.$

SCHOLIUM II.

By proceeding in like manner the sums of

$\frac{1}{1} + \frac{1}{8} + \frac{1}{27} + \frac{1}{64}$, &c. ad infin. = 1,20205691

$\frac{1}{1} + \frac{1}{27} + \frac{1}{125} + \frac{1}{343}$, &c. ad infin. = 1,05179980

$\frac{1}{8} + \frac{1}{64} + \frac{1}{216} + \frac{1}{312}$, &c. ad infin. = 0,15025711

And by quest. 184,

$7 : 1 :: \frac{1}{1} + \frac{1}{27} + \frac{1}{125}, \&c. : \frac{1}{8} + \frac{1}{64} + \frac{1}{216}, \&c.$

But $1,05179980 = 0,15025711 \times 7$

Which is a verification of the process used in both classes of questions.

The two following questions were designed for the first part, but were omitted for want of room.

QUESTION CC.

'Tis required to find the values of x , y , and z , in the following equations, viz.

$$\left. \begin{aligned} xx + xy &= 10 \\ yy + yz &= 21 \\ zz + zx &= 24 \end{aligned} \right\} ?$$

Substitute $y = nx$; And $z = mx$:

$$\text{Then } \left\{ \begin{aligned} xx + nxx &= 10, \\ n^2x^2 + nm x^2 &= 21, \\ m^2x^2 + mx^2 &= 24, \end{aligned} \right\} \div \left\{ \begin{aligned} xx &= \frac{10}{1+n}, \\ xx &= \frac{21}{n^2+nm}, \\ xx &= \frac{24}{m^2+m}. \end{aligned} \right.$$

$$\text{Th. } \frac{24}{m^2+m} = \frac{10}{1+n}; \text{ Th. } n = \frac{5m \cdot m + 1 - 12}{12}:$$

$$\text{Also } \frac{24}{m^2+m} = \frac{21}{n^2+nm}; \text{ Th. } 8n^3 + 8nm = 7m \cdot m + 1:$$

$$\text{Now } n^3 = \frac{25m^3 \cdot m + 2^2 - 120m \cdot m + 1 + 144}{144}$$

$$\text{And } nm = \frac{5m^2 \cdot m + 1 - 12m}{12};$$

Th

QUESTION CC. [Continued]

QUES.

$$\text{Th. } 7m \cdot m + 1 = \frac{25m^2 \cdot m + 1}{18} - \frac{120m \cdot m + 1 + 144}{3} + \frac{10m^2 \cdot m + 1 - 24m}{3},$$

$$\text{Or } 126m \cdot m + 1 = 25m^2 \cdot m + 1 - 120m \cdot m + 1 + 144 + 60m^2 \cdot m + 1 - 144m,$$

$$\text{Or } 246m \cdot m + 1 = 25m^2 \cdot m + 1 + 144 + 60m^2 \cdot m + 1 - 144m,$$

$$\text{Or } 246m^2 + 246m = 25m^2 + 50m^3 + 25m^2 + 144 + 60m^3 + 60m^2 - 144m,$$

$$\text{Th. } 0 = 25m^4 + 110m^3 - 161m^2 - 390m + 144;$$

$$\text{But } \frac{25m^4 + 110m^3 - 161m^2 - 390m + 144}{m-2} = 25m^3 + 160m^2 + 159m - 72,$$

$$\text{Th. } m = 2; x = \left(\frac{10 \cdot 3 - 12}{12} \right)^{\frac{3}{2}}; \text{ And } x = 2.$$

QUESTION CCI.

How many ways may 200 be divided into five whole numbers, x, y, z, e , and x , so that $12x + 3y + z + \frac{e}{2} + \frac{x}{3} = 200$?

Now first $y + z + e + x = 200 - x$
And $3y + z + \frac{e}{2} + \frac{x}{3} = 200 - 12x$ } by transposition

Also $\begin{cases} 18y + 6z + 3e + 2x = 1200 - 72x \\ 2y + 2z + 2e + 2x = 400 - 2x \\ 18y + 18z + 18e + 18x = 3600 - 18x \end{cases}$ } by multiplicatⁿ.

But $\begin{cases} 16y + 4z + e = 800 - 70x \\ 12z + 15e + 16x = 54x + 2400 \end{cases}$ } by subtr.

Therefore $\begin{cases} x = \left(\frac{800}{70} \right) = 11 \frac{2}{7} \\ x = 0: \end{cases}$

Secondly $\begin{cases} 6z + 3e + 2x = 1200 - 72x - 18y \\ 2z + 2e + 2x = 400 - 2x - 2y \end{cases}$ } by transp.

And $6z + 6e + 6x = 1200 - 6z - 6y$ by multip.

But $\begin{cases} 4z + e = 800 - 70x - 16y \\ 3e + 4z = 12y + 66x \end{cases}$ } by substr.

Th. $\begin{cases} y = \left(\frac{800 - 70x}{16} \right) = 50 - \frac{35}{8}x \\ y = 0: \end{cases}$

From

QUESTION CCI. [Continued]

From which limits it will appear; that when

$$\begin{aligned} x &= 1; y = 1 \frac{1}{2} \\ x &= 10; y = 6 \frac{2}{3} \\ x &= 9; y = 10 \frac{2}{3} \\ x &= 8; y = 15 \\ x &= 7; y = 19 \frac{1}{3} \end{aligned}$$

Th. y will have

$$\left\{ \begin{array}{l} 1 \\ 6 \\ 10 \\ 14 \\ 19 \end{array} \right\}$$

values; and when

$$\begin{aligned} x &= 6; y = 23 \frac{3}{8} \\ x &= 5; y = 28 \frac{1}{2} \\ x &= 4; y = 32 \frac{1}{4} \\ x &= 3; y = 36 \frac{1}{2} \\ x &= 2; y = 41 \frac{1}{4} \\ x &= 1; y = 45 \frac{1}{8} \end{aligned}$$

Th. y will have

$$\left\{ \begin{array}{l} 23 \\ 28 \\ 32 \\ 36 \\ 41 \\ 45 \end{array} \right\}$$

values.

Thirdly

$$\left\{ \begin{array}{l} 3x + 2x = 1200 - 72x - 18y - 6x \\ 2x + 2x = 400 - 2x - 2y - 2x \end{array} \right\} \text{ by transp.}$$

And

$$3x + 3x = 600 - 3x - 3y - 3x, \text{ by multip. by } \frac{2}{3}:$$

But by subtr.

$$\left\{ \begin{array}{l} x = 800 - 70x - 16y - 4x \\ x = 3x + 15y + 69x - 600; \end{array} \right.$$

$$\begin{aligned} \text{Th. } \left\{ \begin{array}{l} x = \left(\frac{800 - 70x - 16y}{4} \right) = 200 - \frac{35x}{2} - 4y \\ x = \left(\frac{600 - 69x - 15y}{3} \right) = 200 - 23x - 5y; \end{array} \right. \end{aligned}$$

To

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QUESTION CCI. [Continued]

To apply which, let, First $x = 11$; Then
 $y = 1$; $u = 3\frac{x}{2}$; $u = 0$; and u has 3 values.

Secondly, let $x = 10$; Then

$$\left. \begin{array}{l} y = 6; u = 1; \\ y = 5; u = 5; \\ y = 4; u = 9; \\ \&c. \quad \&c. \end{array} \right\} u = 0; \text{ and } u \text{ has } \left\{ \begin{array}{l} 0 \\ 4 \\ 8 \\ \&c. \end{array} \right\} \text{ values;}$$

Put $V =$ the number of values of u

Then $V = (6 \text{ terms of } 0 + 4 + 8, \&c. =) 60$. See qu. 3d.

Thirdly, let $x = 9$; Then

$$\left. \begin{array}{l} y = 10; u = 2\frac{1}{2}; \\ y = 9; u = 6\frac{1}{2}; \\ y = 8; u = 10\frac{1}{2}; \\ \&c. \quad \&c. \end{array} \right\} u = 0; \text{ and } u \text{ has } \left\{ \begin{array}{l} 2 \\ 6 \\ 10 \\ \&c. \end{array} \right\} \text{ values;}$$

Th. $V = (10 \text{ terms of } 2 + 6 + 10, \&c. =) 200$.

Fourthly, let $x = 8$; Then

$$\begin{array}{l} y = 14; u = 4; u = 54; \text{ and } u \text{ has } 3 \text{ values,} \\ y = 13; u = 8; u = 49; \text{ and } u \text{ has } 7 \text{ values,} \\ \&c. \quad \&c. \quad \&c. \quad \&c. \\ y = 4; u = 44; u = 4; \text{ and } u \text{ has } 43 \text{ values,} \\ y = 3; u = 48; u = 1; \text{ and } u \text{ has } 46 \text{ values,} \\ y = 2; u = 52; u = 6; \text{ and } u \text{ has } 45 \text{ values,} \\ y = 1; u = 56; u = 11; \text{ and } u \text{ has } 44 \text{ values,} \end{array}$$

Th. $V = \left\{ \begin{array}{l} 11 \text{ terms of } 3 + 7 + 11, \&c. \\ + 3 \text{ terms of } 46 + 45 + 44 \end{array} \right\} = 388$.

Fifthly,

QUESTION CCI. [Continued.]

Fifthly, let $x = 7$; Then

$$y = 19; u \sqsupset 1\frac{1}{2}; u \sqsubset - 56; \text{ and } u \text{ has } 1 \text{ value,} \\ \&c. \quad \&c. \quad \&c. \quad \&c.$$

$$y = 8; u \sqsupset 45\frac{1}{2}; u \sqsubset - 1; \text{ and } u \text{ has } 45 \text{ values,}$$

$$y = 7; u \sqsupset 49\frac{1}{2}; u \sqsubset - 4; \text{ and } u \text{ has } 45 \text{ values,}$$

$$y = 6; u \sqsupset 53\frac{1}{2}; u \sqsubset - 9; \text{ and } u \text{ has } 44 \text{ values,} \\ \&c. \quad \&c. \quad \&c. \quad \&c.$$

$$\text{Th. V.} = \left\{ \begin{array}{l} 12 \text{ terms of } 1 + 5 + 9, \&c. \\ + 7 \text{ terms of } 45 + 44 + 43, \&c. \end{array} \right\} = 750.$$

Sixthly, let $x = 6$; Then

$$y = 23; u \sqsupset 3; u \sqsubset - 53; \text{ and } u \text{ has } 2 \text{ values,} \\ \&c. \quad \&c. \quad \&c. \quad \&c.$$

$$y = 13; u \sqsupset 43; u \sqsubset - 3; \text{ and } u \text{ has } 42 \text{ values,}$$

$$y = 12; u \sqsupset 47; u \sqsubset - 2; \text{ and } u \text{ has } 44 \text{ values,} \\ \&c. \quad \&c. \quad \&c. \quad \&c.$$

$$\text{Th. V} = \left\{ \begin{array}{l} 11 \text{ terms of } 2 + 6 + 10, \&c. \\ + 12 \text{ terms of } 44 + 43 + 42, \&c. \end{array} \right\} = 704.$$

Seventhly, let $x = 5$; Then

$$y = 28; u \sqsupset \frac{1}{2}; u \sqsubset - 55; \text{ and } u \text{ has } 0 \text{ value,} \\ \&c. \quad \&c. \quad \&c. \quad \&c.$$

$$y = 17; u \sqsupset 44\frac{1}{2}; u \sqsubset - 0; \text{ and } u \text{ has } 44 \text{ values,}$$

$$y = 16; u \sqsupset 48\frac{1}{2}; u \sqsubset - 5; \text{ and } u \text{ has } 43 \text{ values,} \\ \&c. \quad \&c. \quad \&c. \quad \&c.$$

$$\text{Th. V} = \left\{ \begin{array}{l} 12 \text{ terms of } 0 + 4 + 8, \&c. \\ + 16 \text{ terms of } 43 + 42 + 41, \&c. \end{array} \right\} = 832.$$

Eightly,

QUESTION CCI. [Continued.]

Eighthly, let $x = 4$; Then

$$y = 32; \quad n \sqsupset 2; \quad n \sqsubset - 52; \quad \text{and } n \text{ has } 1 \text{ value,}$$

&c. &c. &c. &c.

$$y = 22; \quad n \sqsupset 42; \quad n \sqsubset - 2; \quad \text{and } n \text{ has } 41 \text{ values,}$$

$$y = 21; \quad n \sqsupset 46; \quad n \sqsubset 3; \quad \text{and } n \text{ has } 42 \text{ values,}$$

&c. &c. &c. &c.

$$\text{Th. V} = \left\{ \begin{array}{l} 11 \text{ terms of } 1 + 5 + 9, \text{ \&c.} \\ + 21 \text{ terms of } 42 + 41 + 40, \text{ \&c.} \end{array} \right\} = 903.$$

Ninthly, let $x = 3$; Then

$$y = 36; \quad n \sqsupset 3\frac{1}{2}; \quad n \sqsubset - 49; \quad \text{and } n \text{ has } 3 \text{ values,}$$

&c. &c. &c. &c.

$$y = 27; \quad n \sqsupset 39\frac{1}{2}; \quad n \sqsubset - 4; \quad \text{and } n \text{ has } 39 \text{ values,}$$

$$y = 26; \quad n \sqsupset 43\frac{1}{2}; \quad n \sqsubset 1; \quad \text{and } n \text{ has } 42 \text{ values,}$$

&c. &c. &c. &c.

$$\text{Th. V} = \left\{ \begin{array}{l} 10 \text{ terms of } 3 + 7 + 11, \text{ \&c.} \\ + 26 \text{ terms of } 42 + 41 + 40, \text{ \&c.} \end{array} \right\} = 977.$$

Tenthly, let $x = 2$; Then

$$y = 41; \quad n \sqsupset 1; \quad n \sqsubset - 51; \quad \text{and } n \text{ has } 0 \text{ value,}$$

&c. &c. &c. &c.

$$y = 31; \quad n \sqsupset 41; \quad n \sqsubset - 1; \quad \text{and } n \text{ has } 40 \text{ values,}$$

$$y = 30; \quad n \sqsupset 45; \quad n \sqsubset 4; \quad \text{and } n \text{ has } 40 \text{ values,}$$

&c. &c. &c. &c.

$$\text{Th. V} = \left\{ \begin{array}{l} 11 \text{ terms of } 0 + 4 + 8, \text{ \&c.} \\ + 30 \text{ terms of } 40 + 39 + 38, \text{ \&c.} \end{array} \right\} = 985.$$

Lastly,

QUESTION CCL. [Continued.]

Lastly, Let $x=1$; Then

$$y=45; u \sqsupset 2\frac{1}{2}; u \sqsubset -48; \text{ and } u \text{ has 2 values;}$$

&c. &c. &c.

$$y=36; u \sqsupset 38\frac{1}{2}; u \sqsubset -3; \text{ and } u \text{ has 38 values;}$$

$$y=35; u \sqsupset 42\frac{1}{2}; u \sqsubset -2; \text{ and } u \text{ has 40 values;}$$

$$\text{Th. V.} = \left\{ \begin{array}{l} 10 \text{ terms of } 2+6+10, \&c. \\ +35 \text{ terms of } 40+39+38, \&c. \end{array} \right\} = 1005;$$

Th. When $x=11$; The question has 3 answers;

$x=10$	;	-	-	-	-	60
$x=9$	;	-	-	-	-	200
$x=8$	;	-	-	-	-	388
$x=7$	;	-	-	-	-	570
$x=6$	;	-	-	-	-	704
$x=5$	;	-	-	-	-	832
$x=4$	;	-	-	-	-	903
$x=3$	;	-	-	-	-	977
$x=2$	;	-	-	-	-	985
$x=1$	;	-	-	-	-	1005

Th. the question admits of - - - 6627 answers.

F I N I S.



PAGE 68, line ult. for $138x$, read $138x^2$, p. 94,
 l. 13, for 15y, read $-15y$. l. 16, for 90y, read
 $-90y$. p. 95, for 117964, read 116964. p. 114,
 l. 4, for $x8x$, read $x8$. p. 125, l. 1, for $x=$, read
 $xx=$. p. 127, l. 12, for $+$, read $-$. p. 131, l. 17,
 read 1, 2, 3, 4, 6, 9. l. 19, for $=119$, read -119 .
 p. 132, l. 13, for $2x$, read $2xx$. p. 133, l. 16, for $+$,
 read $-$. p. 138, l. 8, for xy^3 , read $xy^{\frac{3}{2}}$. p. 142, l. ult.
 for Newron, read Newton. p. 146, l. 15, for $12y=$,
 read $12x=$. l. 17, for y , read xy . p. 149, l. 12, for
 $Th.y^{\frac{3}{2}}$, read $Th.y^{\frac{3}{2}}$. p. 152, l. 13, for 560, read 56.
 p. 160, l. 16, for $\frac{1-20}{45}x^3$, read $\frac{1-20x^3}{45}$. p. 176, l. 8,
 for A , read C . p. 177, l. 19, dele O . p. 178, l. 5, for
 $\frac{Am+Bn+Cp+Dq+Er&c.}{Ax \times B \times C \times D \times E \&c.}$, read $\frac{Am+Bn+Cp+Dq+Er&c.}{M \times N \times P \times Q \times R \&c.}$.
 l. 15, for F , read E . p. 180, l. 19 and 20, for multi-
 plier, read multiple. p. 217, l. 10, for sum, read sums.
 p. 222, l. 8, for $n-m-1$, read $n-m-1$. p. 223, l. 9,
 for $+ax$, read xxx . p. 232, l. 5, for $\frac{4}{2}-a\frac{1}{2}$, read
 $\frac{4}{2}a-\frac{1}{2}$. p. 240, l. 4, for 4, read 3. p. 245, l. 5, for 2,
 read 3. p. 257, l. 18, for $m+p$, read $m-p$. p. 264,
 l. 8, for t , read n . p. 285, l. 20, for 2185, read 2125.
 p. 333, l. 7, for mx , read $m \times$.

